Tearing-mediated reconnection in magnetohydrodynamic poorly ionized plasmas
I. Onset and linear evolution

Princeton Astroplasmas
December 2023

Based on work in preparation

Elizabeth A. Tolman,[1,2] Matthew W. Kunz,[3] James Stone,[1] Lev Arzamasskiy[1]

[1]School of Natural Sciences, Institute for Advanced Study, Princeton, NJ USA

[2]Center for Computational Astrophysics, Flatiron Institute, New York, NY USA

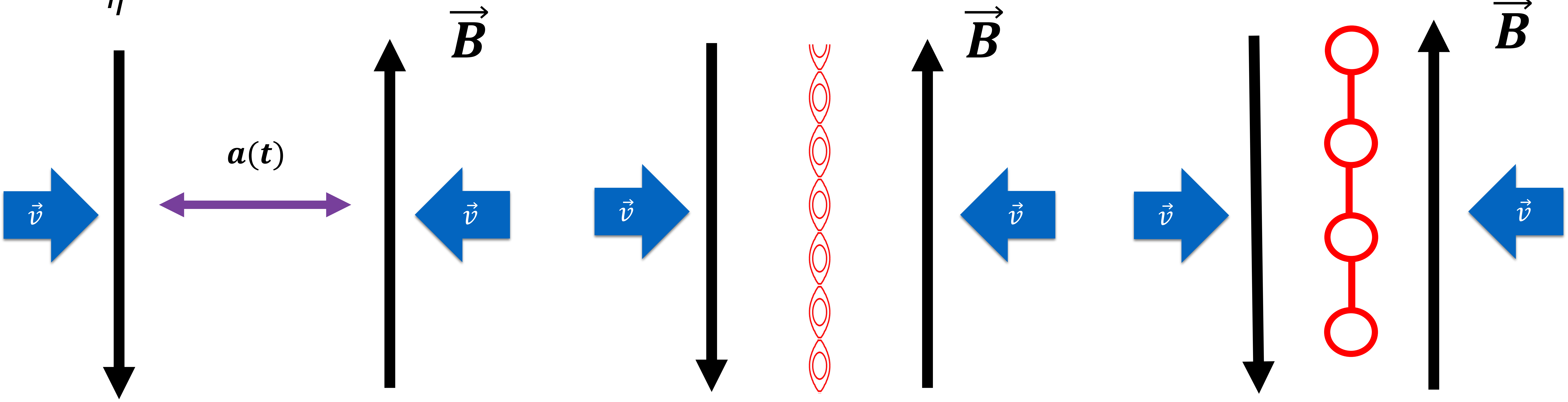
[3]Princeton University, Princeton, NJ USA

FLATIRON INSTITUTE

INSTITUTE FOR ADVANCED STUDY

Tearing initiates reconnection in high S_L plasmas

- Oppositely-directed B field lines can reconnect and release magnetic energy
- Reversing B field forms current sheet of width a, length L, Alfvén speed v_A, resistivity η
- If $\frac{L v_A}{\eta} \equiv S_L \gtrsim 10^4$ reconnection is initiated by onset of tearing instability as a shrinks



1. Stable current sheet

2. Linear tearing

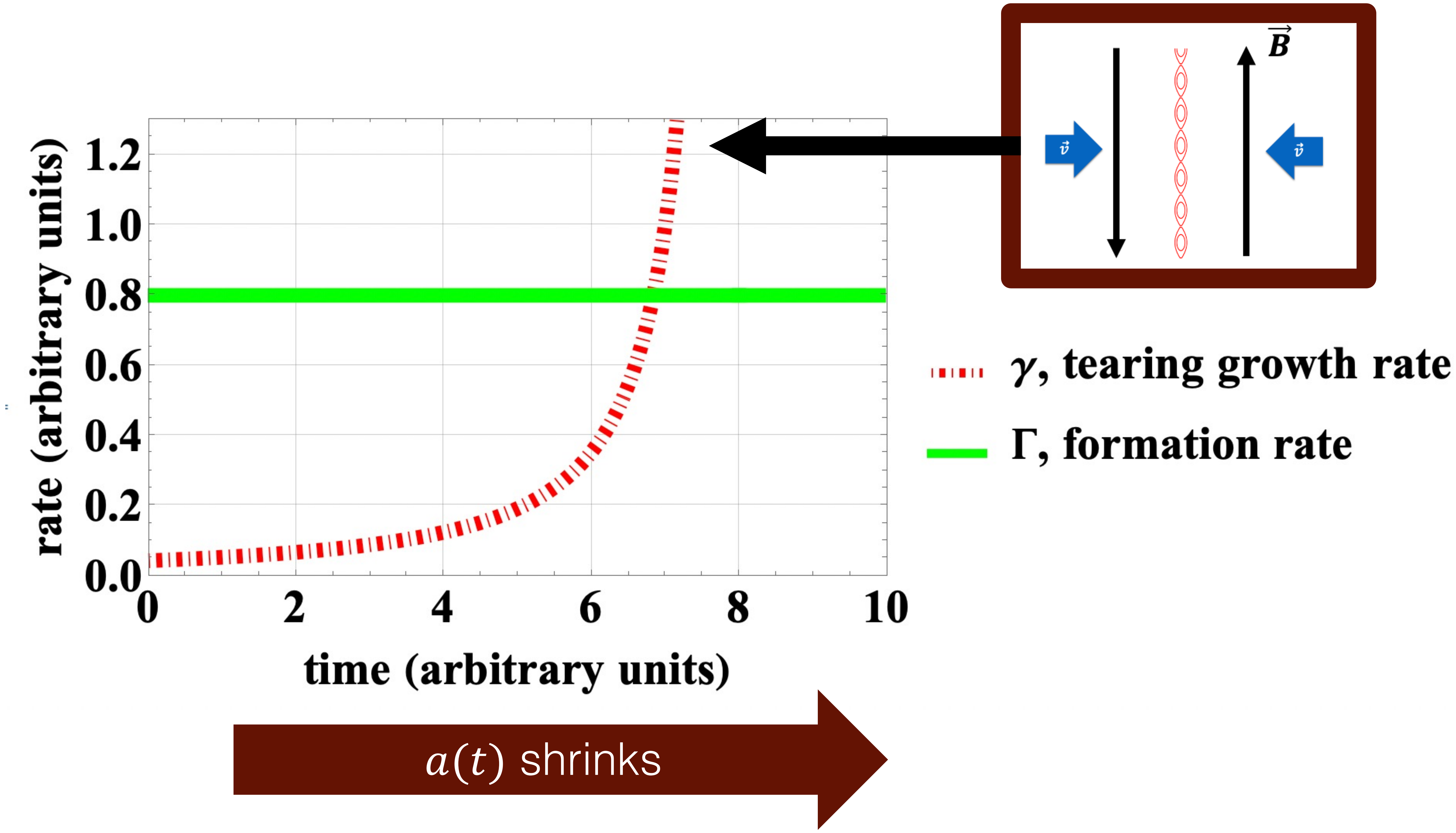
3. Nonlinear tearing (plasmoid chain)

Onset occurs when tearing rate dominates formation

- Recent works examine onset process (e.g. Uzdensky and Loureiro 2016, Tolman et al. 2018)
    - Important for turbulence studies, timescales of solar flares
- Use tearing growth rate (Furth et al. 1963, Coppi et al. 1976)

$$\boxed{\gamma \sim \frac{\eta^{1/2} v_{A,i0}^{1/2}}{a(t)^{3/2}}}$$

- Tearing onsets when linear growth rate γ becomes larger than formation rate Γ
- Formation rate Γ usually modeled as constant



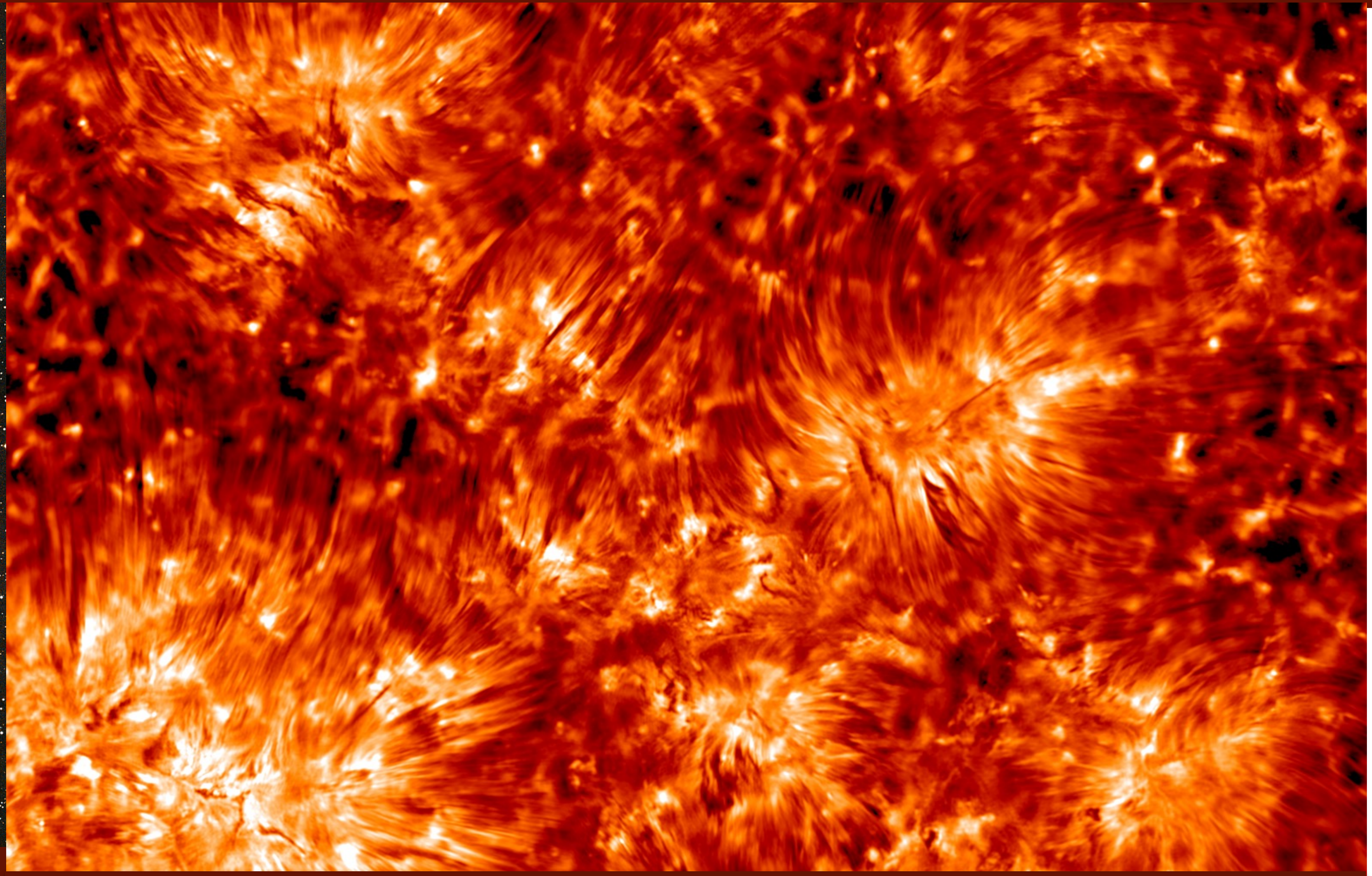
Onset condition

γ (increases as sheet narrows) $\gg$ Γ (often modeled as constant)

Many plasmas are poorly ionized

Molecular cloud: ionization fraction 10^{-4} to 10^{-9}

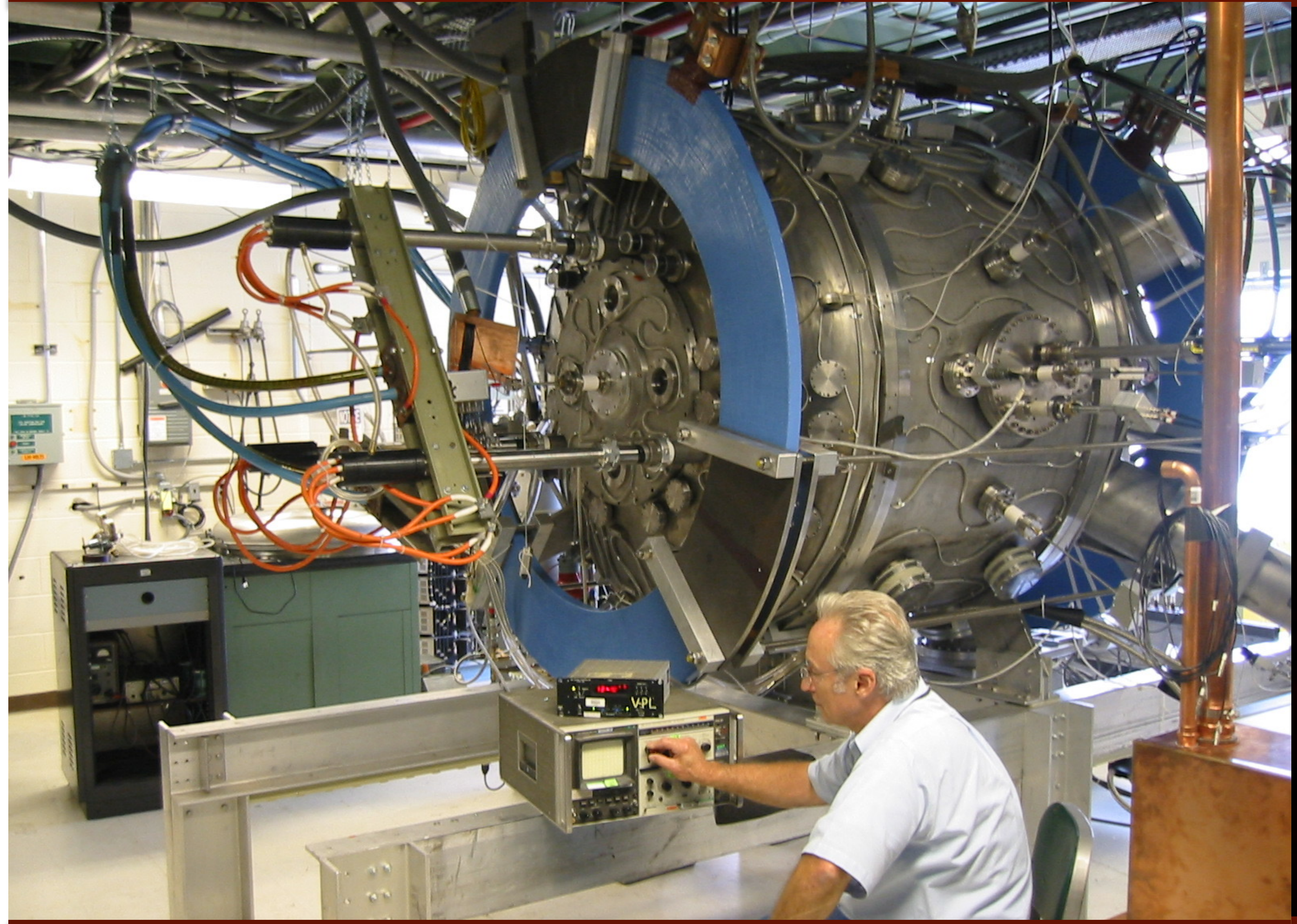
ESO/APEX (MPIfR/ESO/OSO)/A. Hacar et al./Digitized Sky Survey 2. Acknowledgment: Davide De Martin.



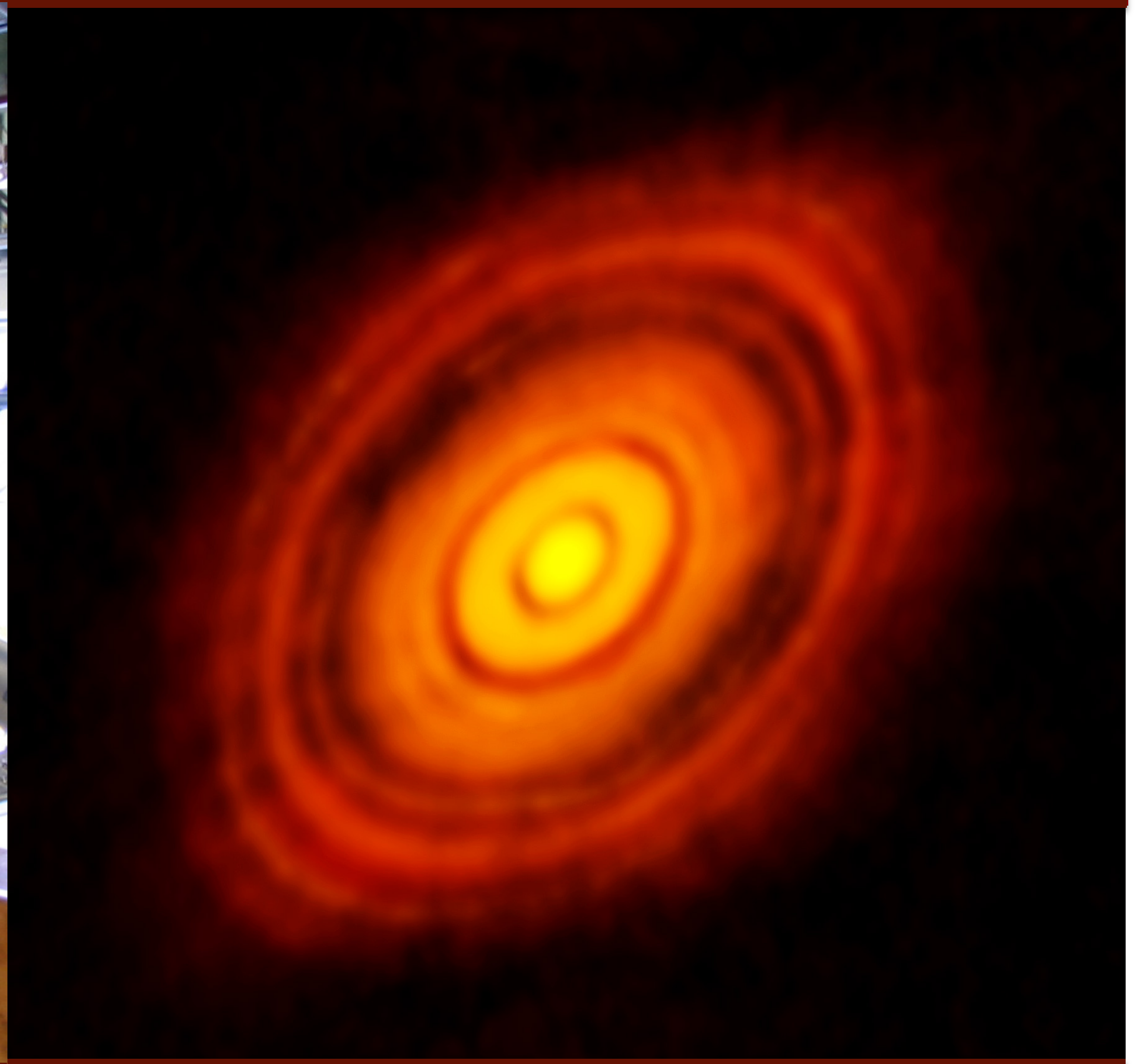
Vasco Henriques and Ainar Drews (ITA, University of Oslo)

Solar chromosphere: ionization fraction 1 to 10^{-4}

Some lab experiments: ionization fraction 10^{-1} to 10^{-2}



Mrx.pppl.gov



ALMA (ESO/NAOJ/NRAO)

Protoplanetary disk: ionization fraction $\sim 10^{-10}$

We study onset in poorly ionized plasmas

- Poorly ionized plasmas are different in multiple ways

- We study a magnetohydrodynamic poorly ionized plasma
  - Neutral fluid + ionized fluid
  - Coupled by ion-neutral collisions, ionization, recombination

- We use:
  - Simulation using AthenaK, astrophysical magnetohydrodynamics (MHD) code
  - Analytics

- We ask: When does tearing onset? What type of mode onsets?

Outline

- Tools for studying onset

- Analytic description of formation process, with consideration of onset

- Type of mode that disrupts sheet

- Simulations of all the above

Tools for studying onset

We use MHD and hydrodynamic equations

Neutral momentum equation

$$\rho_n \frac{Dv_n}{dt} + \boldsymbol{\nu_{ni}\rho_n(\vec{v}_n - \vec{v}_i)} = -\nabla \mathrm{P_n} + \boldsymbol{\alpha\rho_i^2(\vec{v}_i - \vec{v}_n)}$$

Ionized momentum equation

$$\rho_i \frac{Dv_i}{dt} + \boldsymbol{\nu_{ni}\rho_n(\vec{v}_i - \vec{v}_n)} = -\nabla \mathrm{P_i} + \left(\nabla\times\vec{B}\right)\times\vec{B} + \boldsymbol{\xi\rho_n(\vec{v}_n - \vec{v}_i)}$$

Induction equation

$$\frac{\partial \vec{B}}{\partial t} = \nabla\times\left(\vec{v}_i \times\vec{B}\right) + \eta\nabla^2\vec{B}$$

Isothermal EOS

$$P_n = \rho_n C_n^2,\ P_i = \rho_i C_i^2$$

Div. B

$$\nabla\cdot\vec{B} = 0$$

Ionized continuity

$$\frac{\partial\rho_i}{\partial t} + \nabla\cdot(\rho_i\vec{v}_i) = \boldsymbol{\xi\rho_n - \alpha\rho_i^2}$$

Neutral continuity

$$\frac{\partial\rho_n}{\partial t} + \nabla\cdot(\rho_n\vec{v}_n) = \boldsymbol{-\xi\rho_n + \alpha\rho_i^2}$$

Quantity	Definition
ρ_n, ρ_i	Neutral and ionized mass density
P_n, P_i	Neutral and ionized pressure
$\vec{v}_n, \vec{v}_i$	Neutral and ionized velocity
ν_{ni}	Neutral-ion collision frequency
α	Recombination coefficient
ξ	Ionization coefficient
$\vec{B}$	Magnetic field
C_i, C_n	Sound speed

Strong coupling approximation gives B field change

Induction equation

$$\frac{\partial \vec{B}}{\partial t} = \nabla\times\left(\vec{v}_i \times \vec{B}\right) + \eta\nabla^2\vec{B}$$

Ionized momentum equation

$$\rho_i \frac{Dv_i}{dt} + \boldsymbol{\nu_{ni}\rho_n(\vec{v}_i - \vec{v}_n)} = -\nabla \mathrm{P_i} + \boldsymbol{\left(\nabla\times\vec{B}\right)\times\vec{B}} + \xi\rho_n\left(\vec{v}_n - \vec{v}_i\right)$$

Strong coupling: ion inertia negligible, drag force balances magnetic force

$$\frac{\partial \vec{B}}{\partial t} = \boldsymbol{\nabla\times\left(\vec{v}_n \times \vec{B}\right)} + \boldsymbol{\nabla\times\left\{\frac{\left[\left(\nabla\times\vec{B}\right)\times\vec{B}\right]\times\vec{B}}{\nu_{ni}\,\rho_n}\right\}} + \eta\nabla^2\vec{B}$$

Magnetic field evolves due to neutral advection, ambipolar diffusion, resistive diffusion

Poorly ionized tearing growth rate somewhat different

- Tearing growth rate is modified in poorly ionized plasma

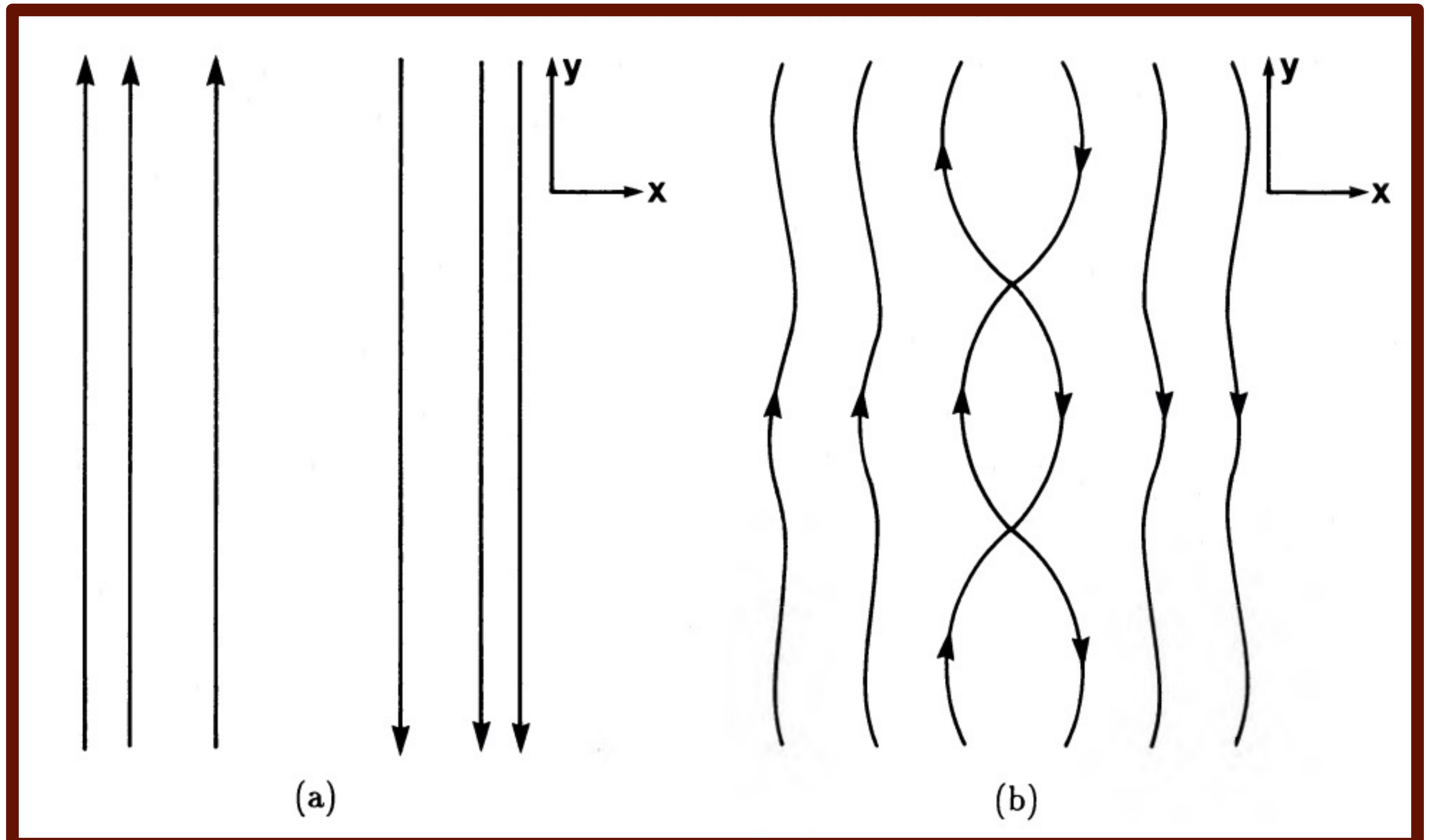
- First considered in Zweibel 1989

- In weakly ionized, very unstable plasmas, rate given approximately by

$$v_{A,i0} \rightarrow v_{A,n0}\left(\frac{\gamma}{\nu_{ni0}}\right)^{1/2}$$

$$\gamma \sim \frac{\eta^{1/2} v_{A,i0}^{1/2}}{a(t)^{3/2}} \rightarrow \gamma \sim \frac{\eta^{2/3} v_{A,n0}^{2/3}}{a(t)^2 \nu_{ni0}^{1/3}}$$

- Tearing "weighed down" by collisions with neutrals

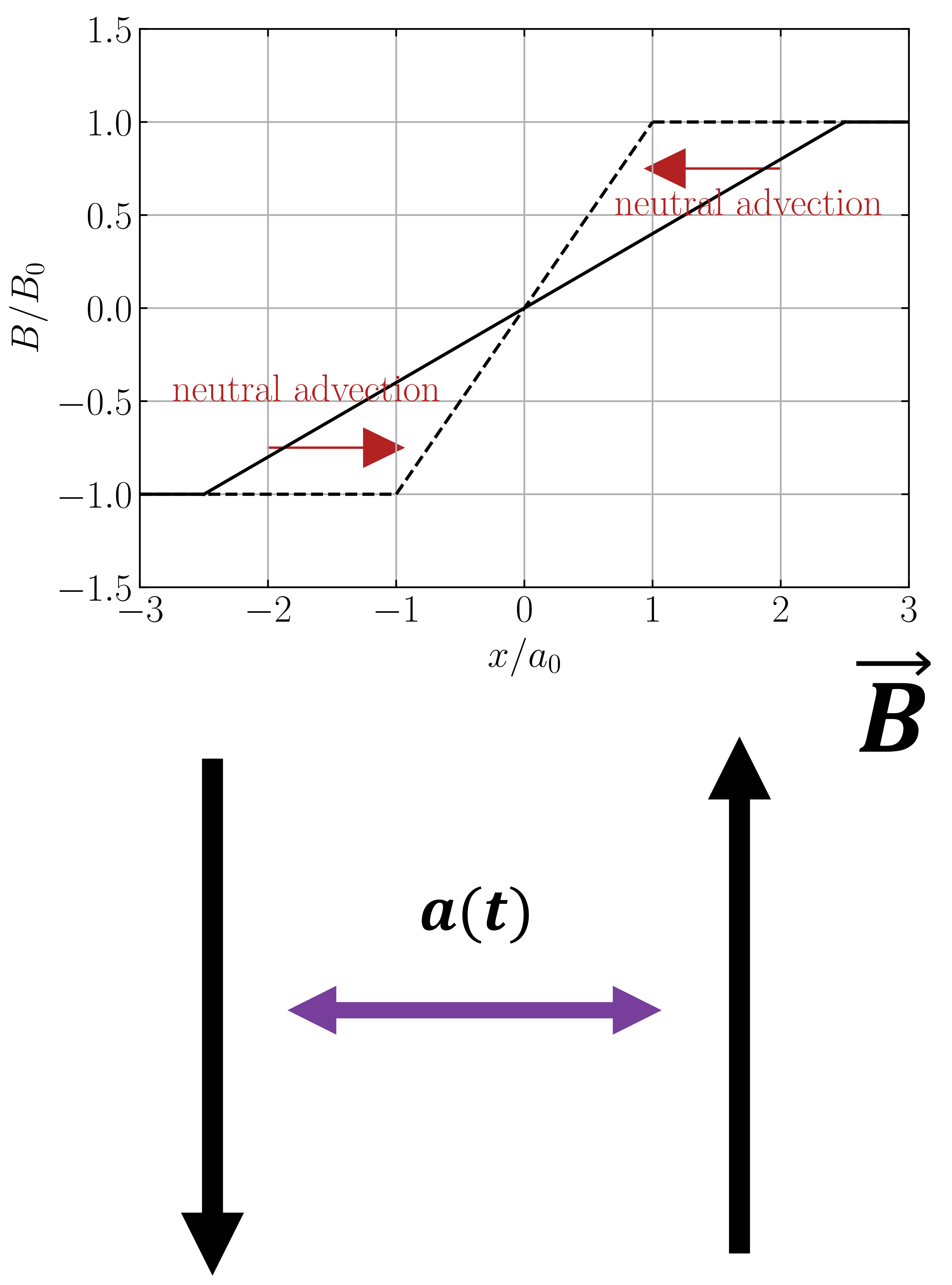


Tearing in a current sheet, from Zweibel ApJ 1989

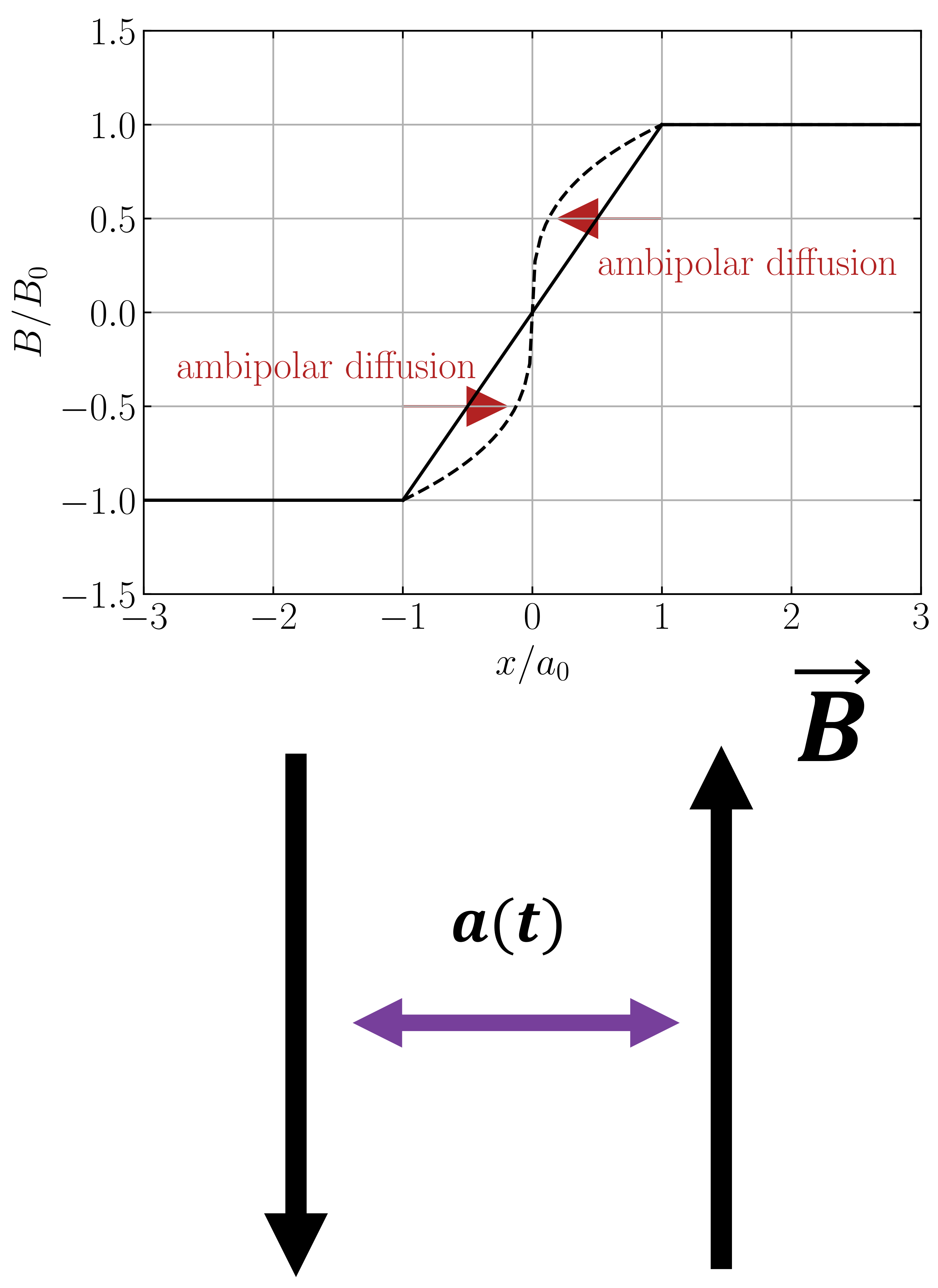
Analytic description of formation process

Current sheet formation has 3 stages

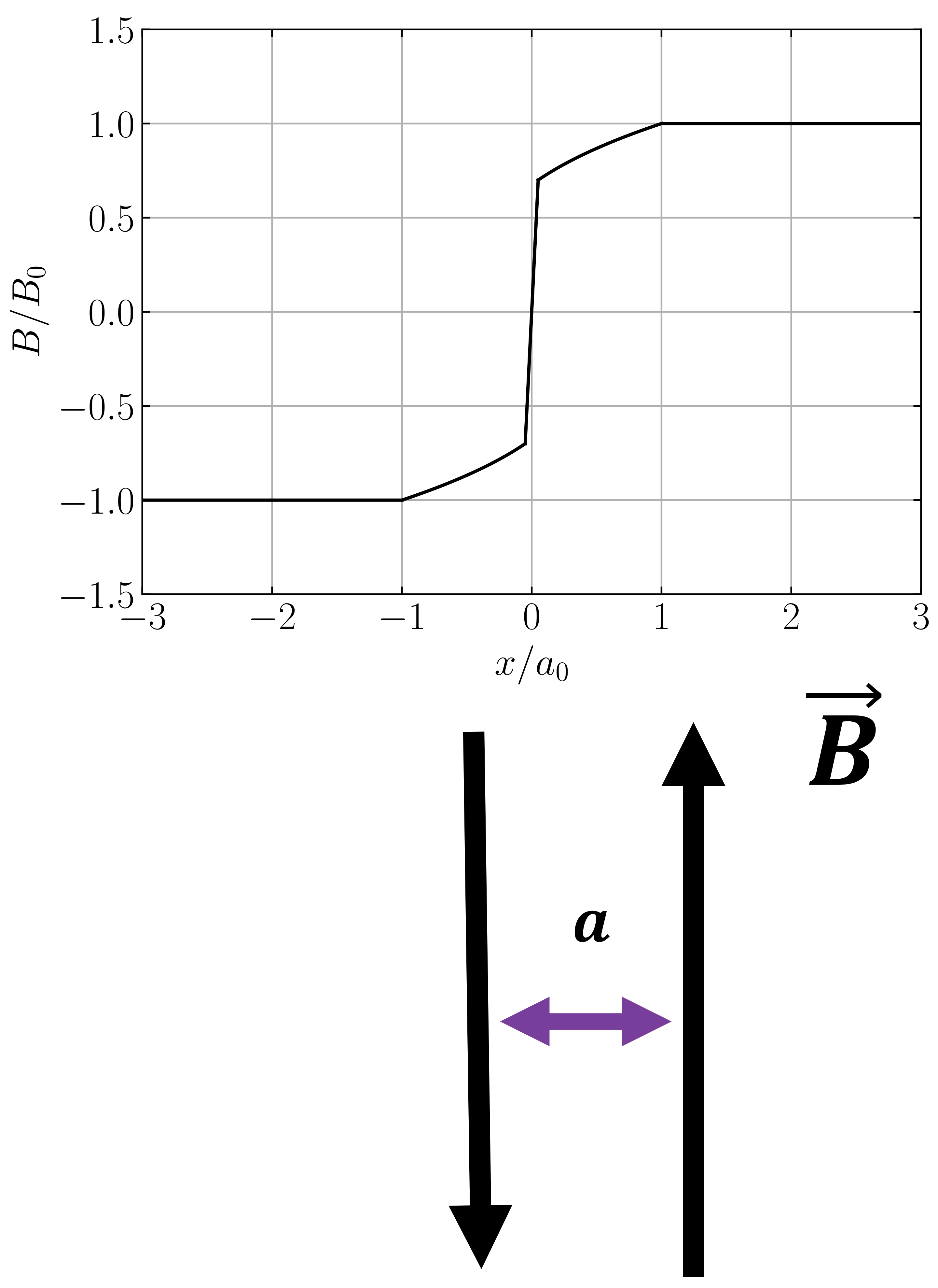
1.



2.



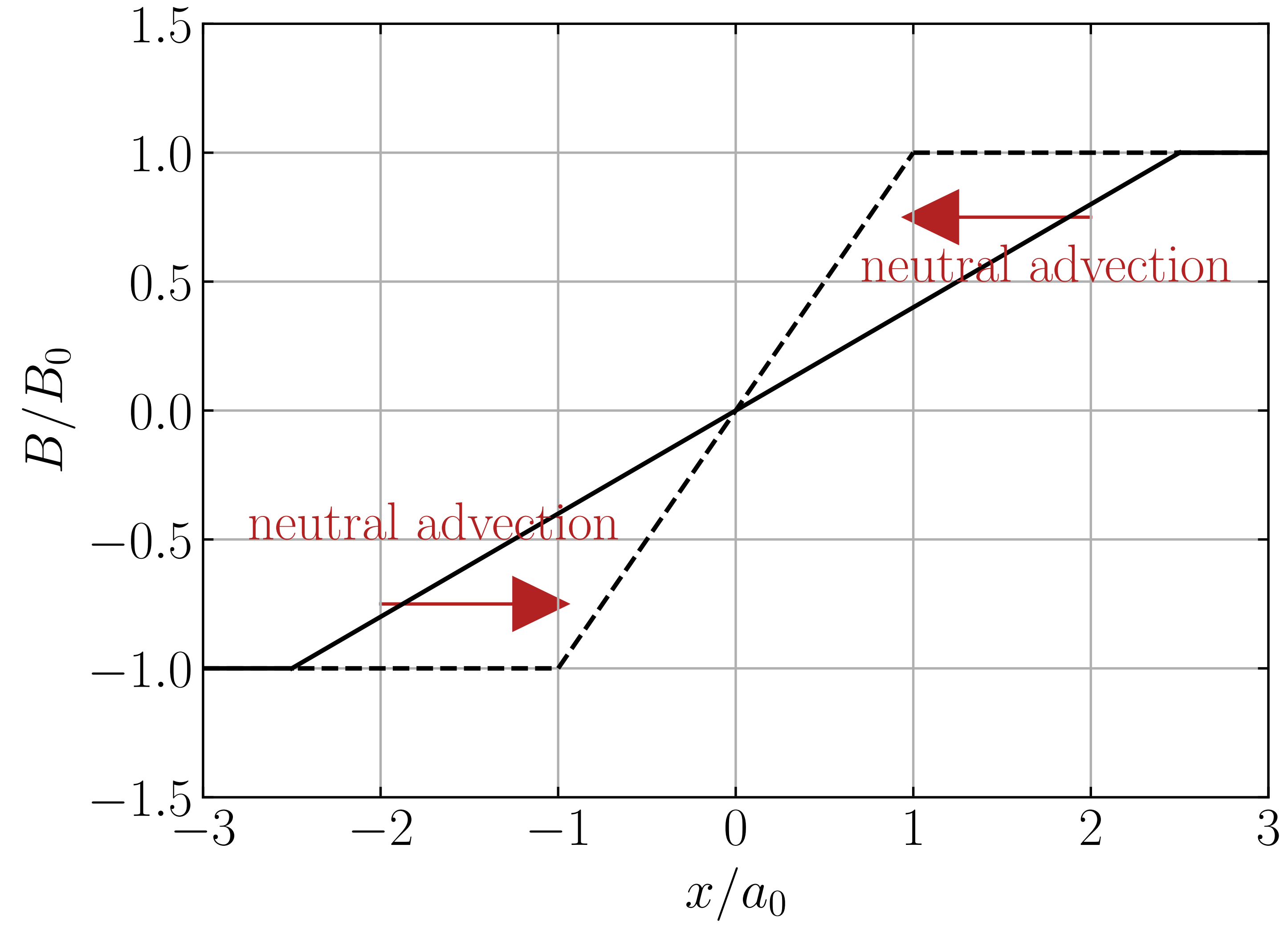
3.



First stage is driven by neutral advection

$$\frac{\partial \vec{B}}{\partial t} = \boldsymbol{\nabla}\times\left(\vec{\boldsymbol{v}}_{\boldsymbol{n}}\times\vec{\boldsymbol{B}}\right) + \boldsymbol{\nabla}\times\left\{\frac{\left\lfloor\left(\boldsymbol{\nabla}\times\vec{\boldsymbol{B}}\right)\times\vec{\boldsymbol{B}}\right\rfloor\times\vec{\boldsymbol{B}}}{\boldsymbol{\nu}_{\boldsymbol{ni}}\,\boldsymbol{\rho}_{\boldsymbol{n}}}\right\} + \eta\nabla^2\vec{B}$$

- Consider neutral advection term $\sim \frac{v_{A,n0}\,B_0}{a}$
- Consider ambipolar diffusion term $\sim \frac{v_{A,n0}^2\,B_0}{a^2\,\nu_{ni0}}$
- On large length scales, formation dominated by neutral advection



$$a(t) > a_0 \equiv \frac{v_{A,n0}}{\nu_{ni0}}$$

Onset cannot occur during first stage

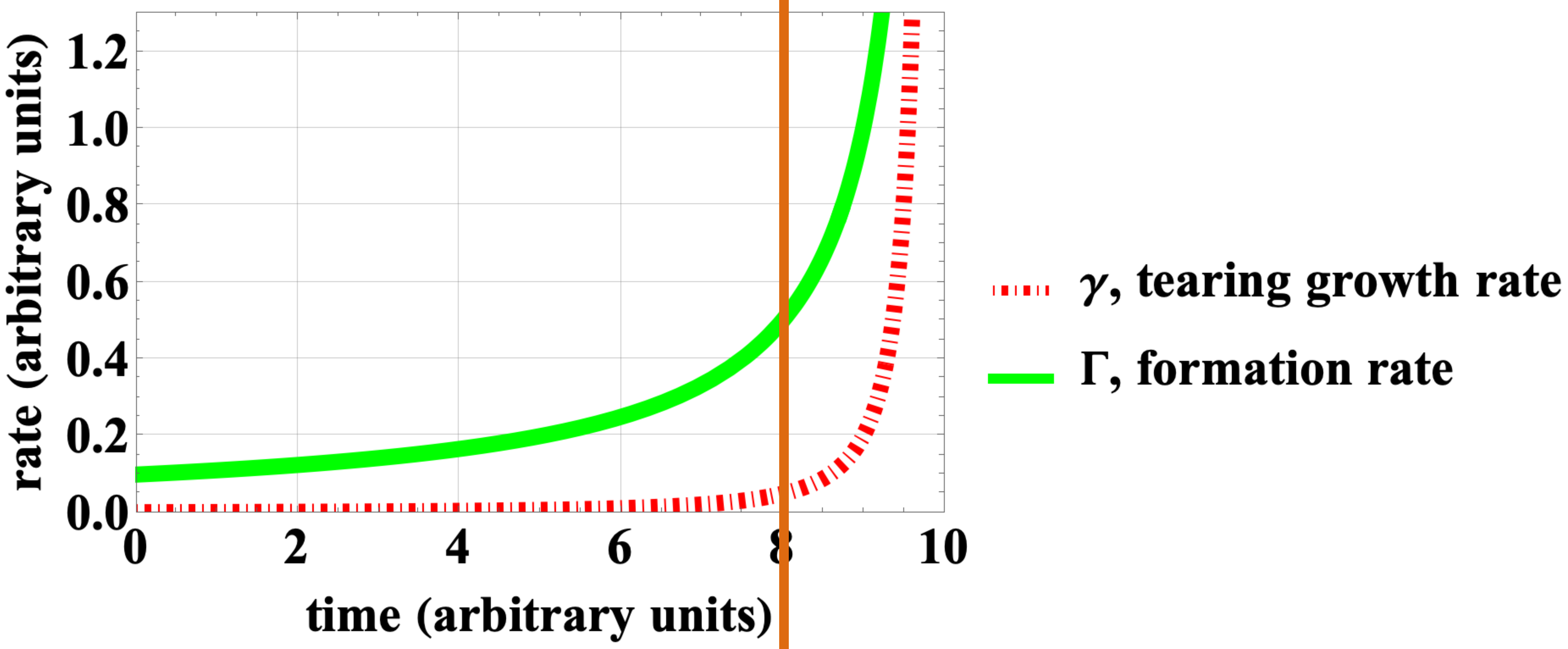
$$\boxed{\frac{\partial \vec{B}}{\partial t} = \nabla\times\left(\vec{v}_n \times \vec{B}\right)}$$

- Formation rate given by $\Gamma_{NA} \sim \frac{v_{A,n0}}{a(t)}$
- Recall tearing rate $\gamma \sim \frac{\eta^{2/3} v_{A,n0}^{2/3}}{a(t)^2 v_{ni0}^{1/3}}$
- Growth rates related by

$$\gamma/\Gamma_{NA} \sim \eta^{\frac{2}{3}}/a(t) v_{A,n}^{\frac{1}{3}} v_{ni0}^{\frac{1}{3}}$$

- Minimum width is $a_0 \equiv v_{A,n0}/v_{ni0}$
- So, we have $\gamma/\Gamma_{NA} < v_{ni0}^{2/3} \eta^{\frac{2}{3}}/v_{A,n}^{\frac{4}{3}}$

End of neutral advection stage



$a(t)$ shrinks

Small parameter (10^{-8} at typical molecular cloud parameters)

Second stage is driven by ambipolar diffusion

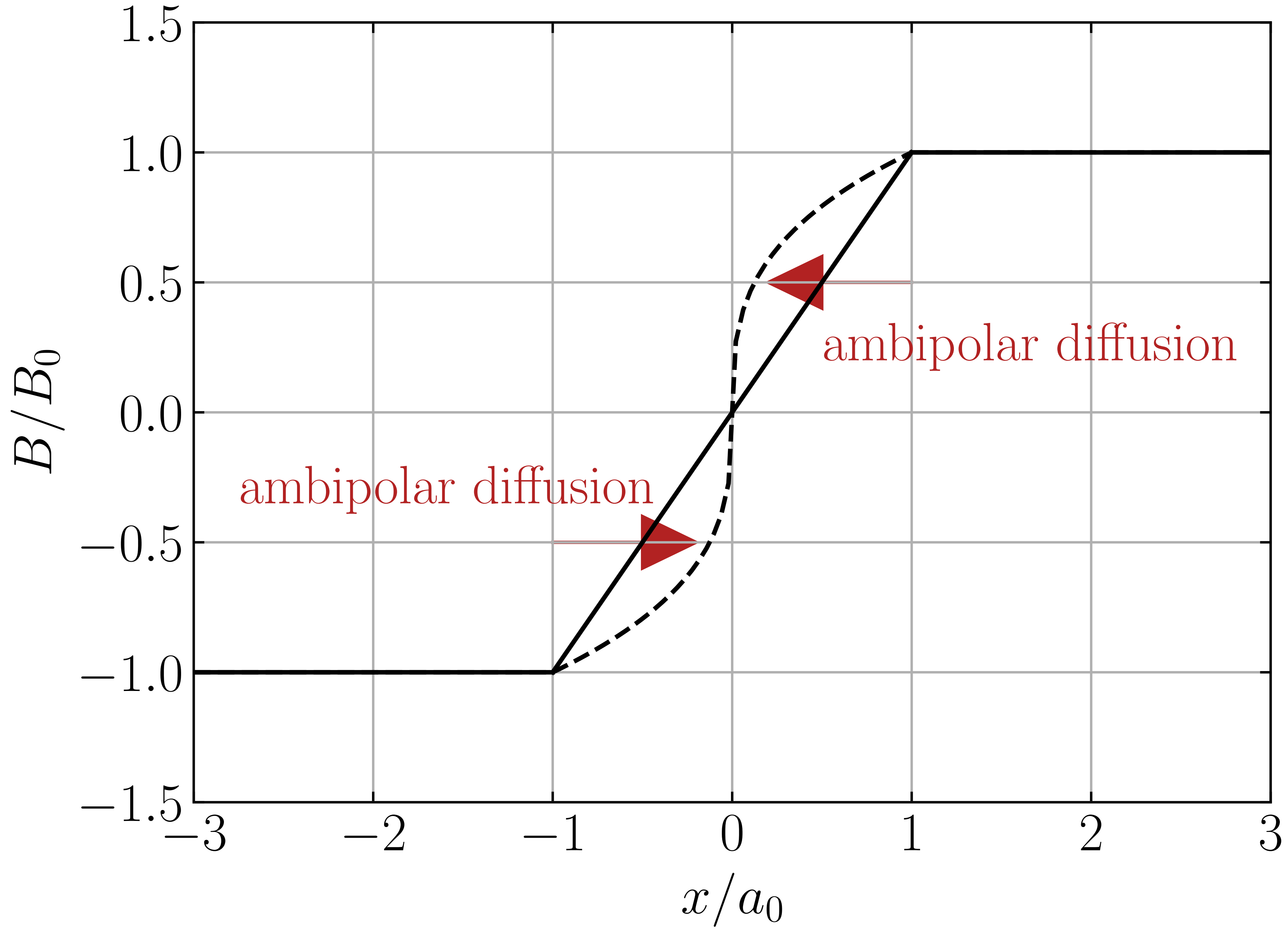
- For a $\hat{y}$-directed field, ambipolar diffusion term reads

$$\boxed{\frac{\partial B_y}{\partial t} = \frac{\partial}{\partial x}\left(\frac{B_y^2}{\nu_{ni}\rho_n}\frac{\partial B_y}{\partial x}\right)}$$

- The steady state of this equation is

$$\boxed{B_y(x) \sim x^{1/3}}$$

- Ambipolar diffusion thus naturally sharpens current sheet
- Sharpening mechanism first proposed in Brandenburg and Zweibel ApJ 1994



Ambipolar rate of formation increases with time

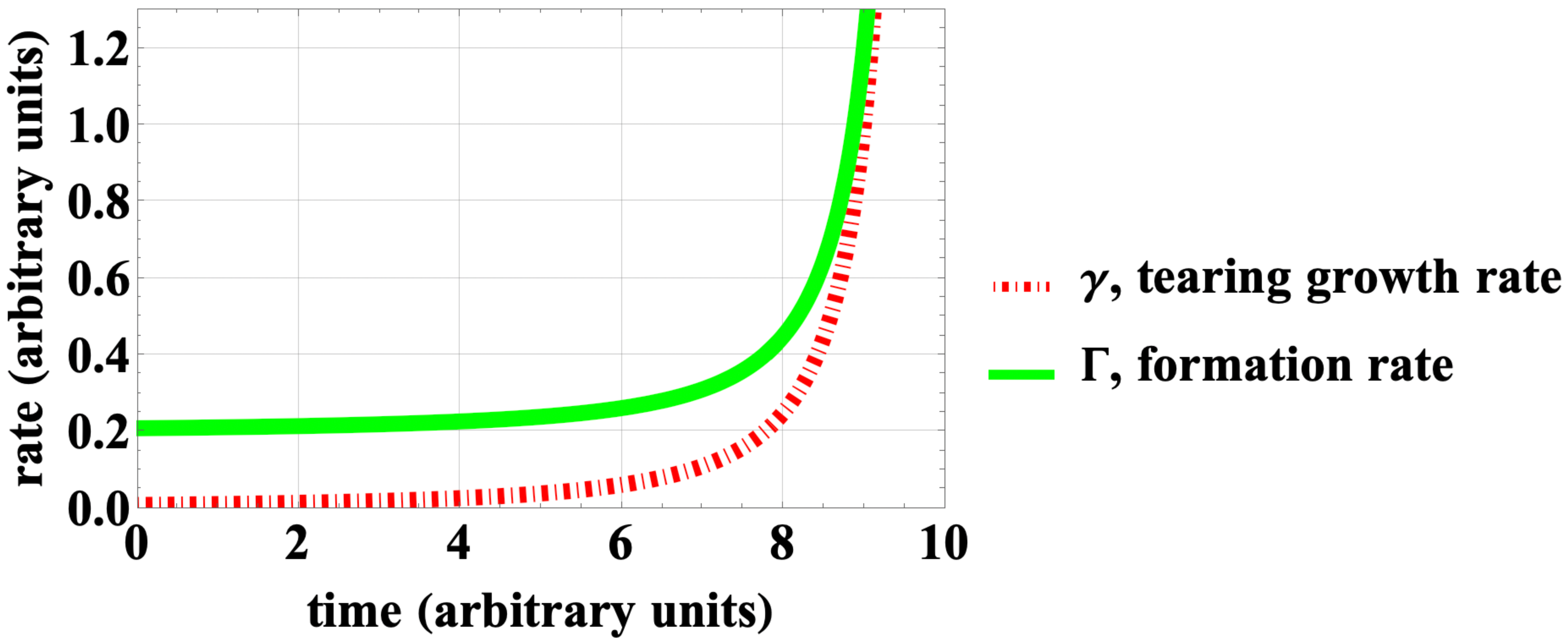
- Can use expression for ambipolar diffusion to estimate a formation rate

$$\boxed{\frac{\partial B_y}{\partial t} = \frac{\partial}{\partial x}\left(\frac{B_y^2}{\nu_{ni}\rho_n}\frac{\partial B_y}{\partial x}\right)} \Rightarrow \boxed{\Gamma_{AD} \sim \frac{v_{A,n0}^2}{a(t)^2\nu_{ni0}}}$$

- Increases with time
- Poorly ionized tearing growth rate is

$$\boxed{\gamma \sim \frac{\eta^{2/3} v_{A,n0}^{2/3}}{a(t)^2 \nu_{ni0}^{1/3}}}$$

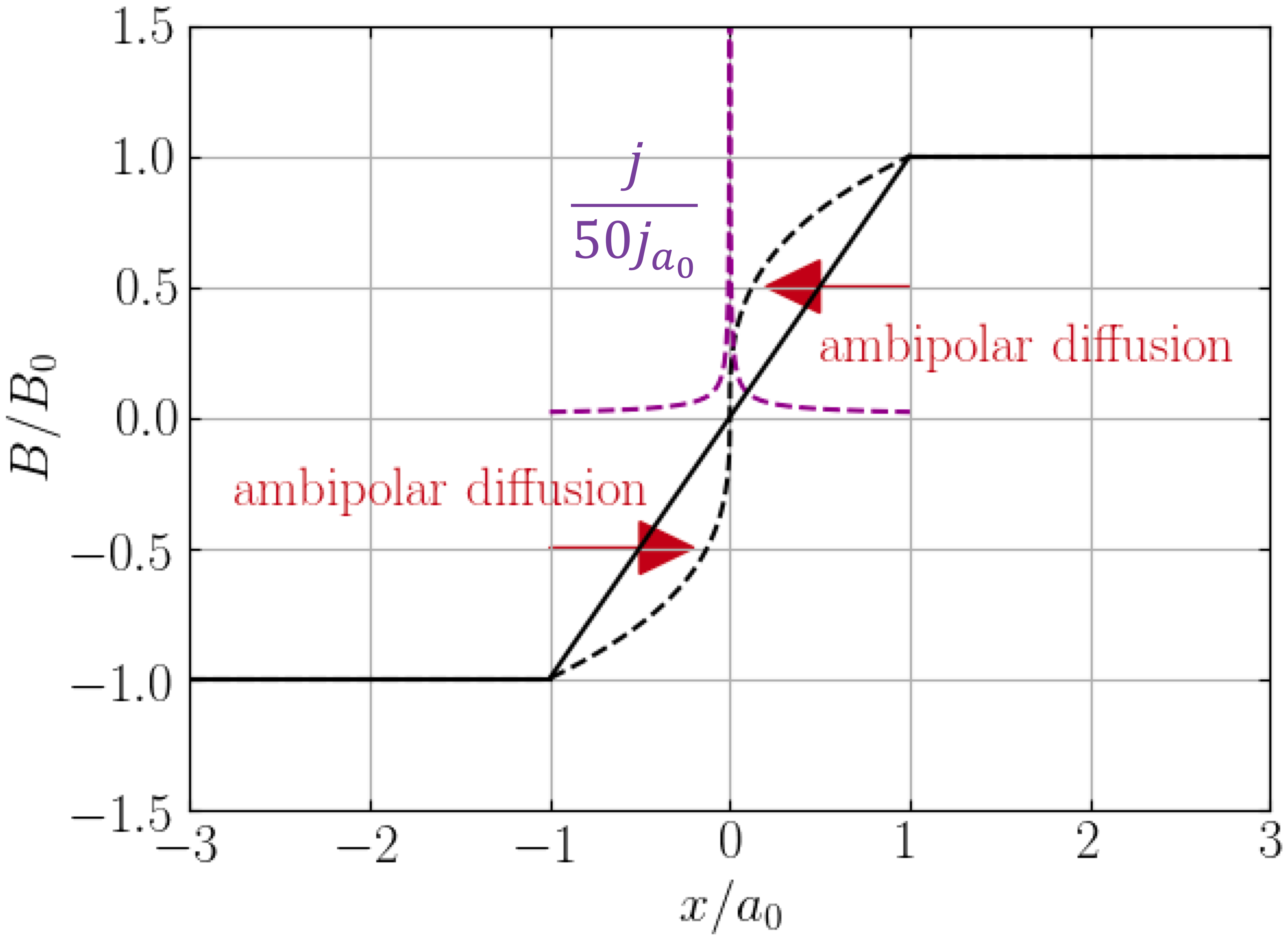
- Recall condition for tearing onset is $\gamma \gg \Gamma$
- Onset cannot occur during formation process



$a(t)$ shrinks

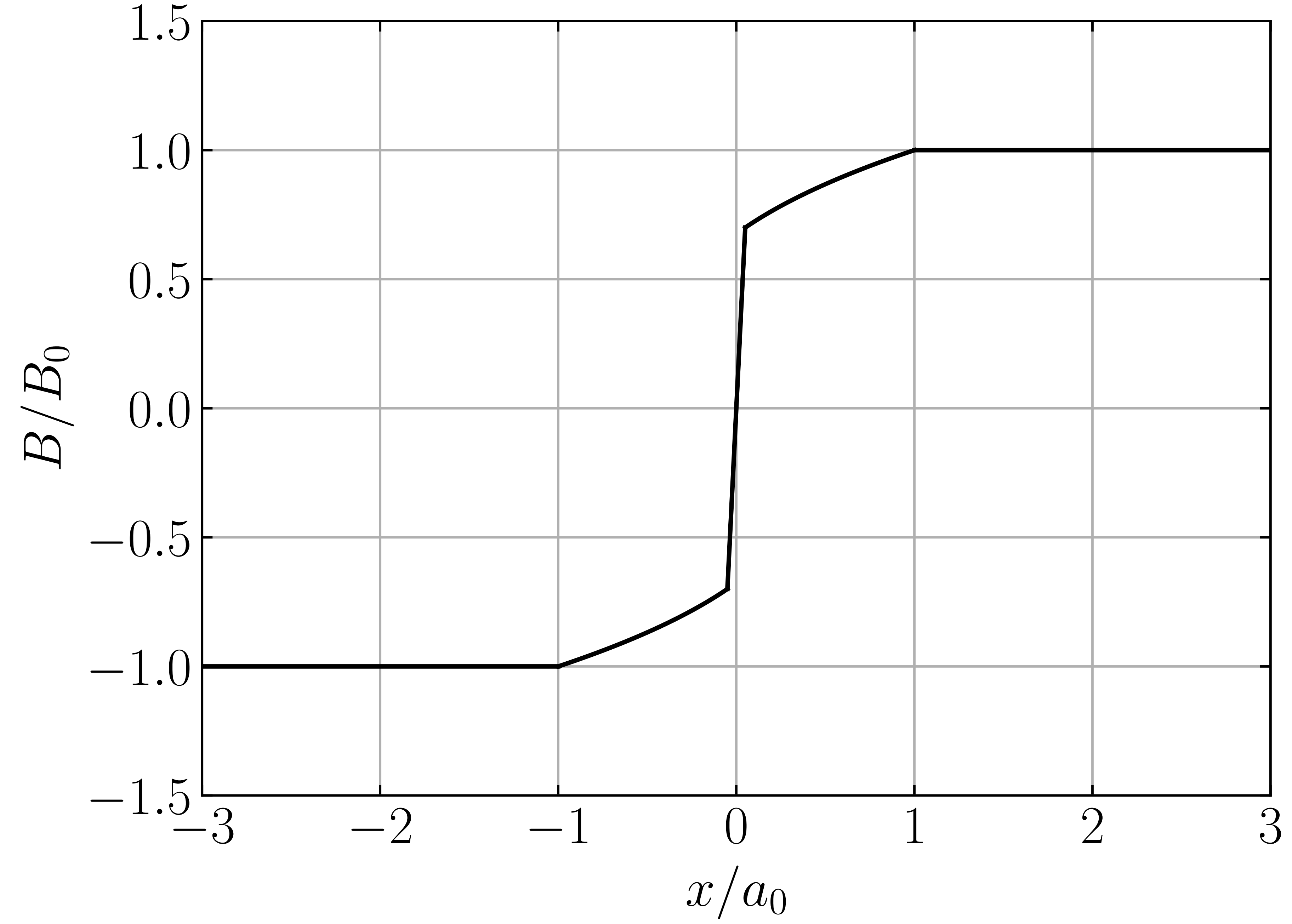
Finite ion pressure and resistivity halt formation process

- Steady state of ambipolar diffusion term, $B_y(x) \sim x^{1/3}$, involves current singularity
- Finite ion pressure, resistivity remove singularity; strong coupling assumption breaks (Brandenburg and Zweibel ApJ 1995)
- Formation process will stop



Final stage is steady-state current sheet

- Steady-state current sheet is final situation

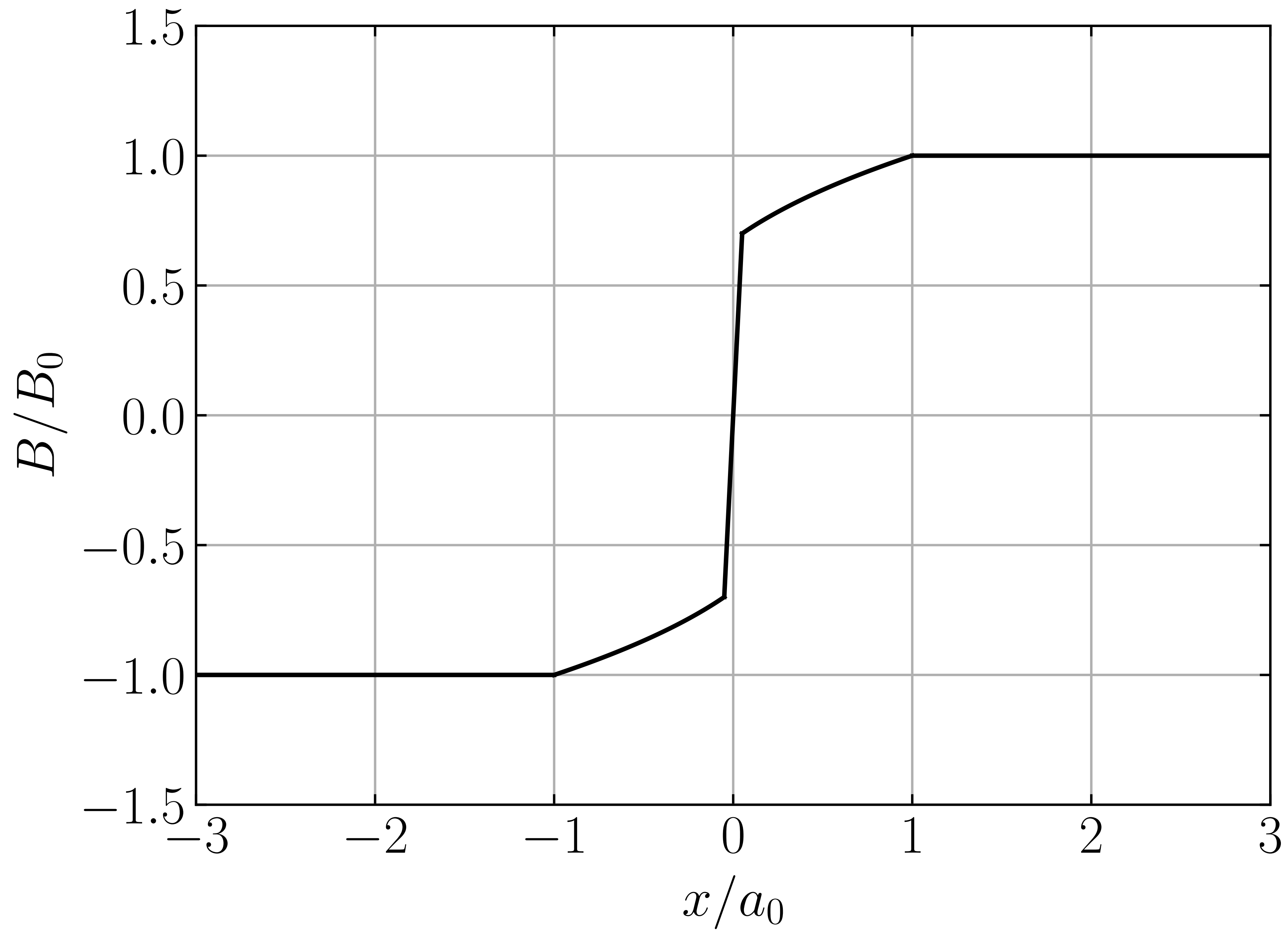


Tearing onset easily occurs in final current sheet

- In stationary current sheet, tearing can easily onset

- Onset time given by time for magnetic field to diffuse inwards: ν_{ni0}^{-1} (inverse neutral-ion collision frequency)

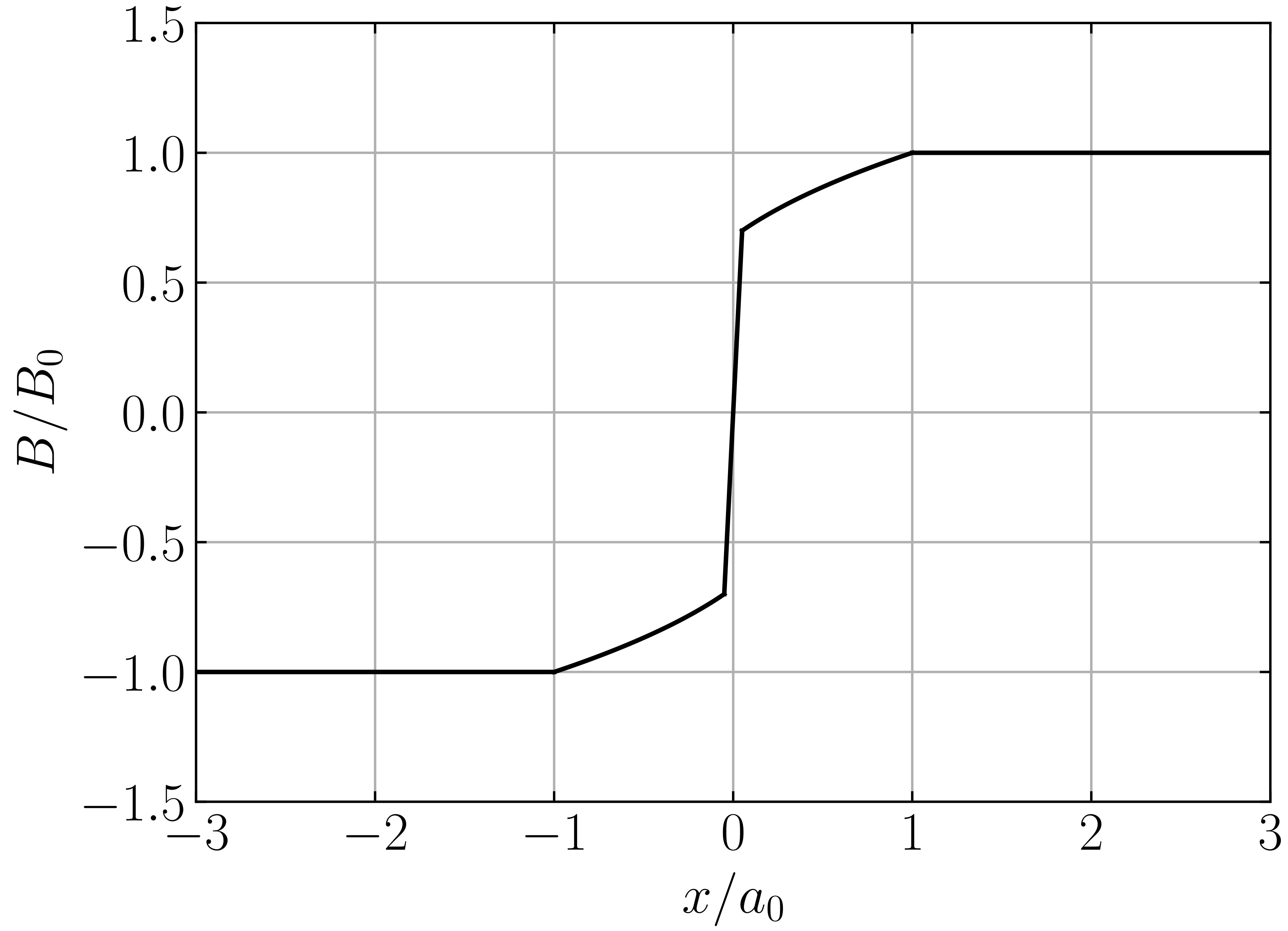
$$\boxed{t_{onset} \sim t_{form} \sim \nu_{ni0}^{-1}}$$



Type of mode that disrupts sheet

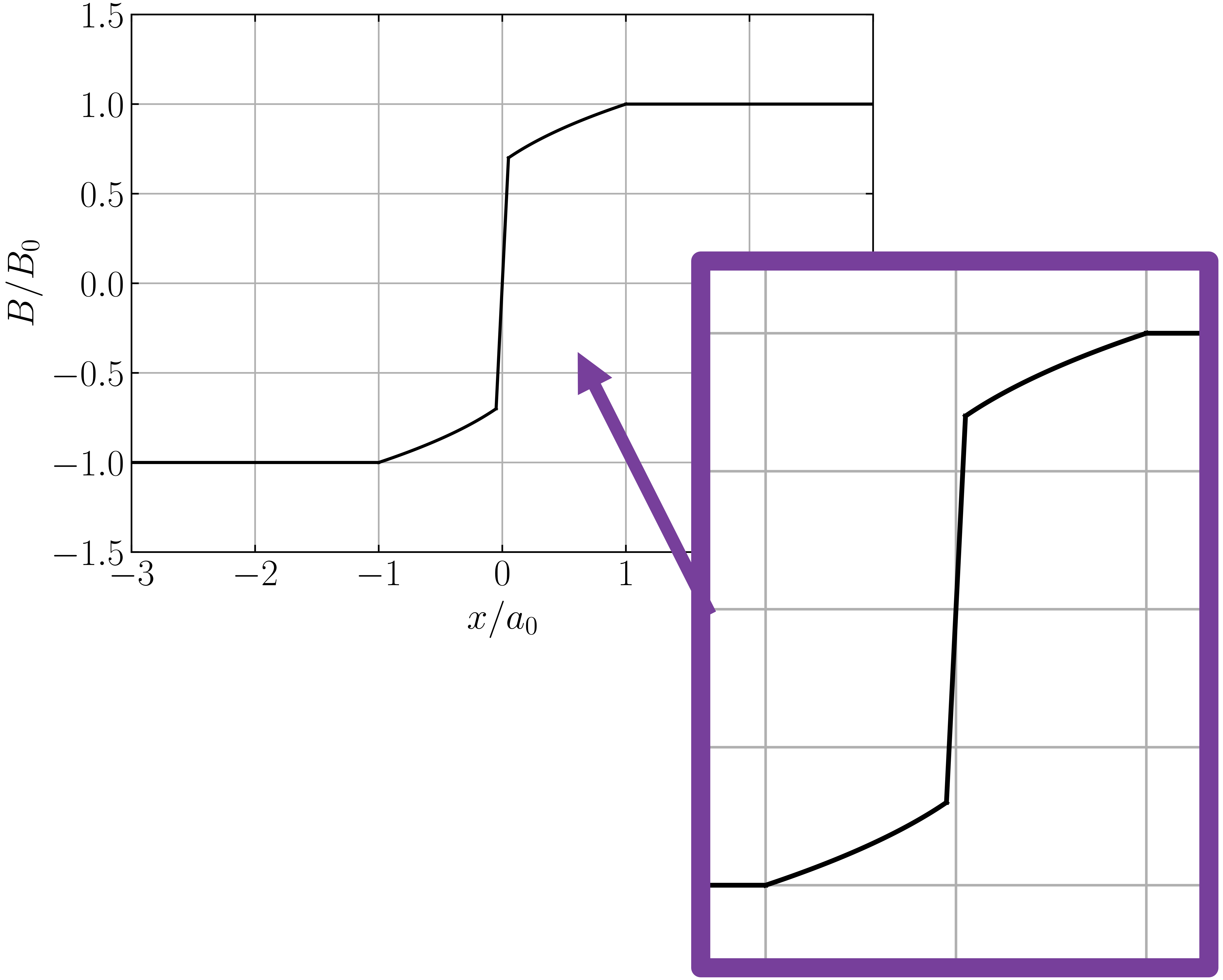
Onset mode, growth rate can be found from steady state

- After strong coupling breaks, current sheet will reach a final steady state
- Tearing onsets as in a stationary current sheet
- Wavelength and growth rate of mode that disrupts sheet can be found by doing stationary linear stability analysis
- Wavelength of the mode that first disrupts the forming sheet decreases as ionization fraction decreases



Width of CS determined by scaling arguments

- In order to determine onset mode, need width of inner layer, other parameters
- We find these using “twiddle algebra”
- Backed up by numerical work: Heitsch & Zweibel (2003)
- Should be thought of like a Sweet-Parker type calculation for a poorly ionized sheet



Width of CS determined by scaling arguments

- Plasma in inner region of current sheet recombines:

Ionized continuity

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \vec{v}_i) = \xi \rho_n - \alpha \rho_i^2$$

$$\frac{\rho_{i,cs}\, v_{i,\eta}}{a} \sim \alpha \rho_{i,cs}^2$$

- Magnetic field in inner region diffuses away

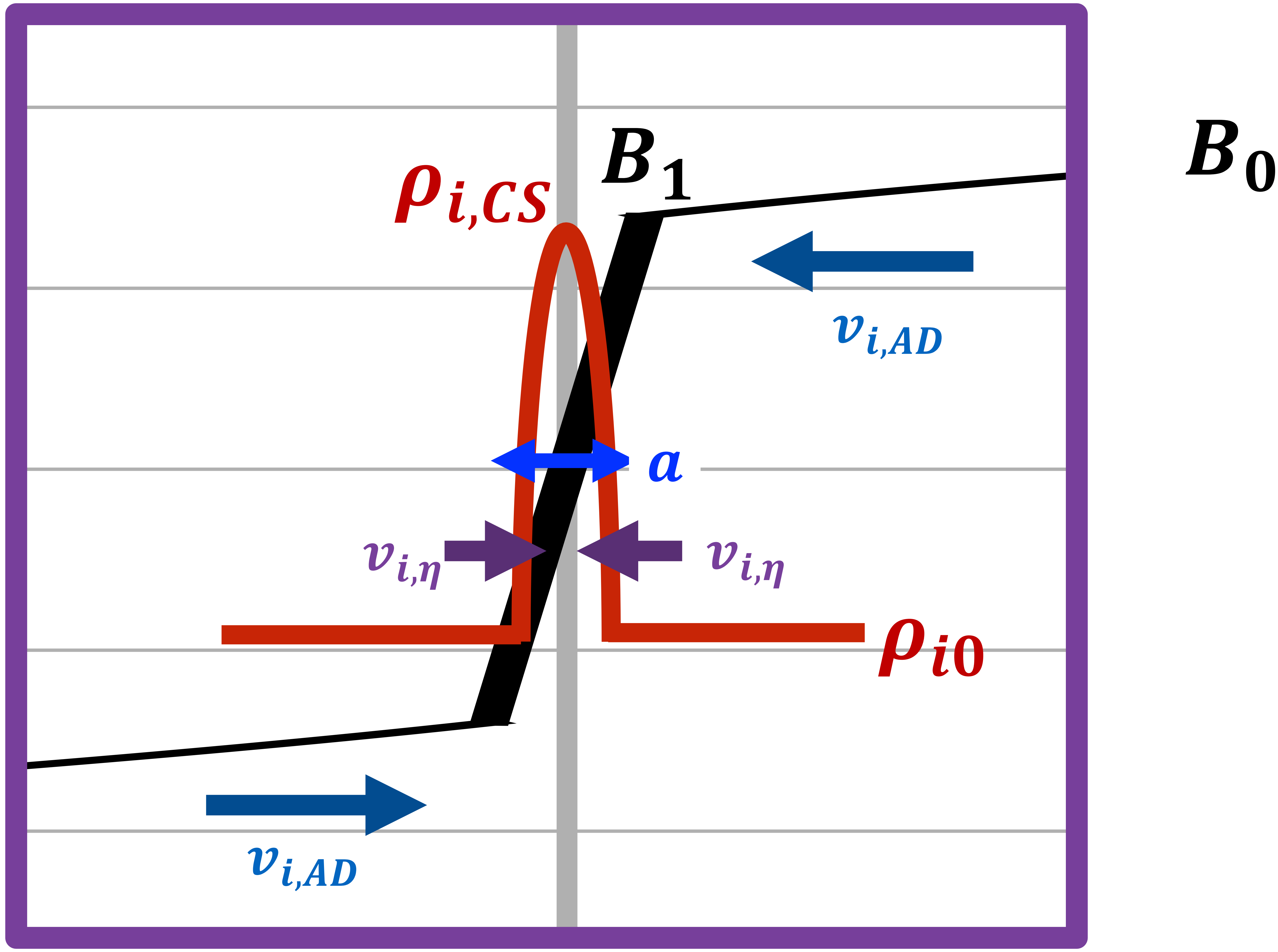
Induction equation

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{v}_i \times \vec{B}\right) + \eta \nabla^2 \vec{B}$$

$$a \sim \frac{\eta}{v_{i,\eta}}$$

- Ionized pressure at center of CS roughly balances magnetic field far from sheet

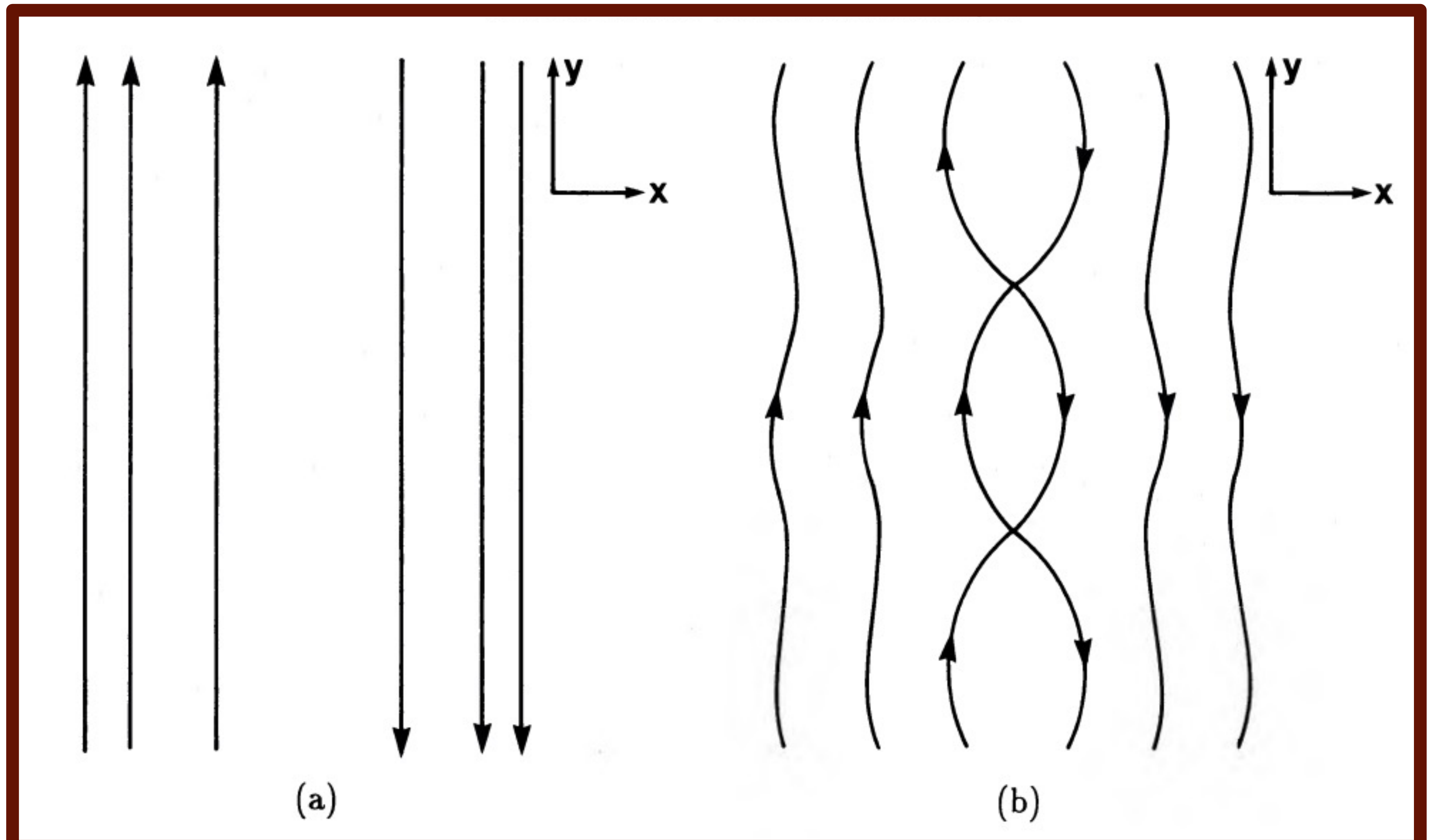
$$\rho_{i,cs} \sim \frac{B_0^2}{C_i^2}$$



We find $a \sim \frac{C_i}{B_0}\sqrt{\frac{\eta}{\alpha}}$ (other parameters in paper)

From parameters, can find overall spectrum

- Earlier, we saw approximate asymptotic expressions for tearing growth
- Actually, tearing growth is more complex
- Spectrum of wavenumber k unstable, with complicated dispersion relation (see paper)
- Function of a, plasma parameters



Tearing in a current sheet, from Zweibel ApJ 1989

From parameters, can find overall spectrum

- Let us consider how spectrum depends on CS parameters
- Fix:

$$\xi = 0.1\,\nu_{ni0}, C_i = C_n = a_0\nu_{ni0}, \eta = 10^{-4}a_0^2\nu_{ni0}$$

- Vary recombination coefficient α
- Corresponds to varying ionization fraction:

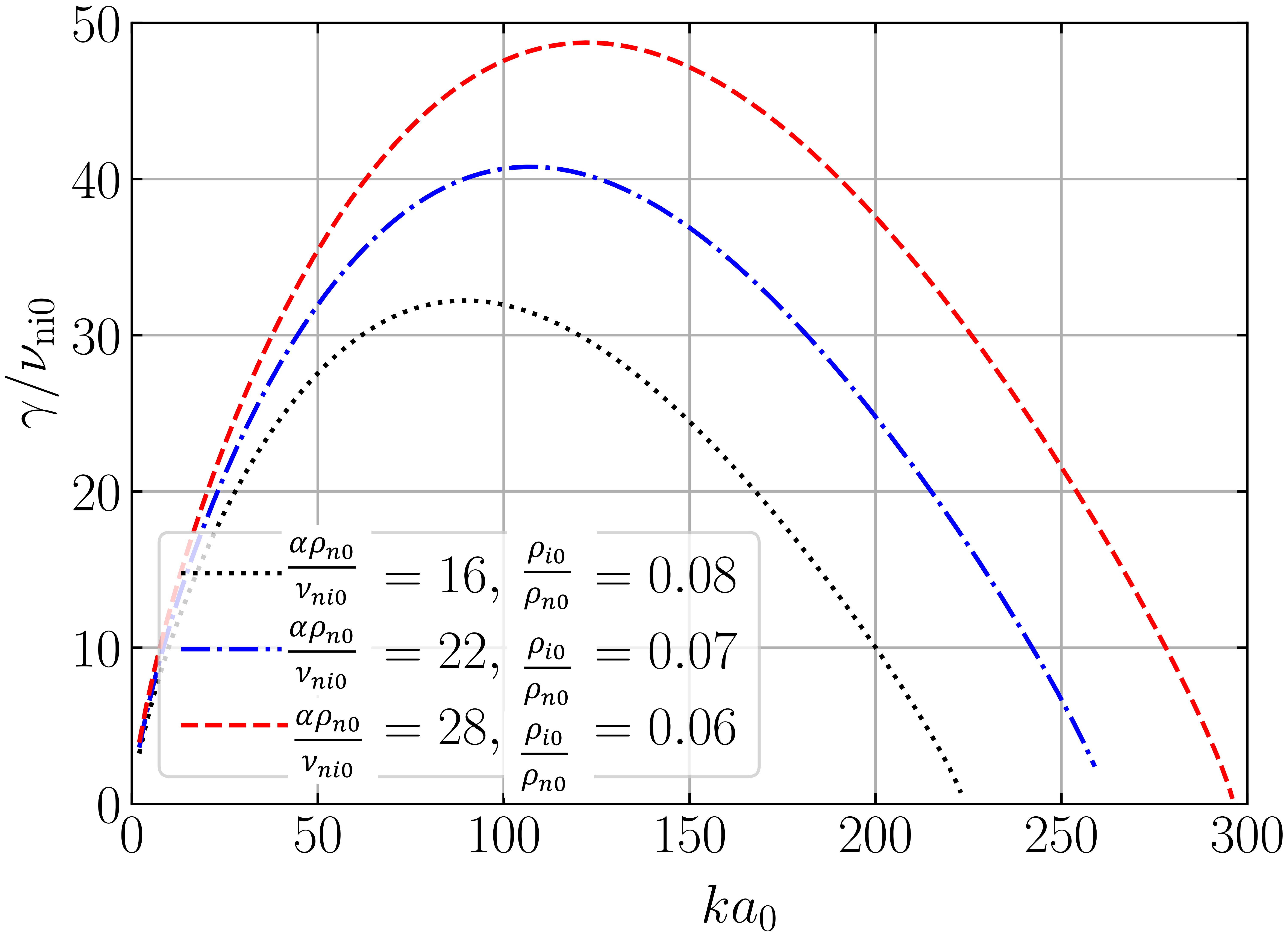
Ionized continuity

$$\frac{\partial\rho_i}{\partial t} + \nabla\cdot(\rho_i\vec{v}_i) = \boldsymbol{\xi\rho_n - \alpha\rho_i^2}$$

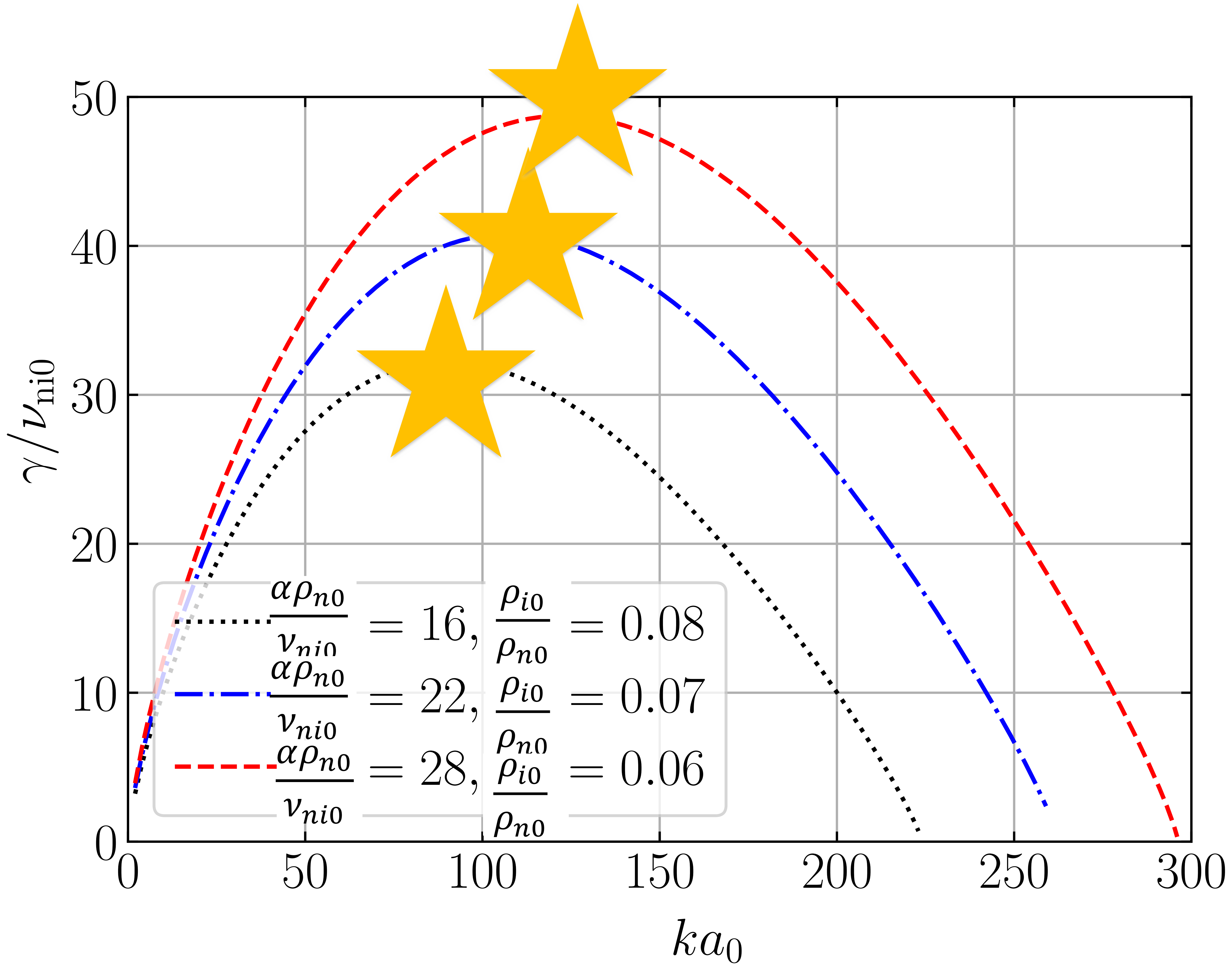
Steady-state

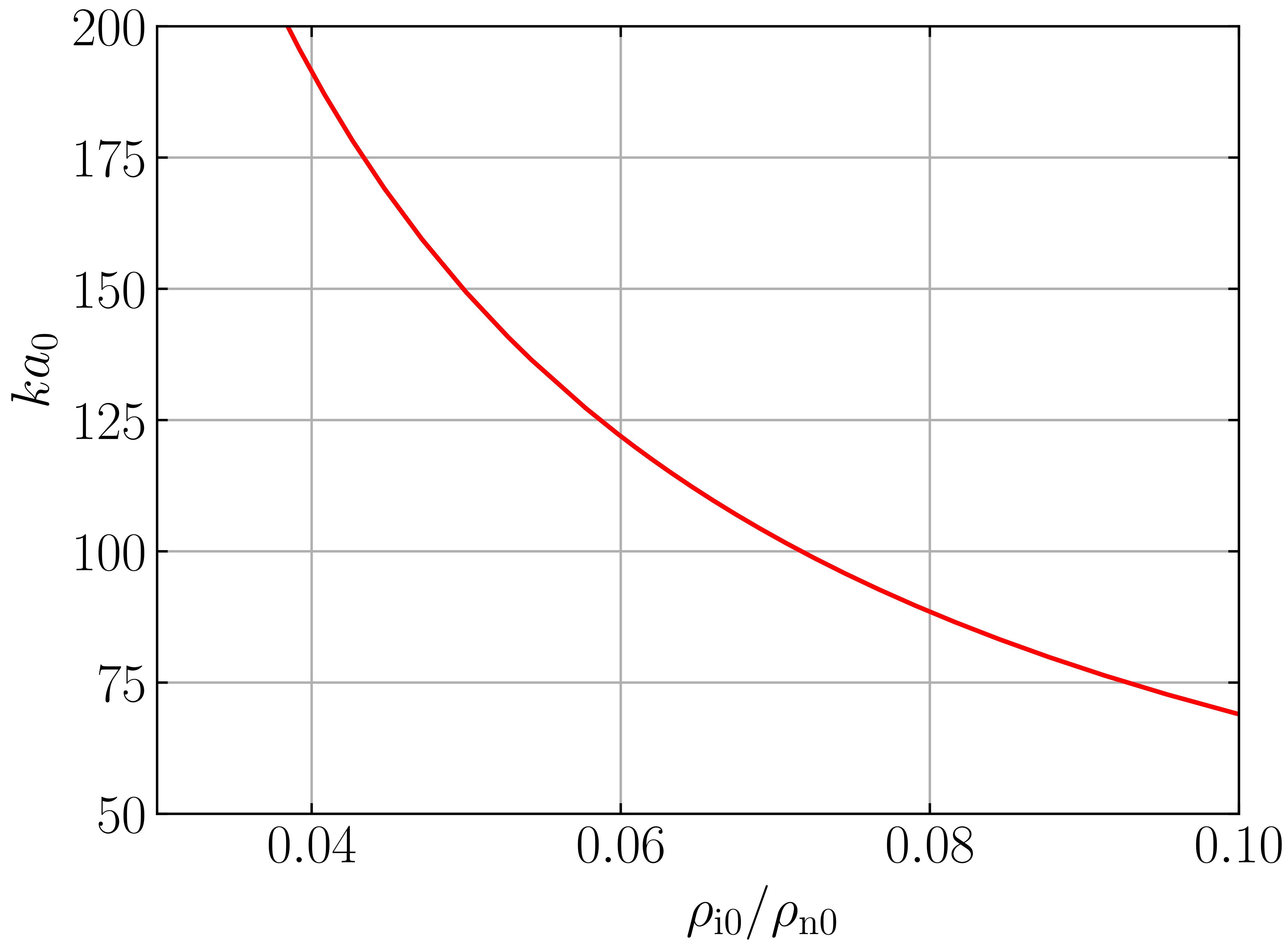
$$\frac{\rho_{i0}}{\rho_{n0}} = \sqrt{\frac{\xi}{\alpha\rho_{n0}}}$$

- Use $\boldsymbol{a} \sim \frac{\boldsymbol{C_i}}{\boldsymbol{B_0}}\sqrt{\frac{\boldsymbol{\eta}}{\boldsymbol{\alpha}}}$



Mode with fastest growth rate will dominate onset

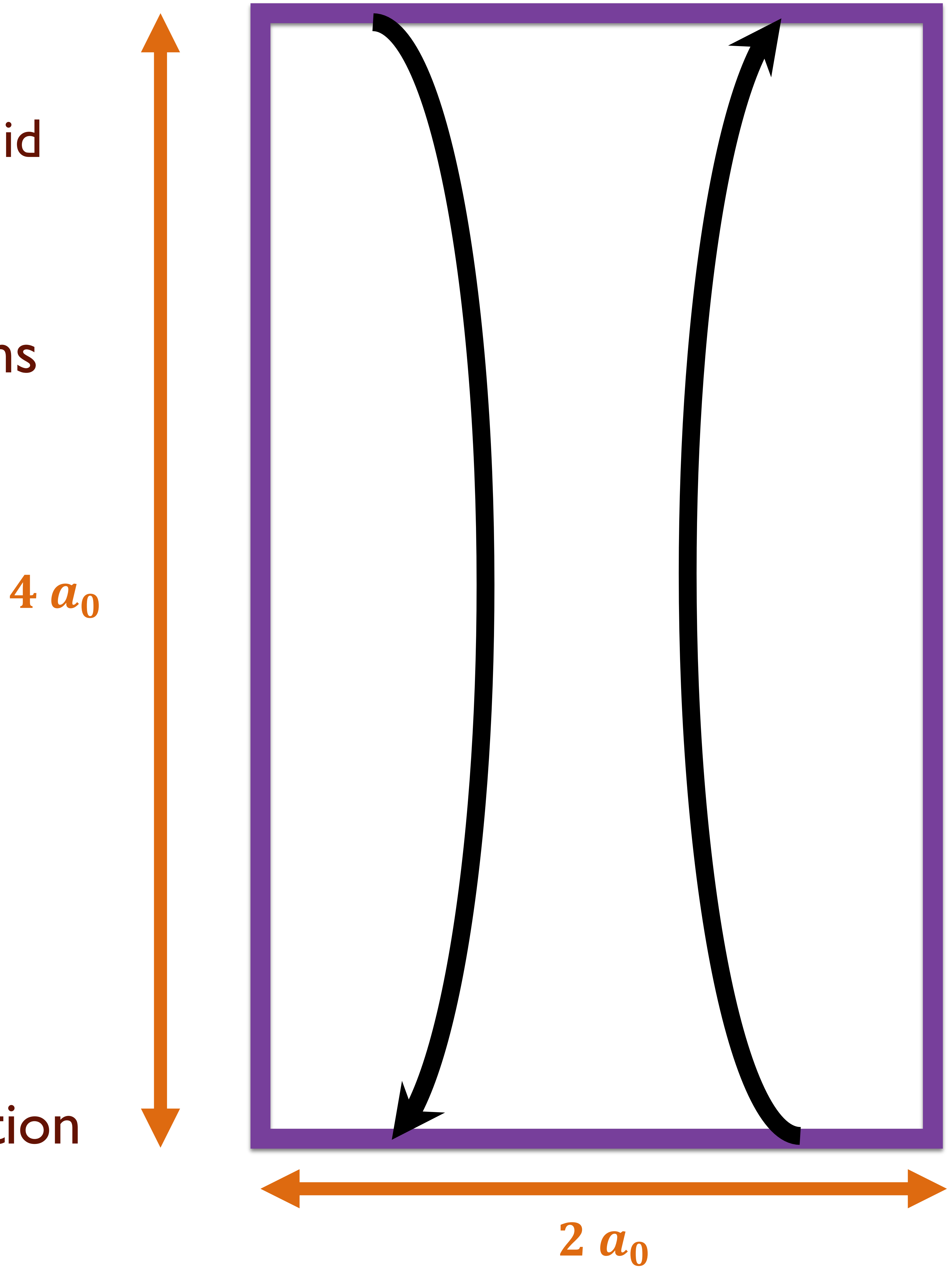




Simulations of all the above

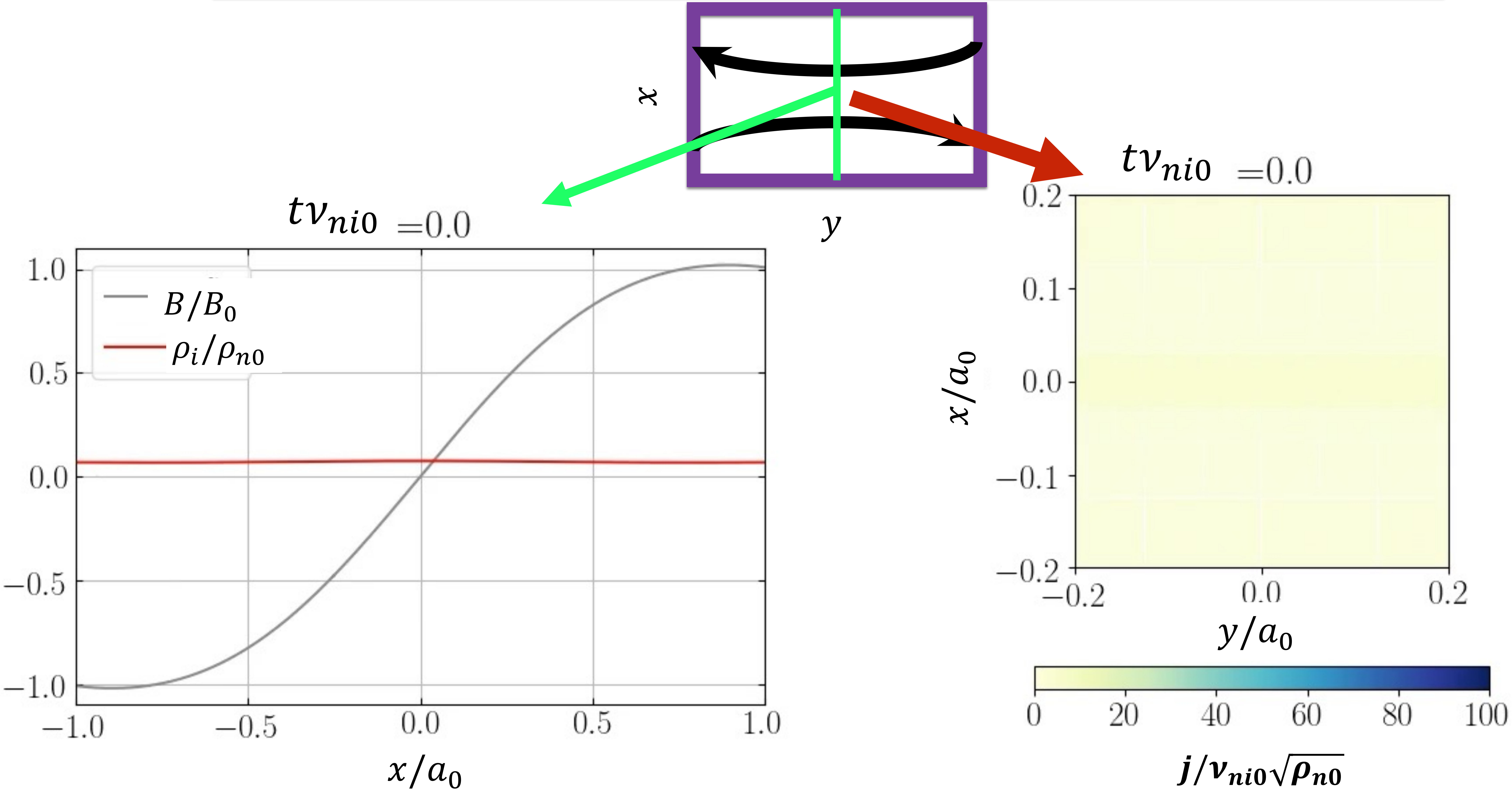
Simulations verify analytics

- We run suite of AthenaK simulations
  - Simulate coupled evolution of neutral and ionized fluid
- Small range of ionization fractions: 4% to 10%
  - Large ionization fractions do not obey conditions necessary for strong coupling
  - Small ionization fractions quickly require huge levels of resolution
- Slightly curved magnetic field
- Seeded with amplitude 10^{-7} Gaussian-random momentum perturbations
- Far from sheet: 256X512 resolution
- Close to center: SMR level 6-7 (equivalent resolution 16384x32768 to 32768x65536)



Breaking of strong coupling, plasmoids observed

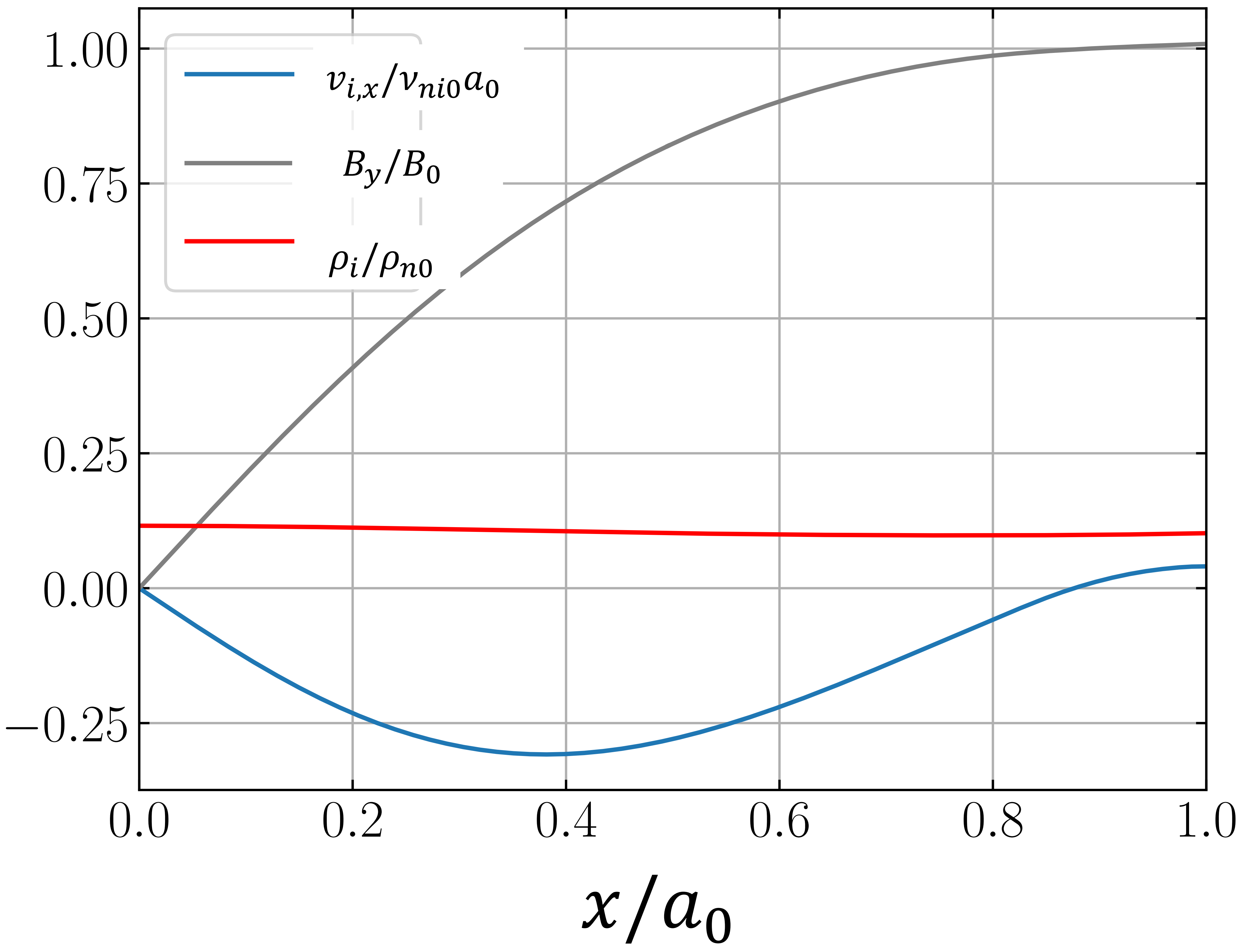
Parameters: $\xi = 0.1\,\nu_{ni0}, C_i = C_n = a_0\nu_{ni0}, \eta = 10^{-4} a_0^2 \nu_{ni0}, \frac{\alpha\rho_{n0}}{\nu_{ni0}} = 22, \frac{\rho_{i0}}{\rho_{n0}} = 0.07$

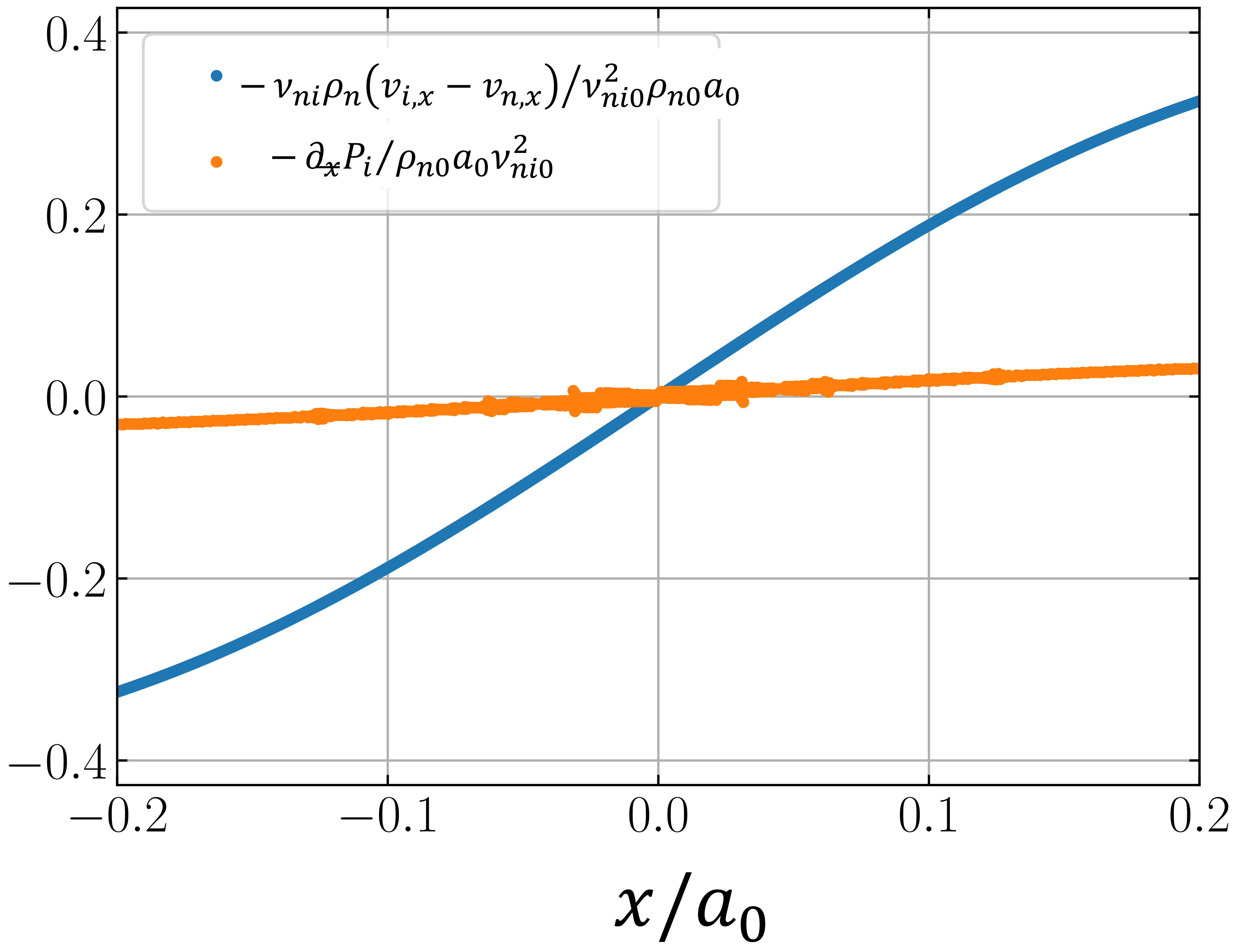


Breaking of strong coupling observed in simulation

$$\rho_i \frac{Dv_i}{dt} + \boldsymbol{\nu_{ni}\rho_n(\vec{v}_i - \vec{v}_n)} = -\nabla \mathrm{P_i} + \boldsymbol{(\nabla\times\vec{B})\times\vec{B}} + \xi\rho_n(\vec{v}_n - \vec{v}_i)$$

$t\nu_{ni0} = 0.05$

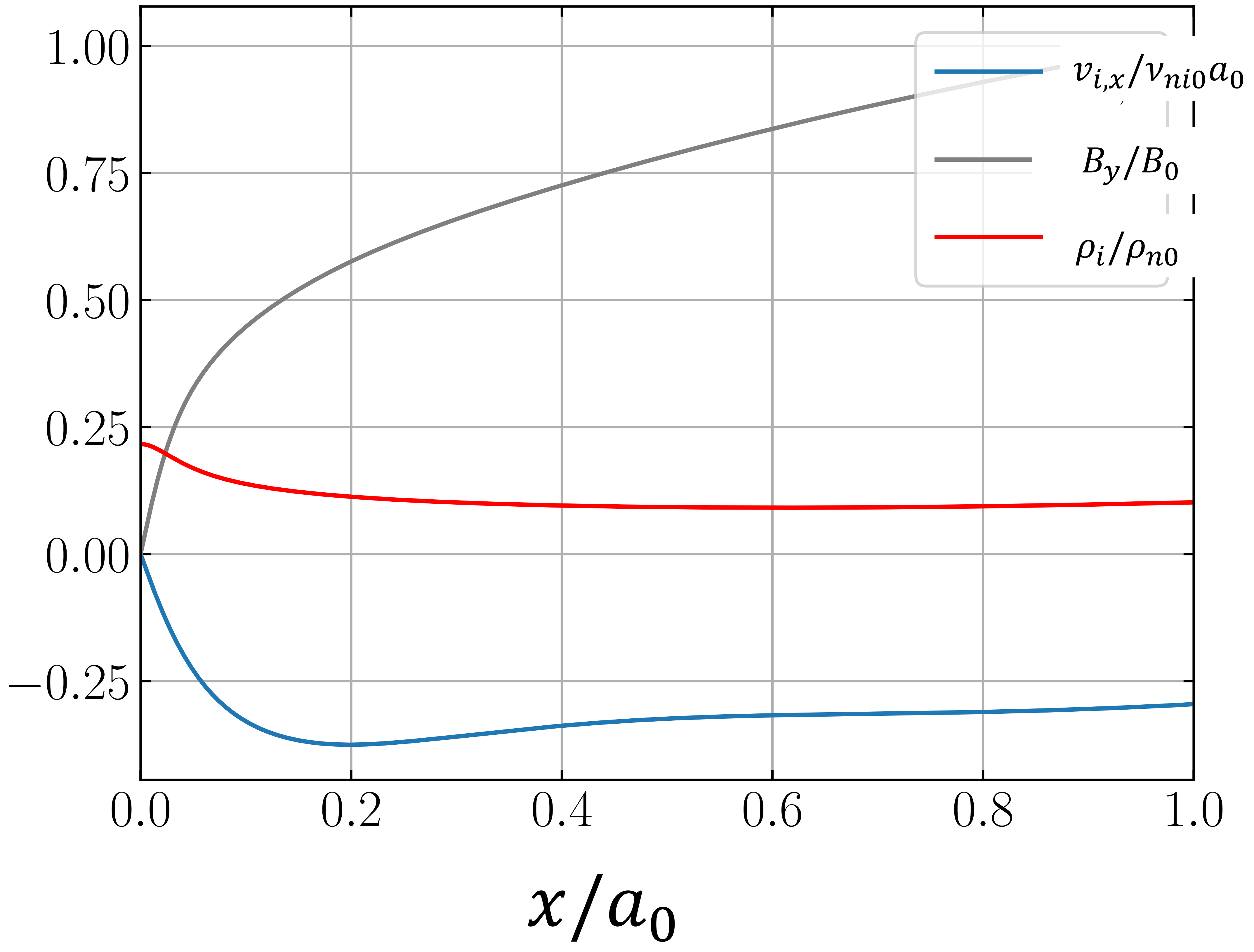


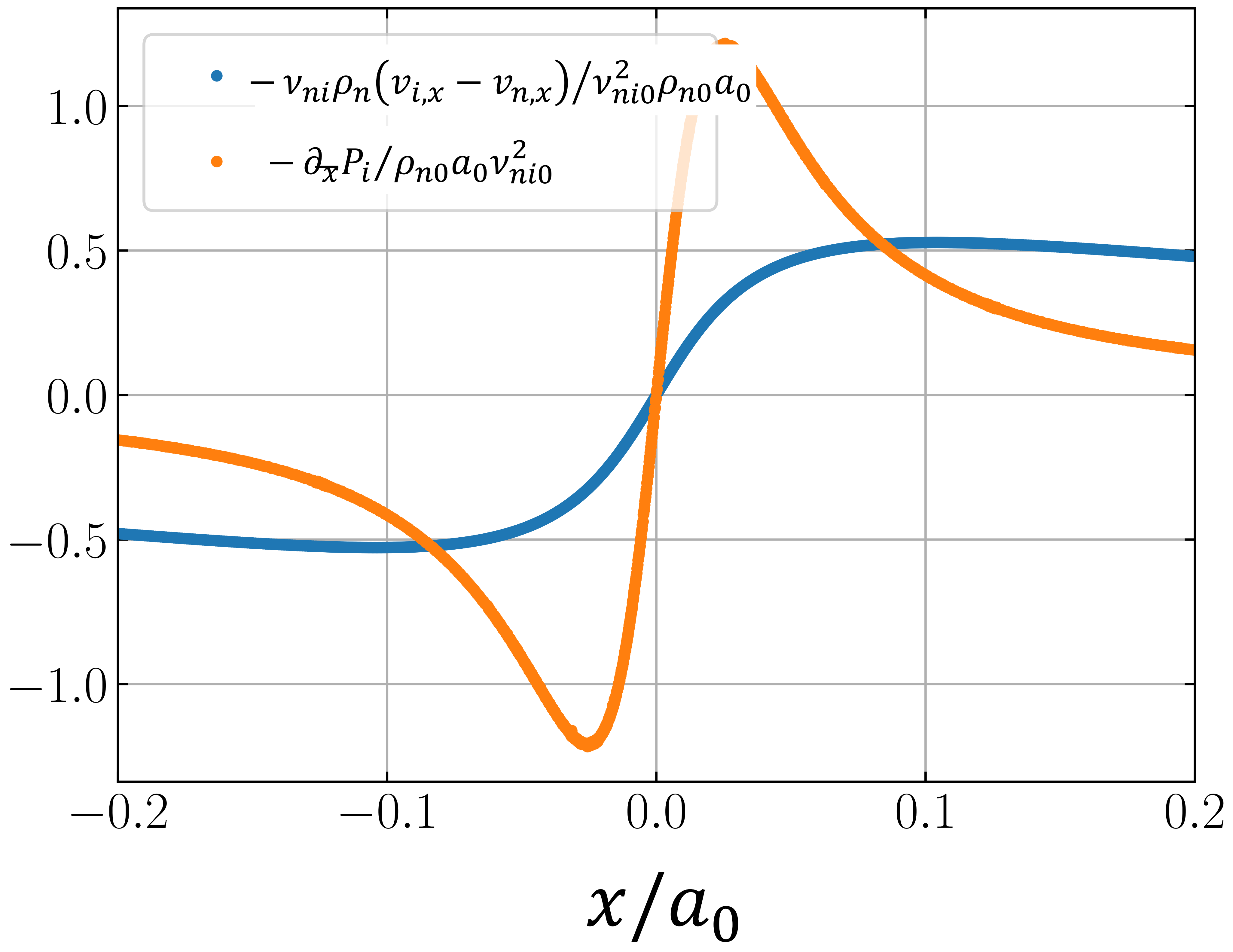


Breaking of strong coupling observed in simulation

$$\rho_i \frac{Dv_i}{dt} + \boldsymbol{\nu_{ni}\rho_n(\vec{v}_i - \vec{v}_n)} = -\nabla \mathrm{P_i} + \boldsymbol{(\nabla\times\vec{B})\times\vec{B}} + \xi\rho_n(\vec{v}_n - \vec{v}_i)$$

$t\nu_{ni0} = 0.25$

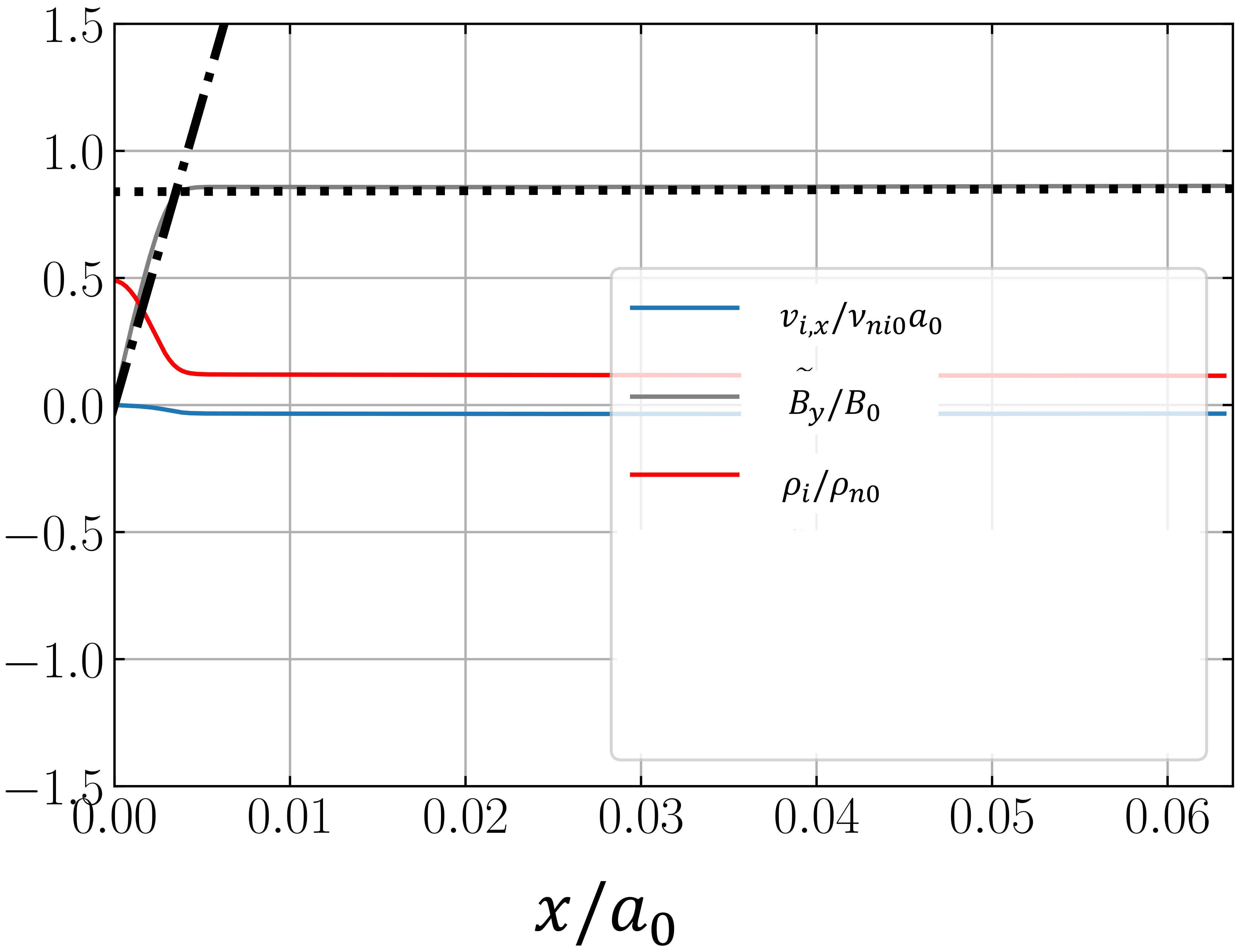


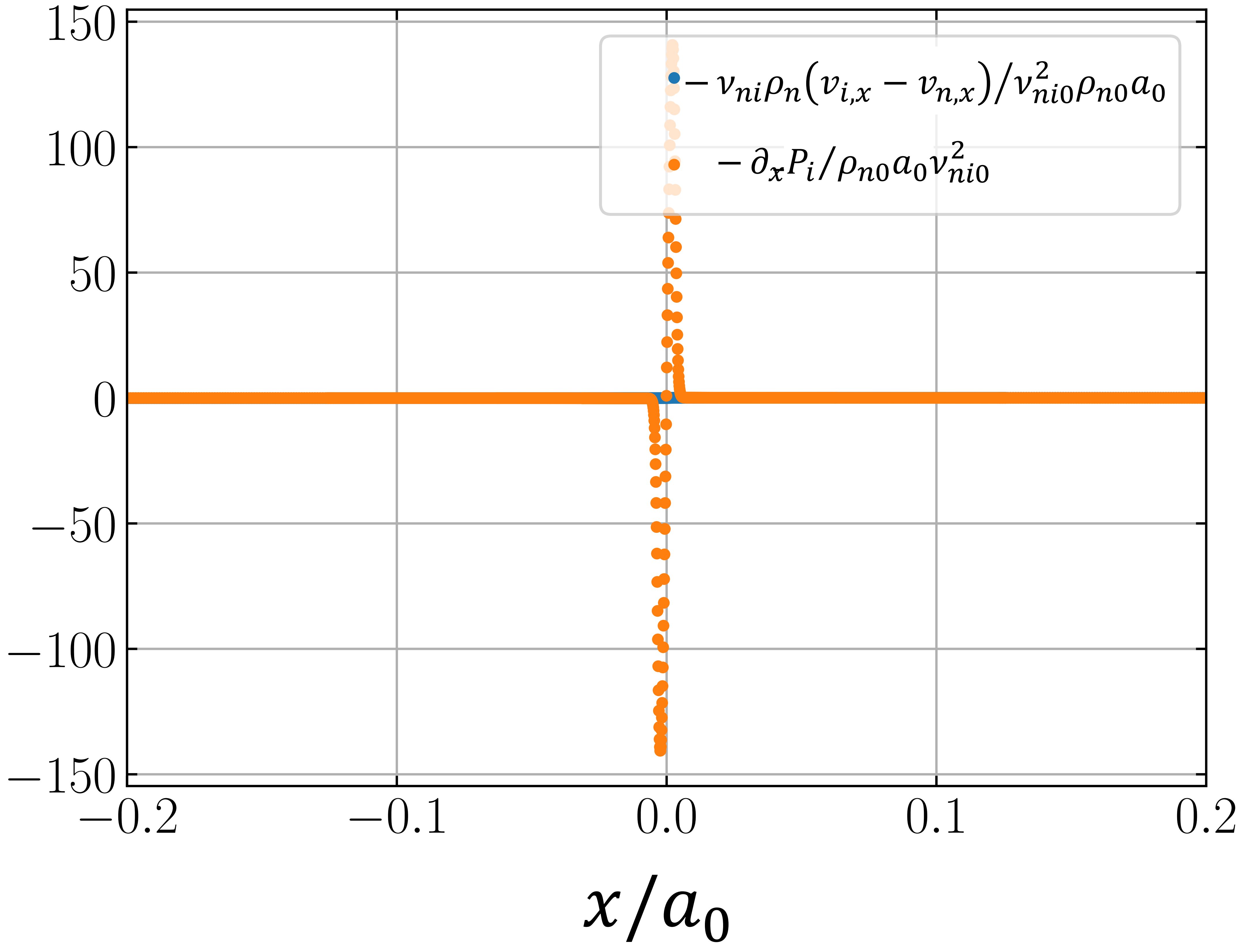


Breaking of strong coupling observed in simulation

$$\rho_i \frac{Dv_i}{dt} + \boldsymbol{\nu_{ni}\rho_n(\vec{v}_i - \vec{v}_n)} = -\nabla \mathrm{P_i} + \boldsymbol{(\nabla\times\vec{B})\times\vec{B}} + \xi\rho_n(\vec{v}_n - \vec{v}_i)$$

$t\nu_{ni0} = 0.25$





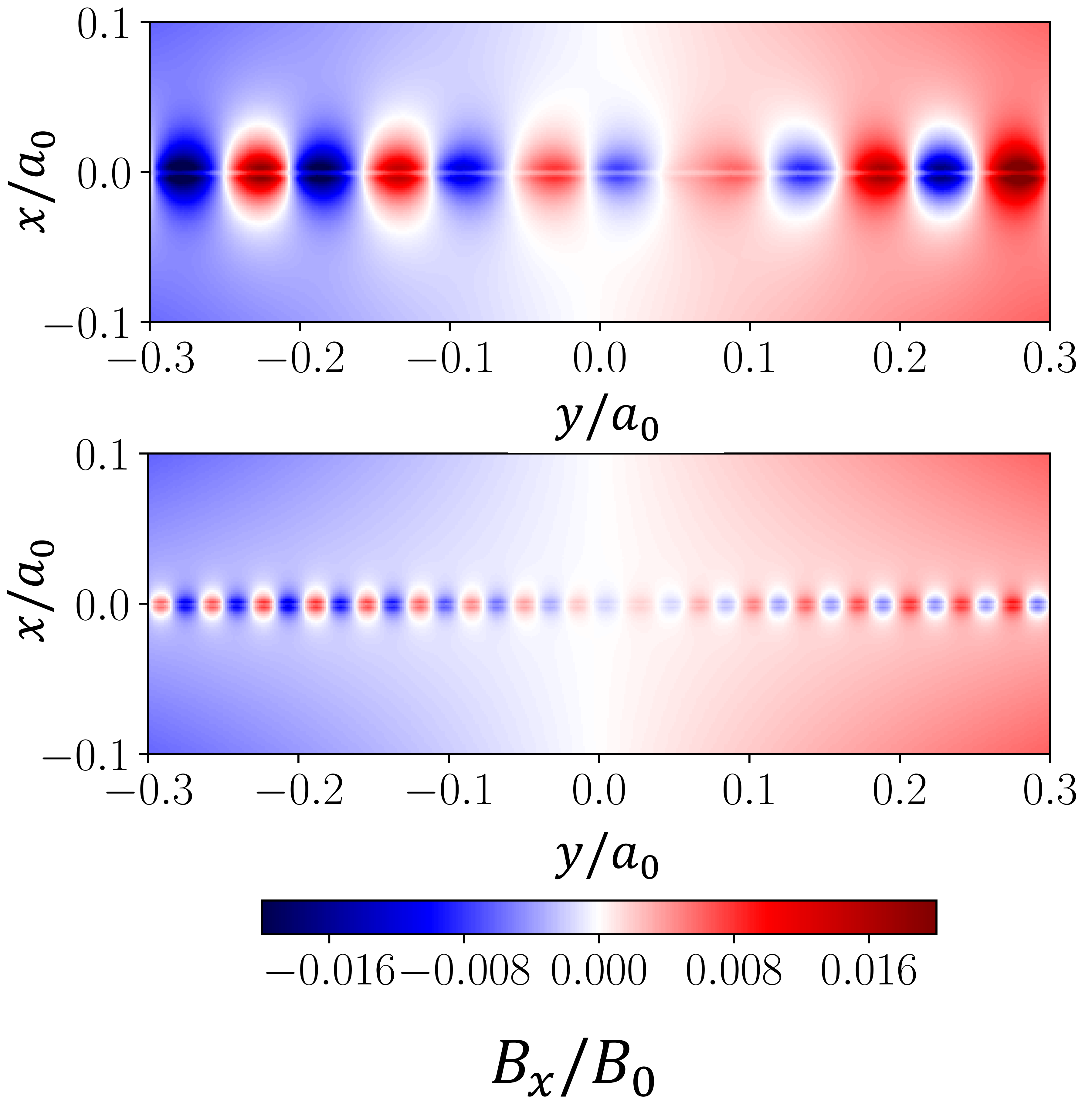
Can observe tearing eigenmodes in simulation

- Tearing first observed in profile of B_x
- We can see these profiles at right for two simulations

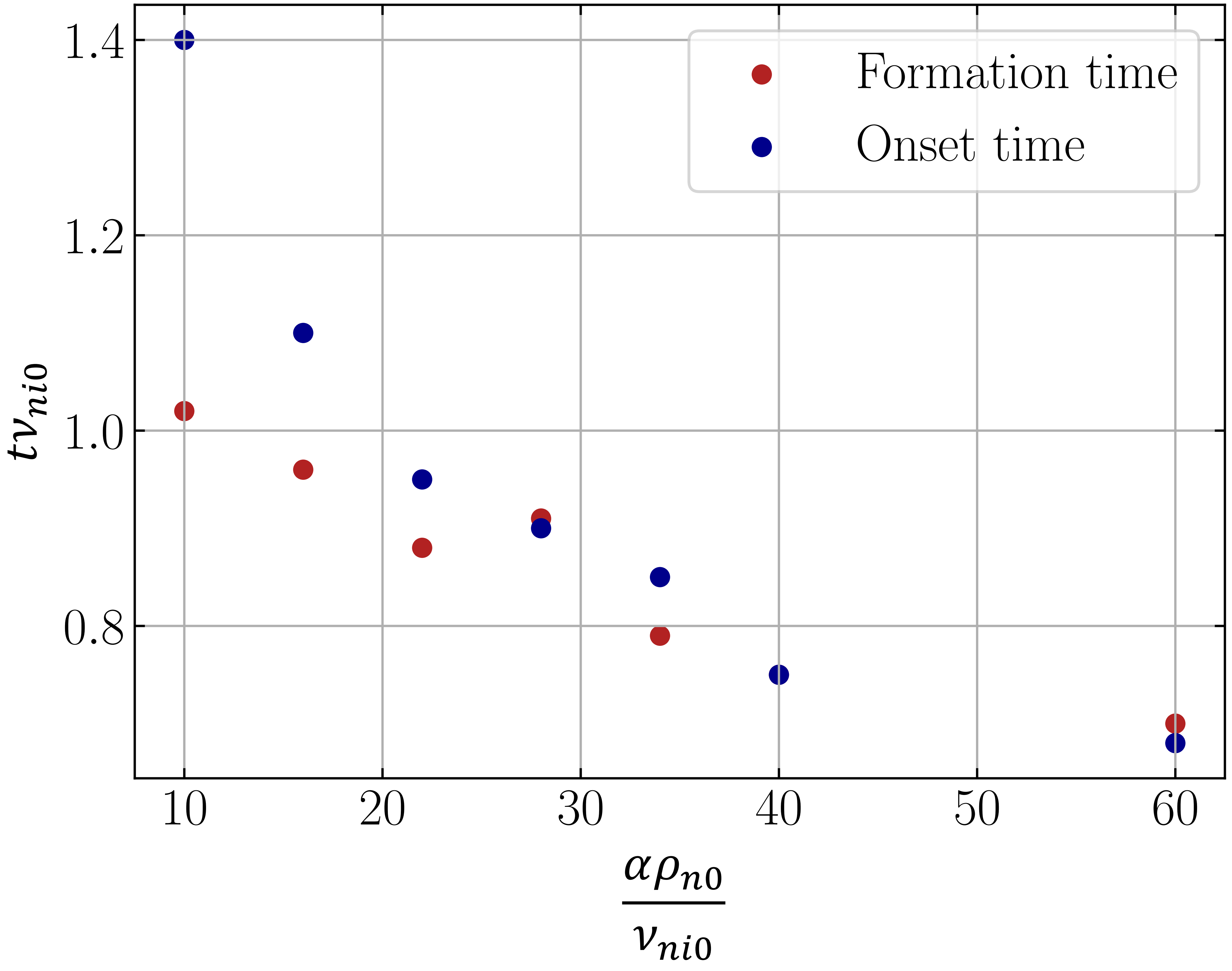
Top parameters: $\frac{\alpha \rho_{n0}}{\nu_{ni0}} = 10, \frac{\rho_{i0}}{\rho_{n0}} = 0.1$

Bottom parameters: $\frac{\alpha \rho_{n0}}{\nu_{ni0}} = 60, \frac{\rho_{i0}}{\rho_{n0}} = 0.04$

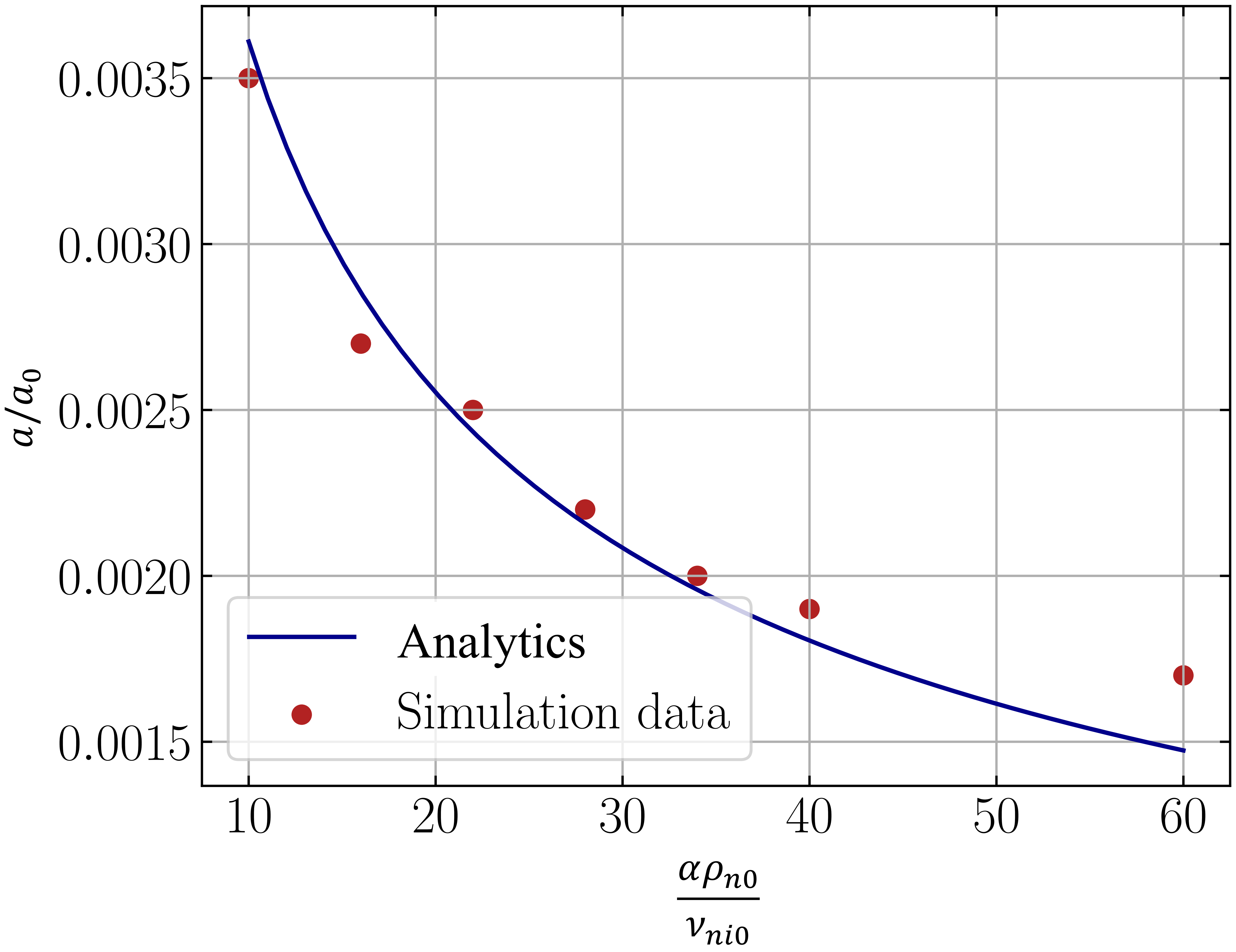
Same for both: $\xi = 0.1\, \nu_{ni0}, C_i = C_n = a_0 \nu_{ni0}, \eta = 10^{-4} a_0^2 \nu_{ni0}$



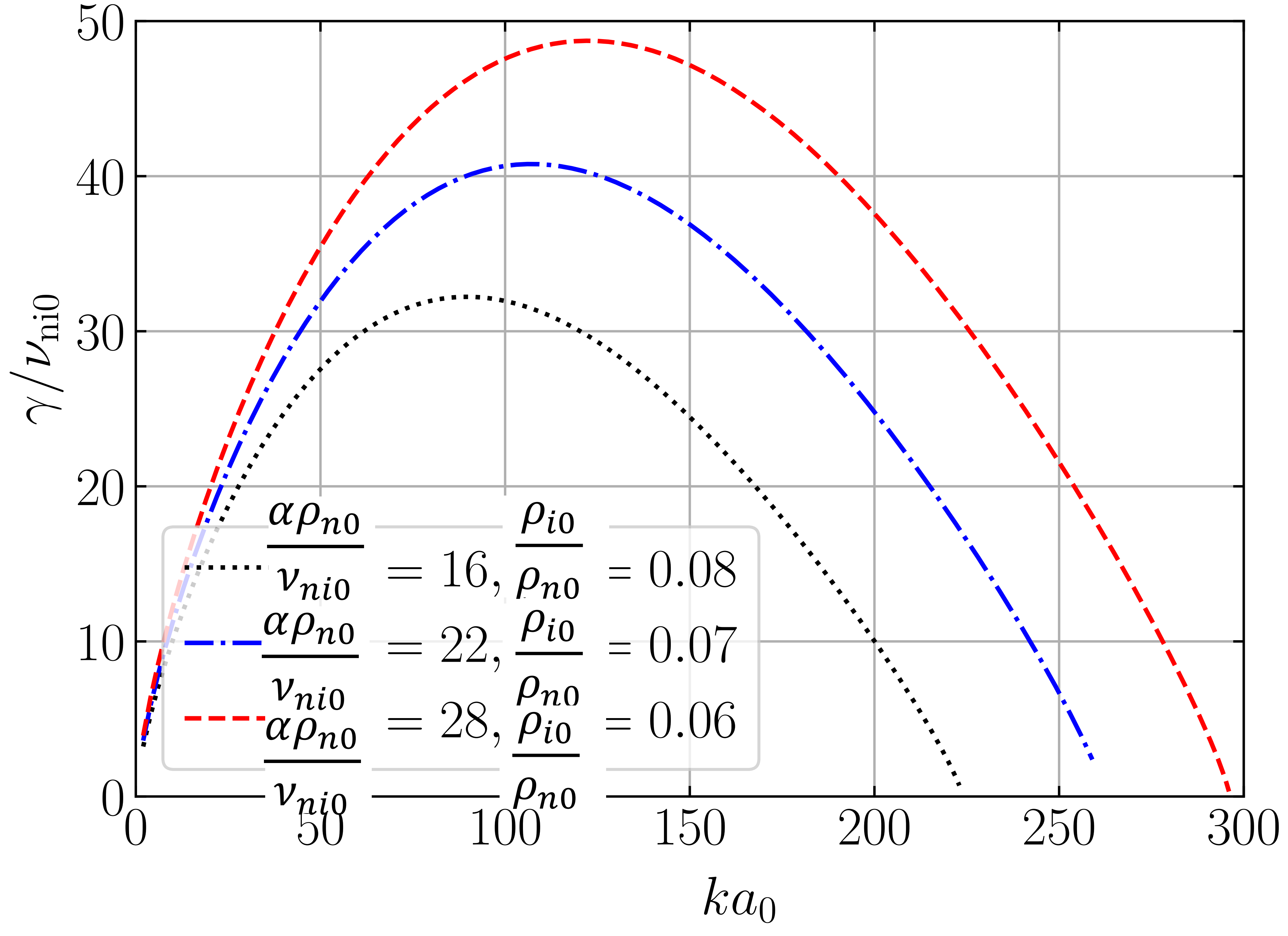
Onset time roughly corresponds to formation time

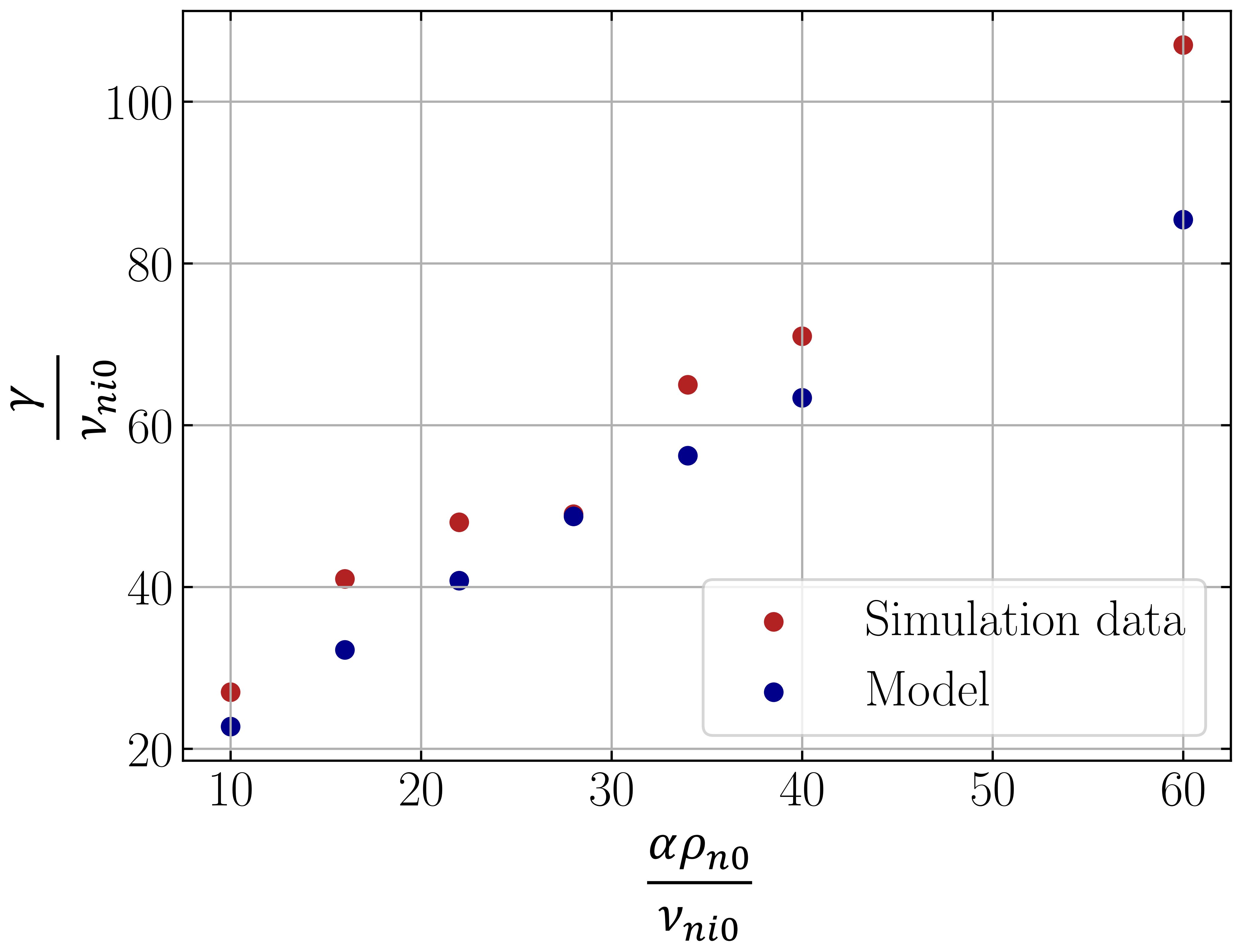


Simulation width roughly agrees with analytics

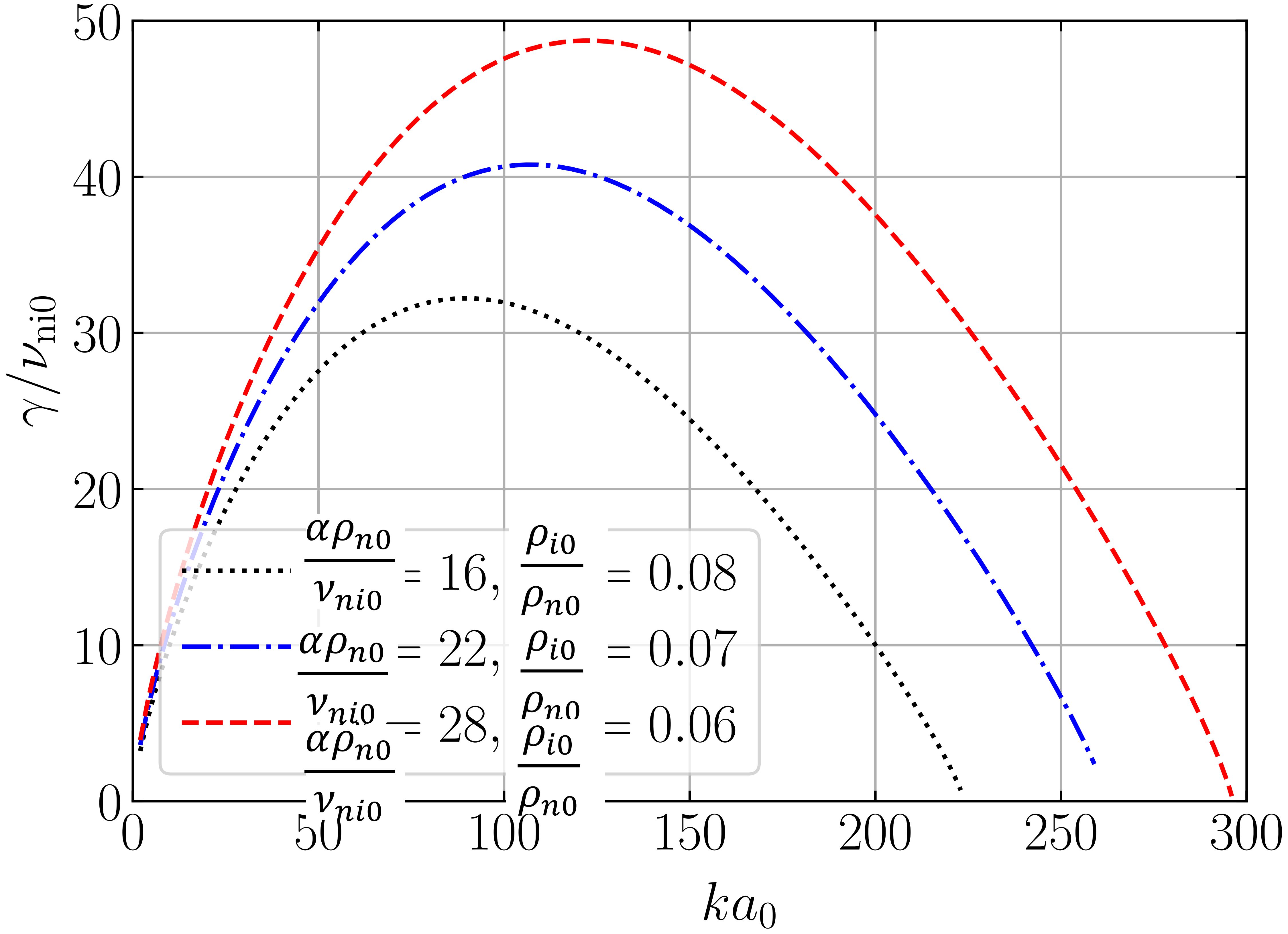


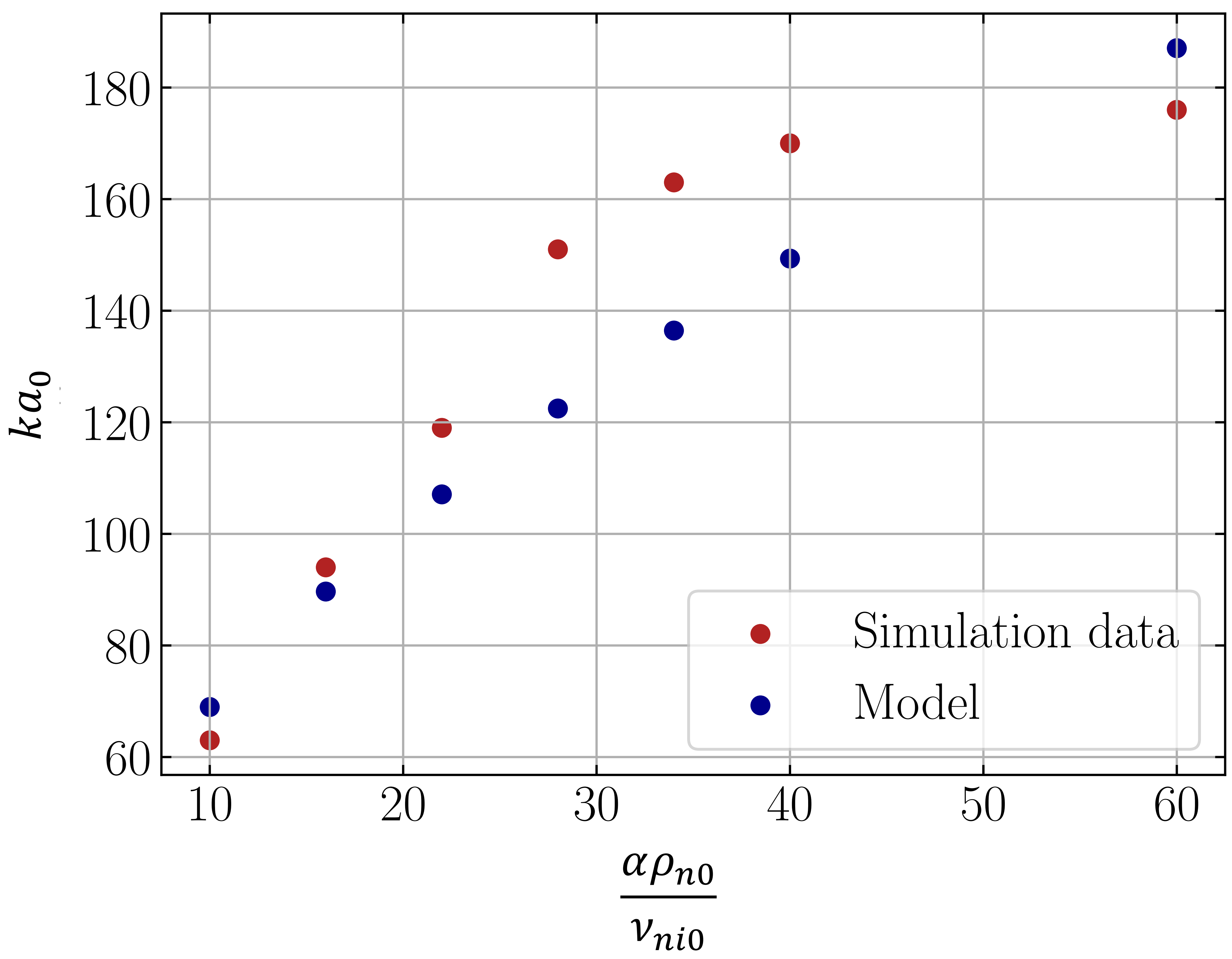
Simulation growth rates roughly agree with analytics



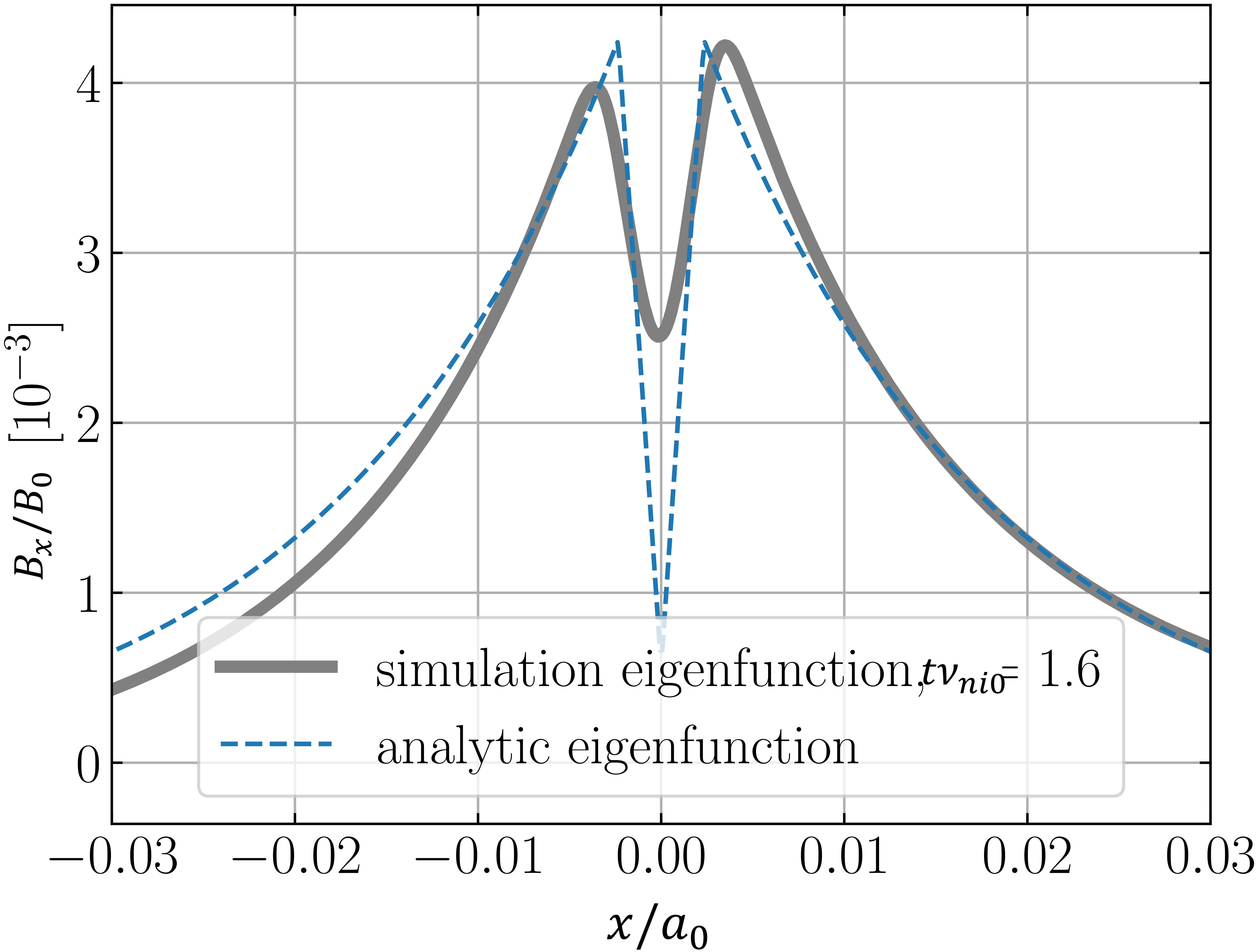


Simulation wavenumbers roughly agree with analytics





Simulation eigenfunction roughly agrees with analytics



Conclusions

- Ambipolar diffusion naturally triggers reconnection in partially ionized plasmas
- Onset time determined by time to diffuse magnetic field inwards: ν_{ni0}^{-1}
- Growth rate and wavenumber of mode that disrupts sheet can be predicted
  - Wavelength of the mode that first disrupts the forming sheet decreases as ionization fraction decreases

Based on work in preparation: stay tuned for paper!

Acknowledgements: We are grateful for helpful conversations with Amitava Bhattacharjee, Alexander Chernoglazov, Yuri Levin, Nuno Loureiro, Brian Metzger, Alexander Philippov, Bart Ripperda, George Wong, Muni Zhou, and Ellen Zweibel. E.T. was supported by the W.M. Keck Foundation Fund at the Institute for Advanced Study. Research at the Flatiron Institute is supported by the Simons Foundation.