Electric field screening in pair discharges and generation of pulsar radio emission

West Virginia University Department of Physics and Astronomy
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Based on E.A. Tolman, A.A. Philippov, and A.N. Timokhin, Electric field screening in pair discharges and generation of pulsar radio emission, ApJL 933 L37 (2022)., arXiv:2202.01303.

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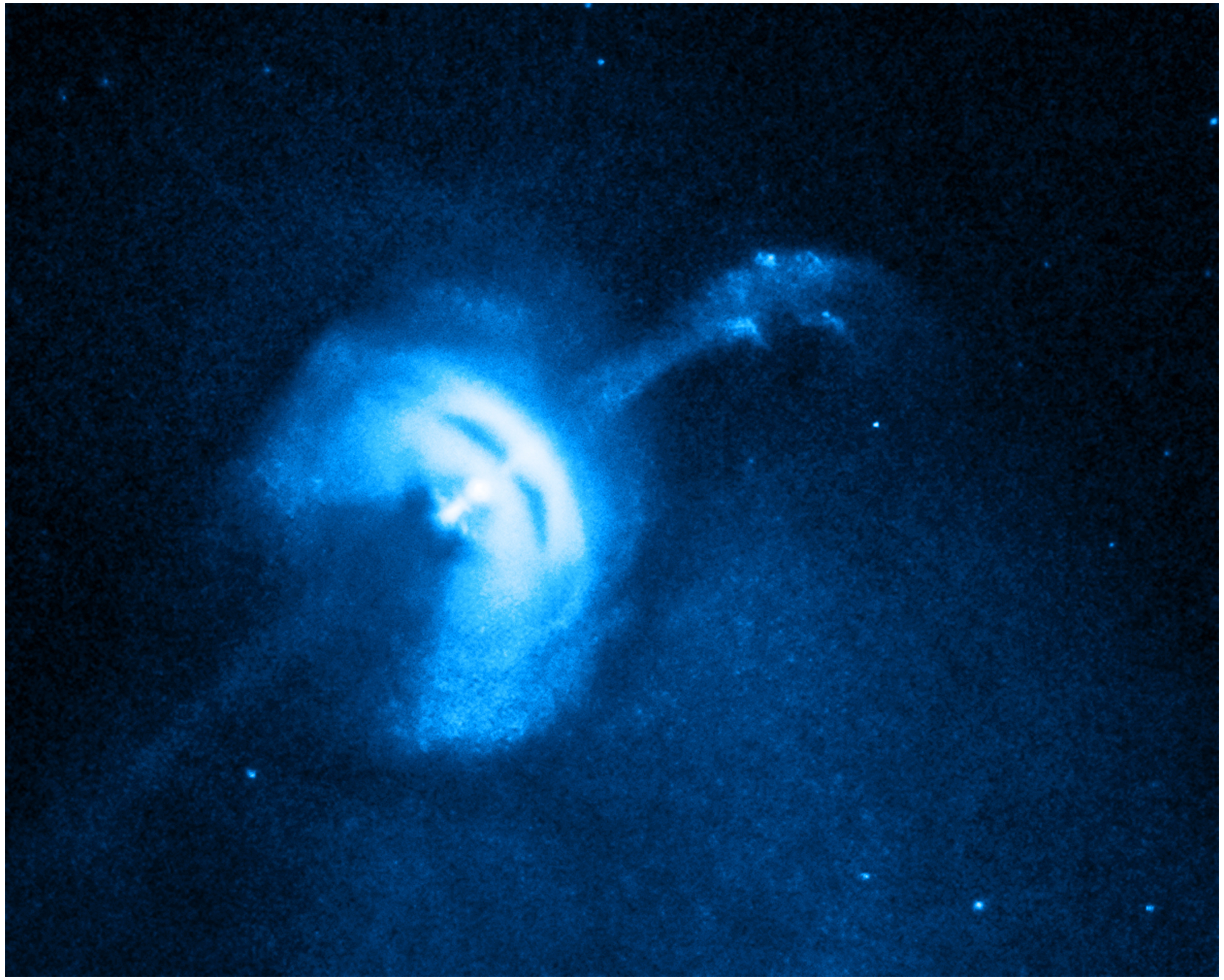
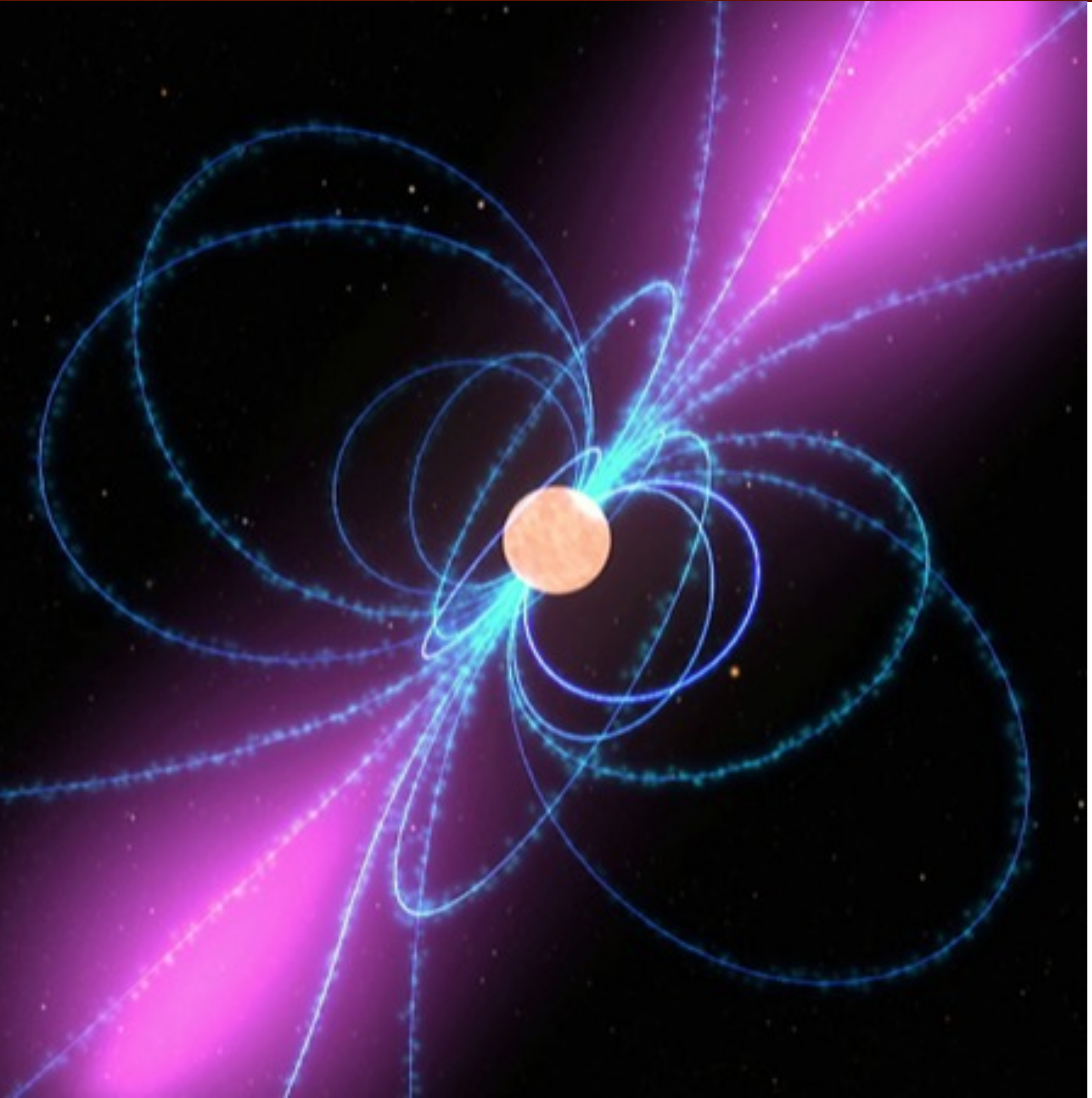
Pulsars are rapidly rotating, magnetized neutron stars



Vela pulsar, in X-rays
Image source: Chandra X-ray Observatory Center
Credit: NASA/CXC/Univ of Toronto/M.Durant et al

- A pulsar is a highly magnetized, rapidly rotating neutron star
  - 10 km radius
  - 1.4 x mass of sun
  - Magnetic field of about 10^{12} G
  - Spin period of 1.4 ms to 8.5 s
- Called a pulsar because it emits pulses of radio emission
- About 3,000 known pulsars

Pulsars emit coherently in radio from polar cap



- Several aspects of pulsar physics are interesting
- One is the coherent radio emission produced in the polar cap
- Many aspects of this emission are unexplained
  - We focus on two in this talk:
    - Luminosity
    - Spectrum

Image source: NASA

Radio luminosity is unexplained

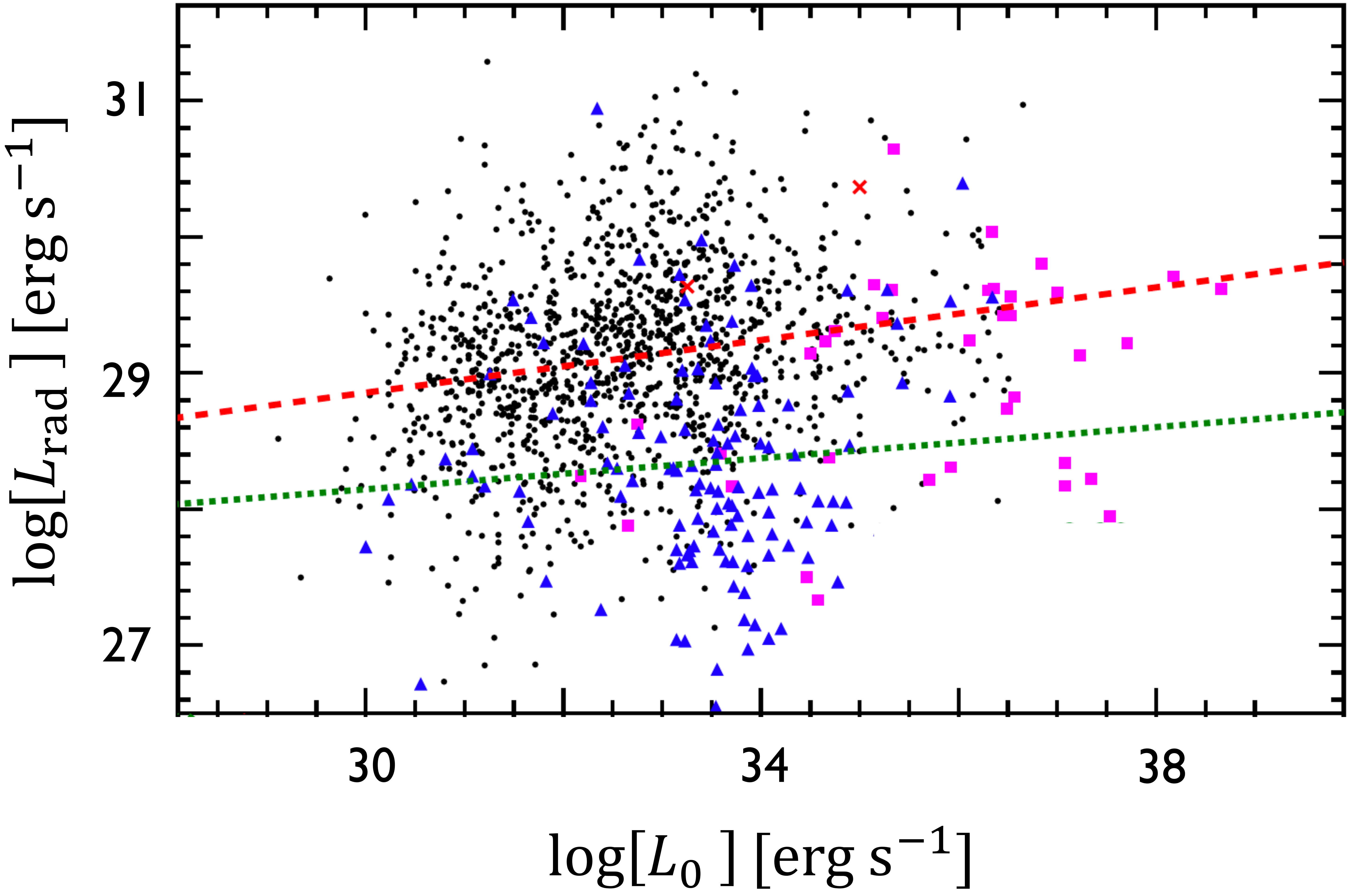
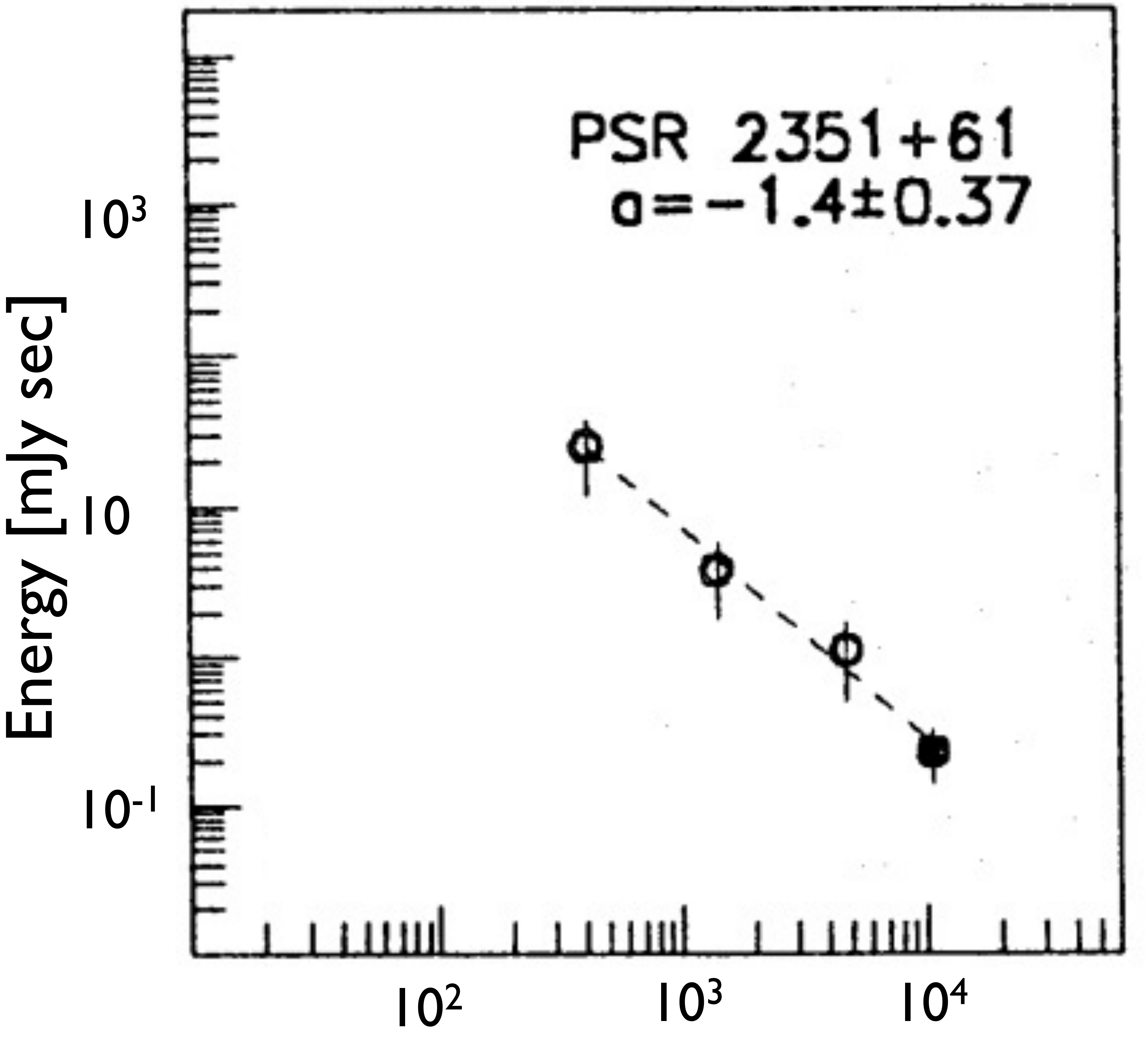


Image source: Szary et al., ApJ 2014

- Radio luminosity $L_{\rm rad}$ has magnitude

$$L_{\rm rad} \sim 10^{27} - 10^{31}\ \text{erg s}^{-1}$$

Pulsar radio spectrum is $S_\omega \sim \omega^{-1.4\pm1.0}$



Frequency [MHz]

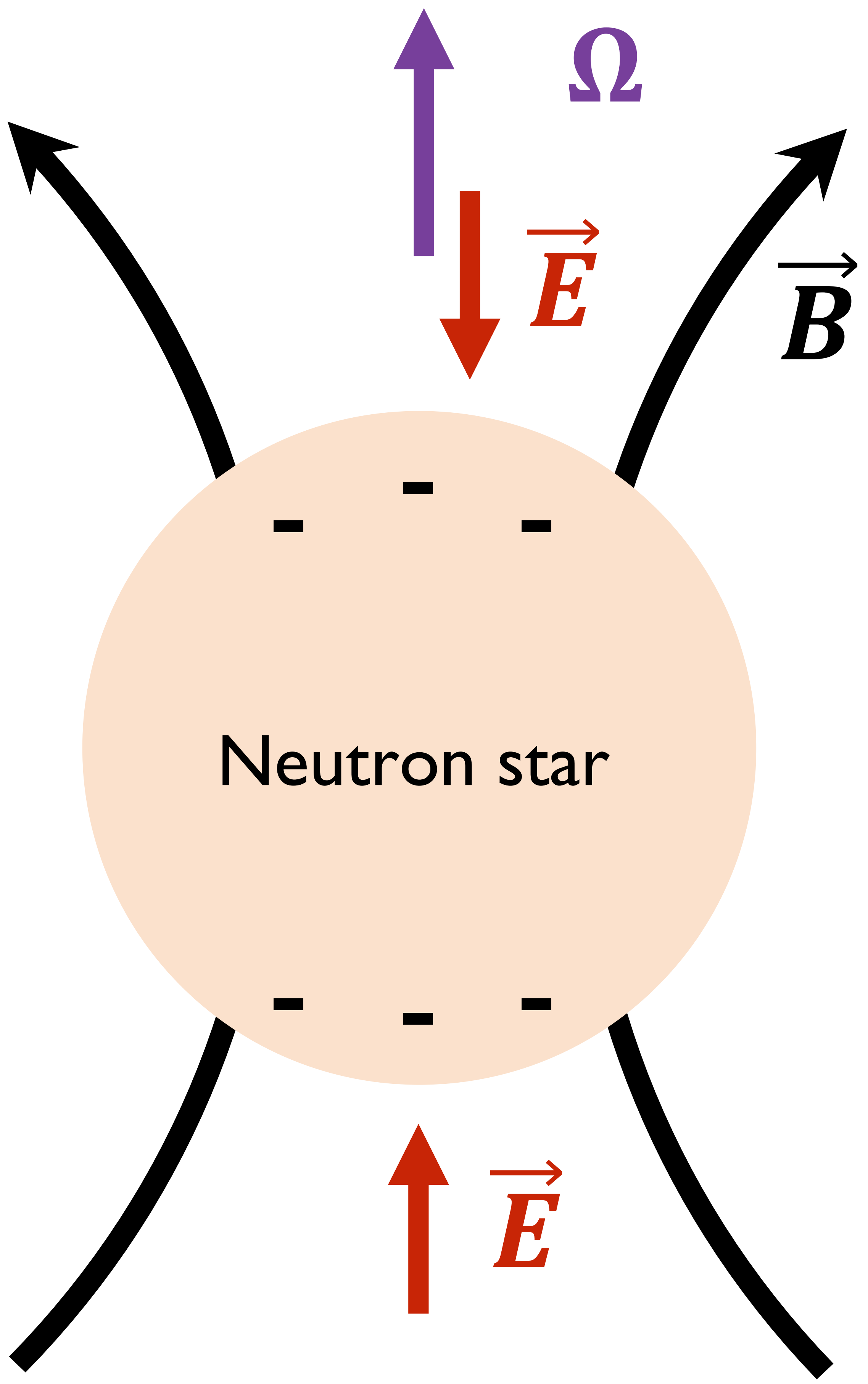
Image source: Malofeev et al., A&A 1994

- Typical radio spectrum across several pulsar observations is [Bates et al. MNRAS 2013]:

$$S_\omega \sim \omega^{-1.4\pm1.0}$$

To understand these aspects of the radio emission, let’s consider what’s happening in the region where the emission is produced.

A pulsar is surrounded by plasma



- Rotating B field causes polarization of conducting neutron star
  - Same mechanism as Faraday generator
- Causes strong E field outside of star
- E field extracts charges from star surface, causes creation of additional charges via cascade (described later)
- Results in creation of plasma around star

Pulsar plasma has several interesting parts

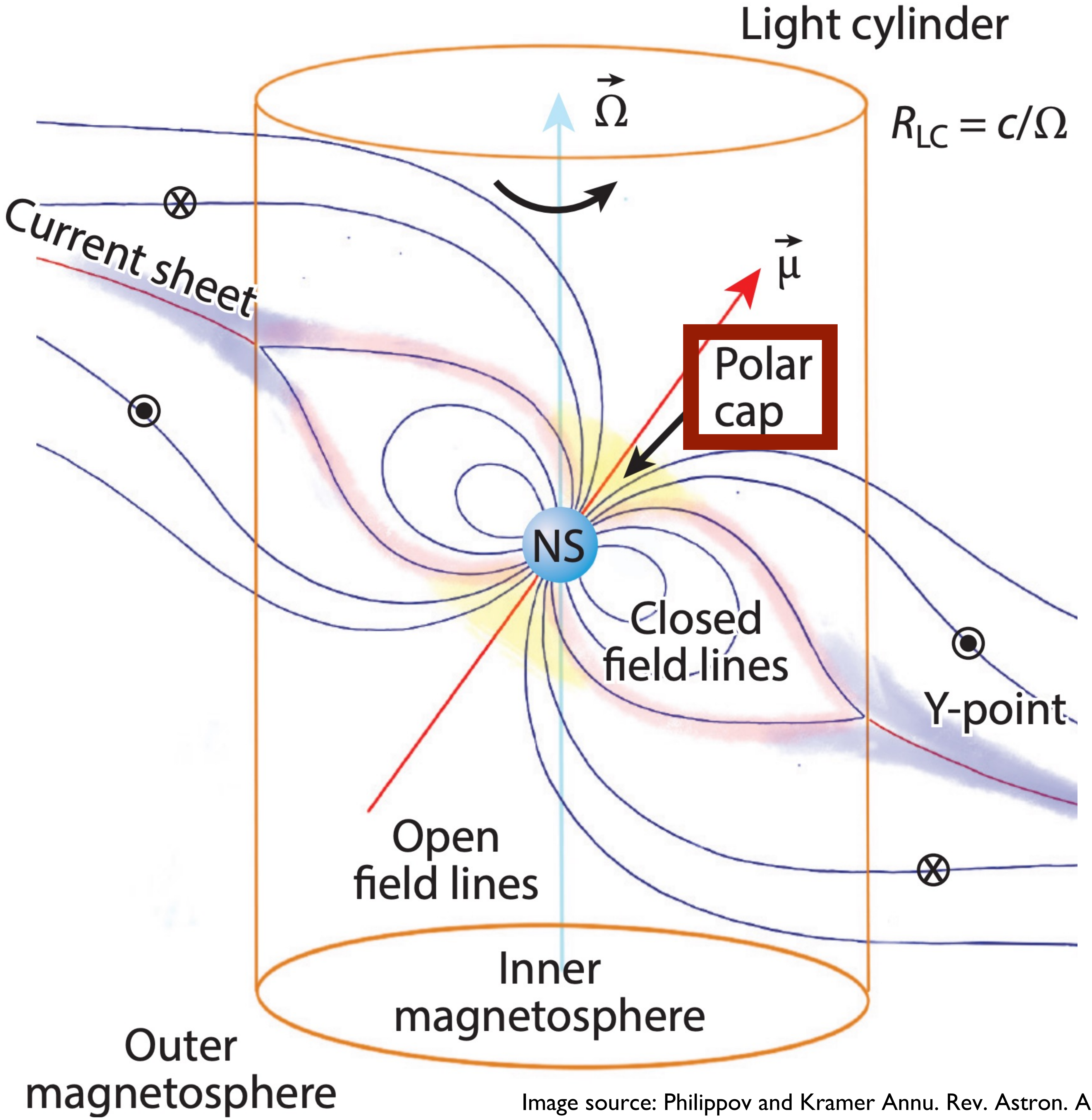


Image source: Philippov and Kramer Annu. Rev. Astron. Astrophys. 2022

- Plasma magnetosphere surrounding pulsar has complex structure, several interesting phenomena
- Radio emission is produced in the polar cap
- Magnetic field lines extend to infinity
- Plasma periodically created and leaves

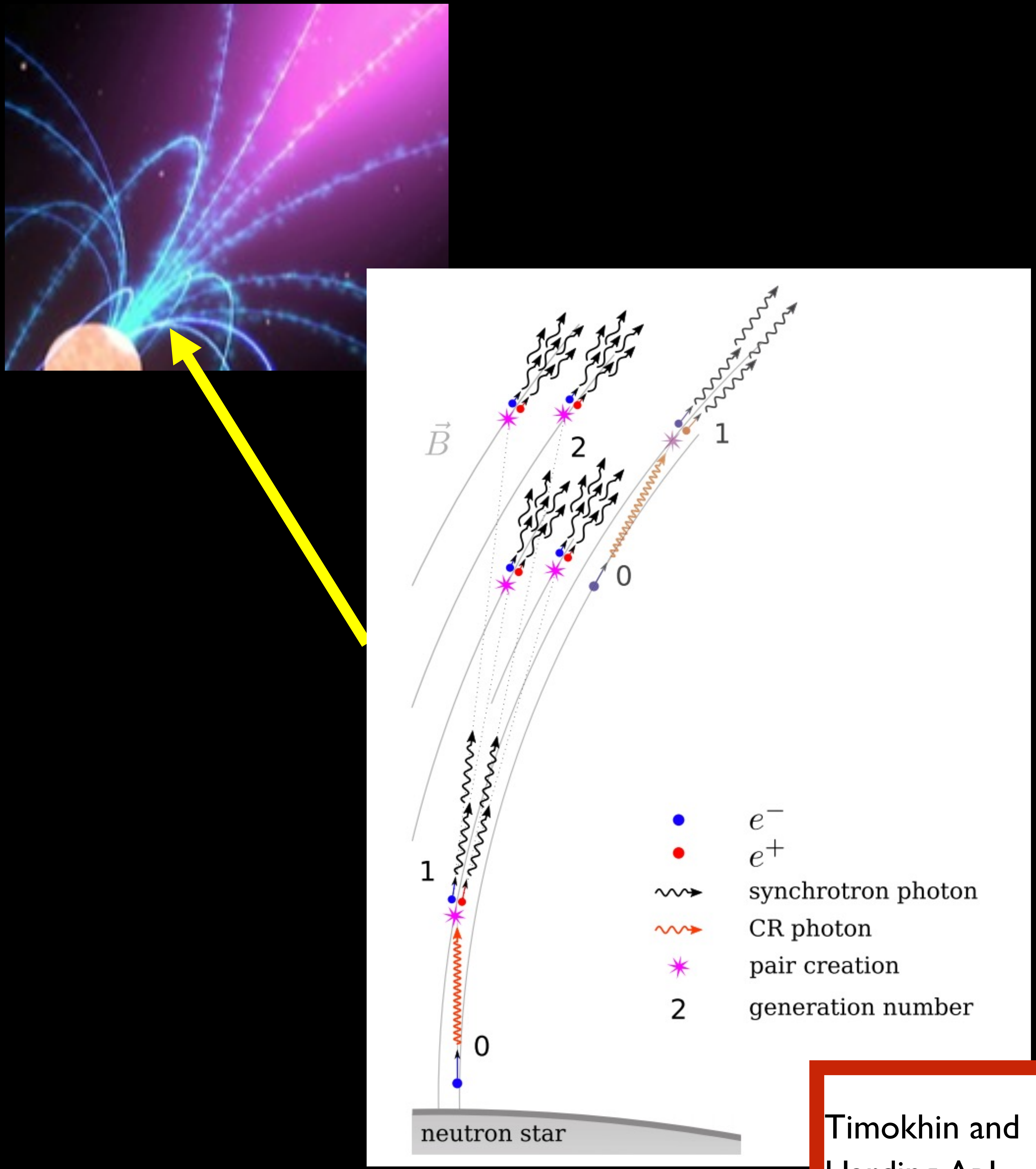
Outline

- This talk will give insight into radio luminosity and spectrum
- The talk has 2 parts:

1. The polar cap discharge where radio emission may be produced through screening of the electric field

2. The three important (and one unimportant) stages of electric field evolution in the polar cap discharge, with implications for observation of each:
    - Screening
    - Nonlinear
    - Frozen
    - Linear

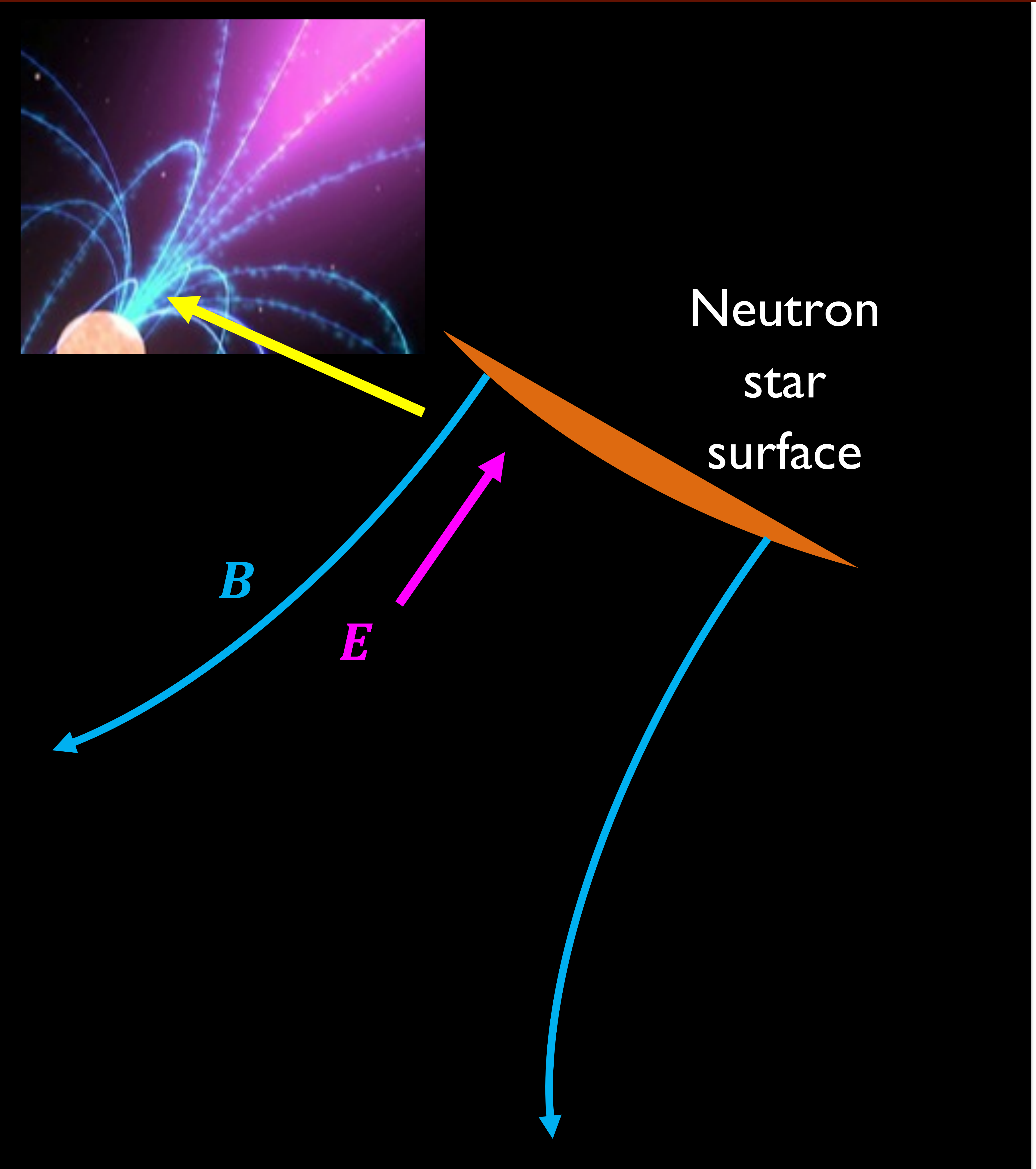
Pair discharge in polar cap may create radio emission



Timokhin and Harding ApJ 2015

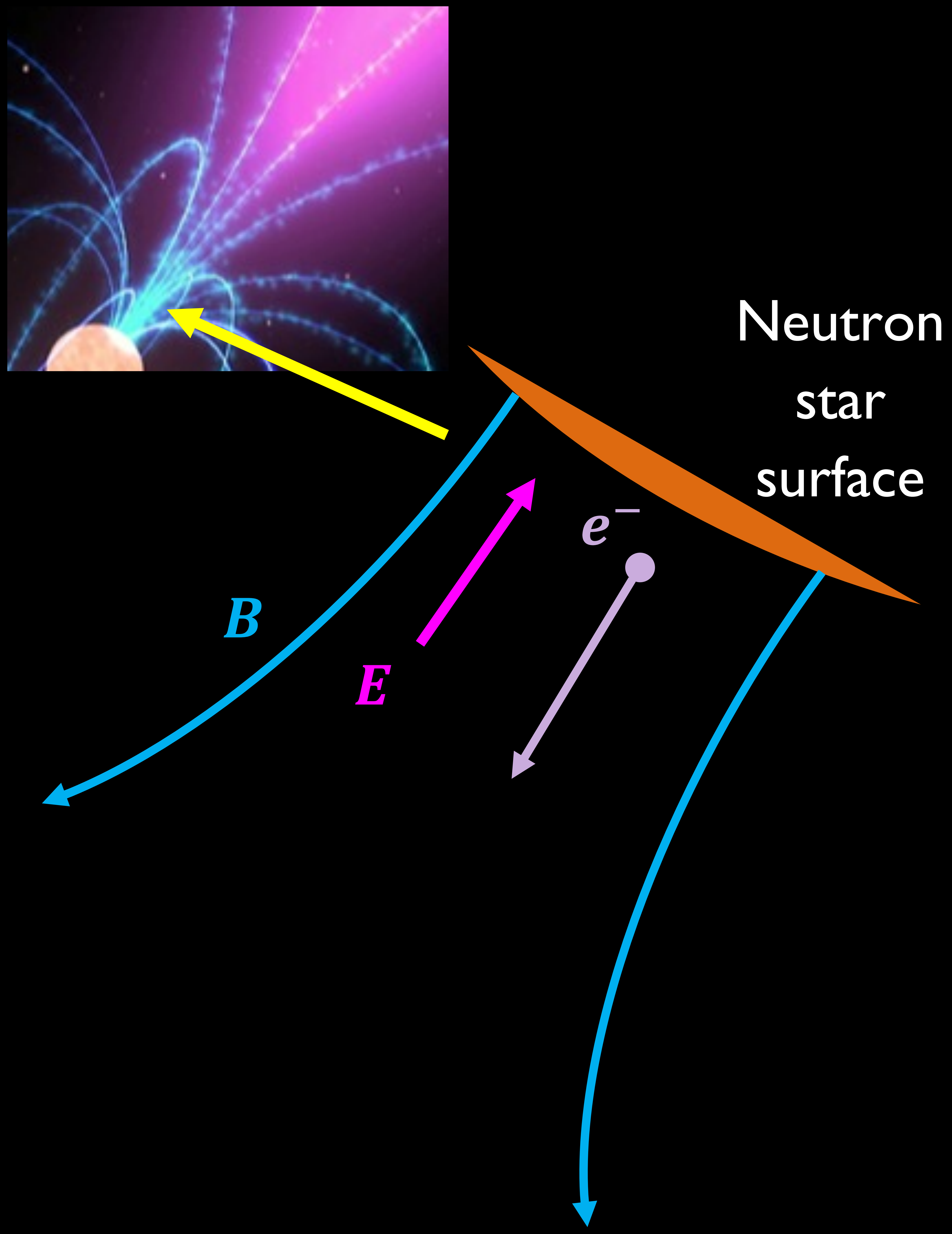
- Polar cap has strong inductive E field which creates complicated pair discharge

Pair discharge in polar cap may create radio emission



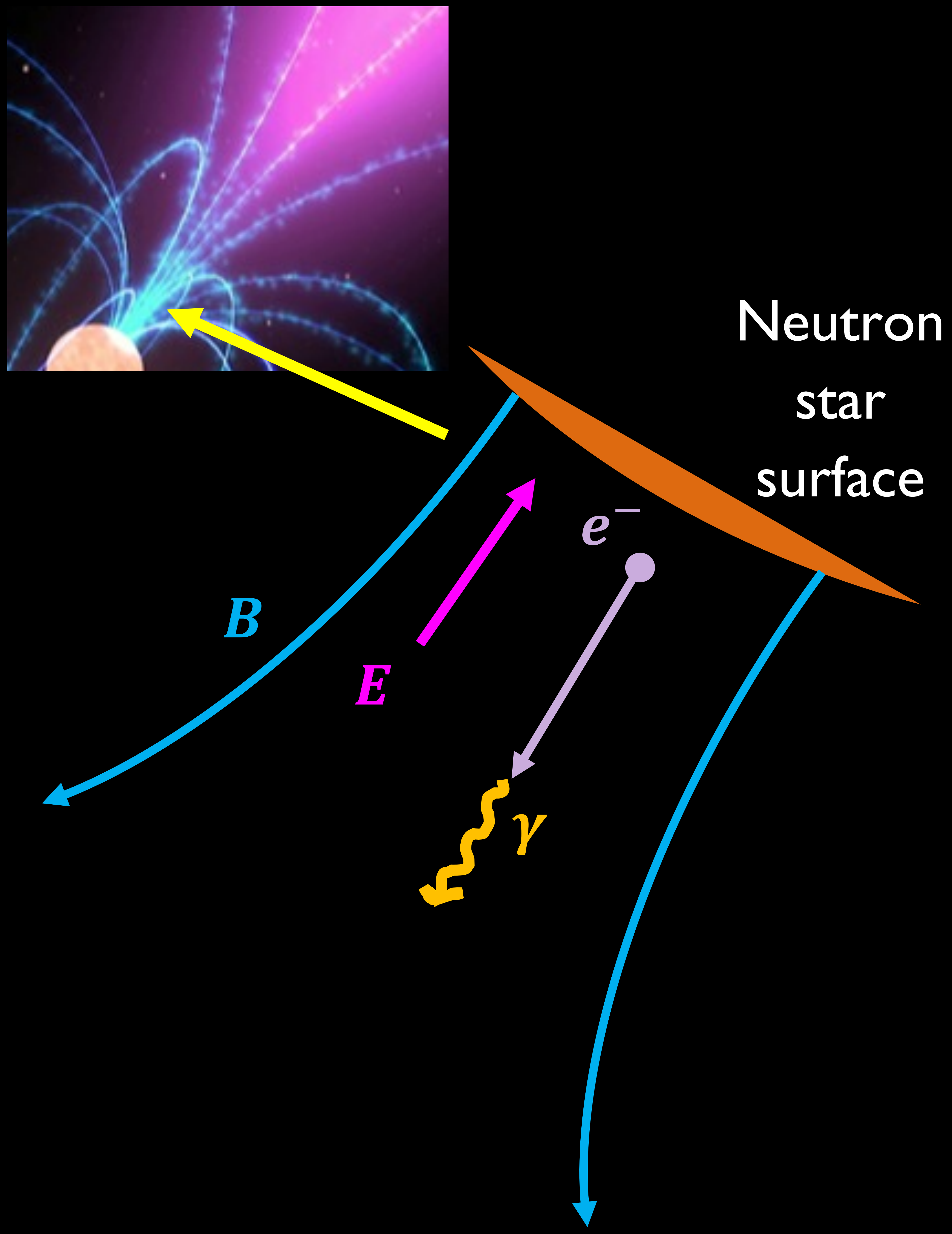
- Let’s take a simplified model of the discharge:

Pair discharge in polar cap may create radio emission



- Let's take a simplified model of the discharge:
  1. E field accelerates e- from surface to $\gamma \sim 10^7$

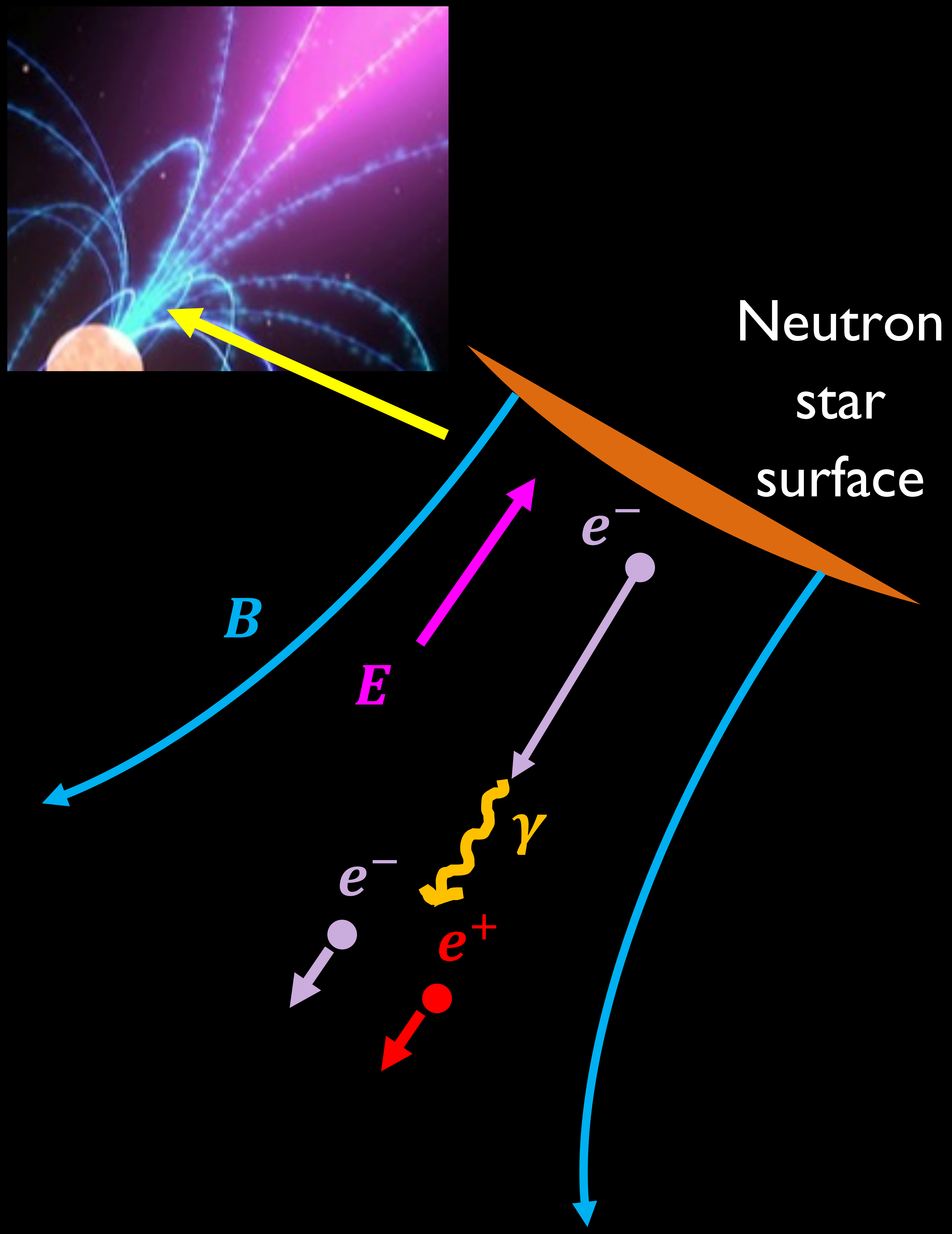
Pair discharge in polar cap may create radio emission



- Let’s take a simplified model of the discharge:

1. E field accelerates e- from surface to $\gamma \sim 10^7$
2. Primary e- continually curvature radiate gamma rays

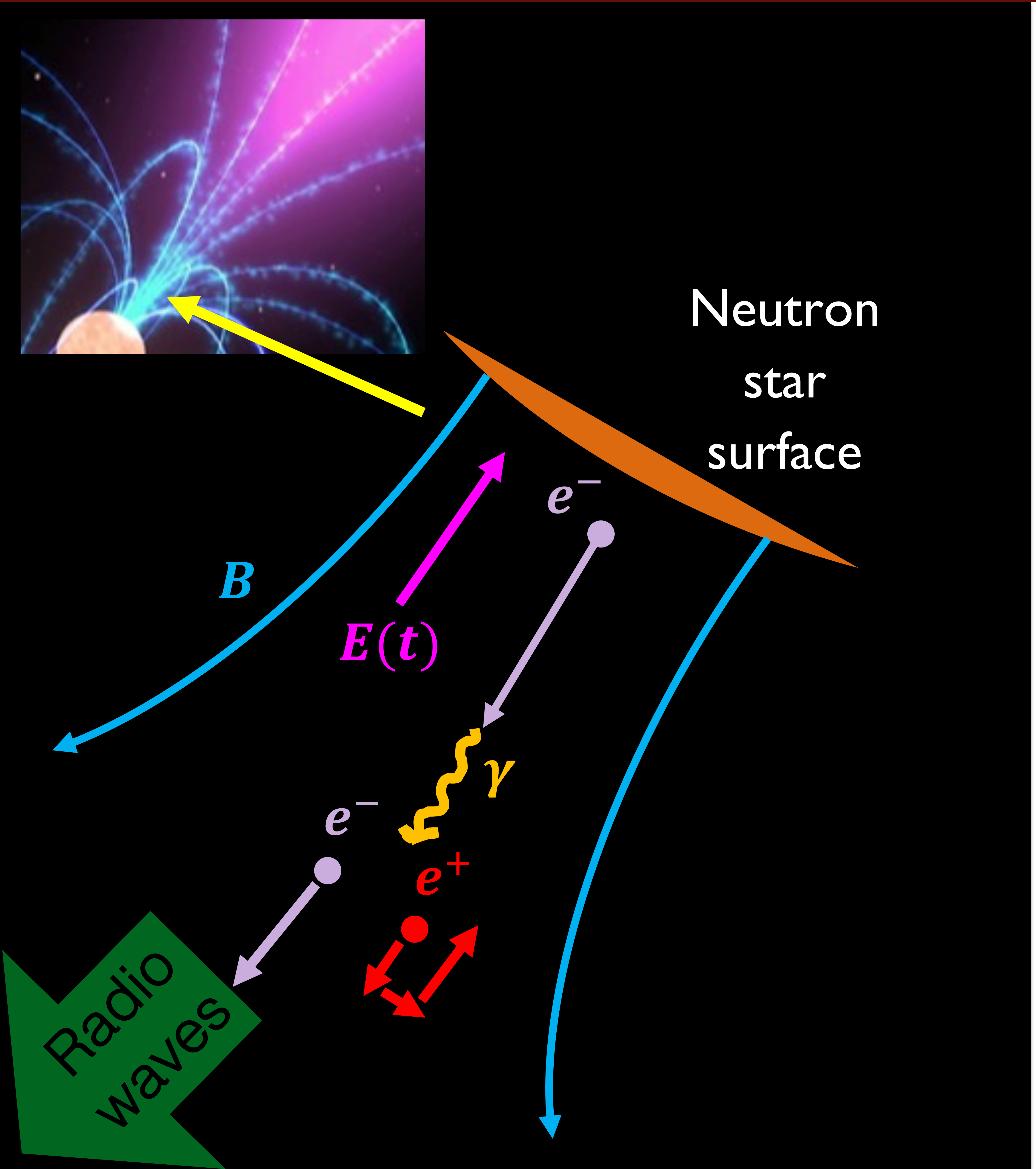
Pair discharge in polar cap may create radio emission



- Let's take a simplified model of the discharge:

1. E field accelerates e- from surface to $\gamma \sim 10^7$
2. Primary e- continually curvature radiate gamma rays
3. Gamma rays are absorbed in magnetic field
4. QED process continually creates lower energy $\gamma \sim 10^2$ pairs

Pair discharge in polar cap may create radio emission



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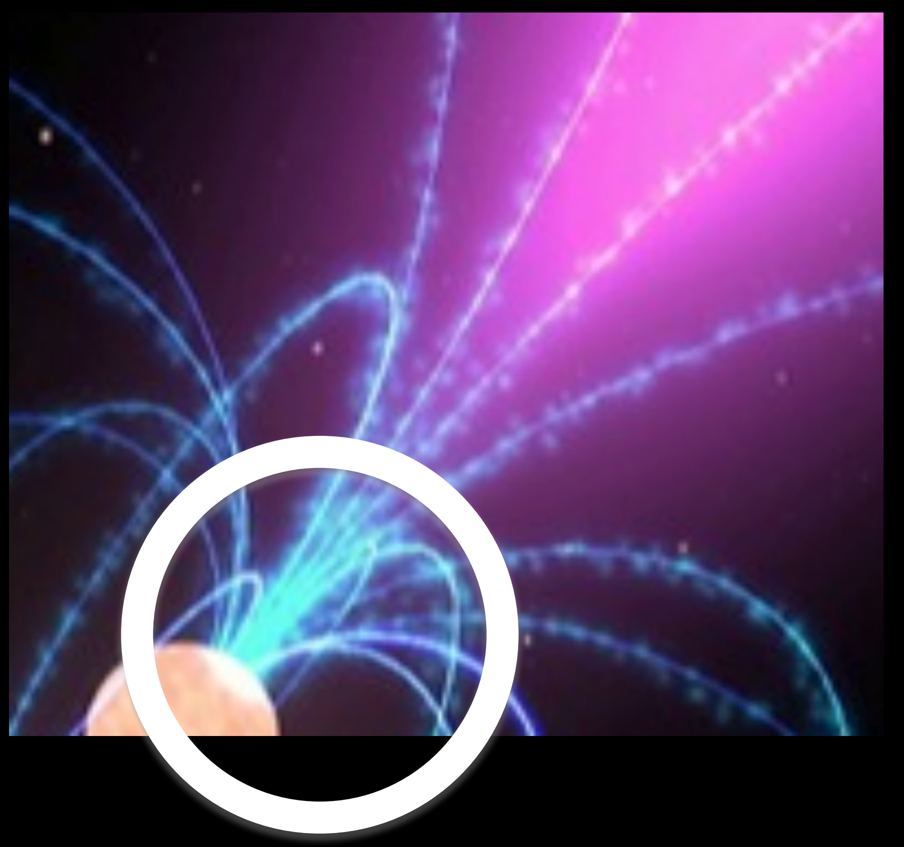
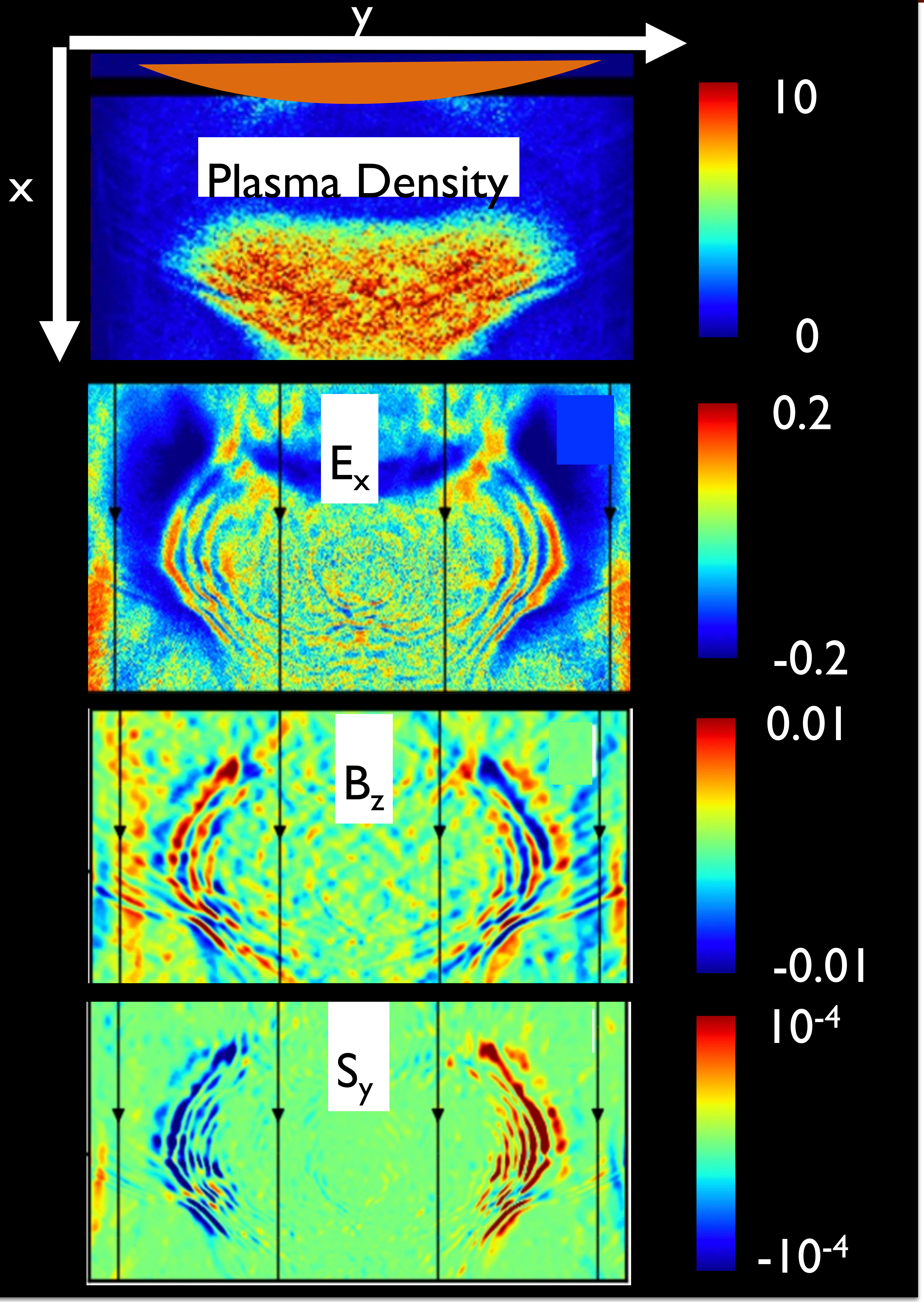
- Continuously created $\gamma \sim 10^2$ pairs screen E, set up waves that are damped by more pairs

- Damped waves become radio emission [Philippov et al. 2020, others]

Parallel dynamics control evolution of radiation





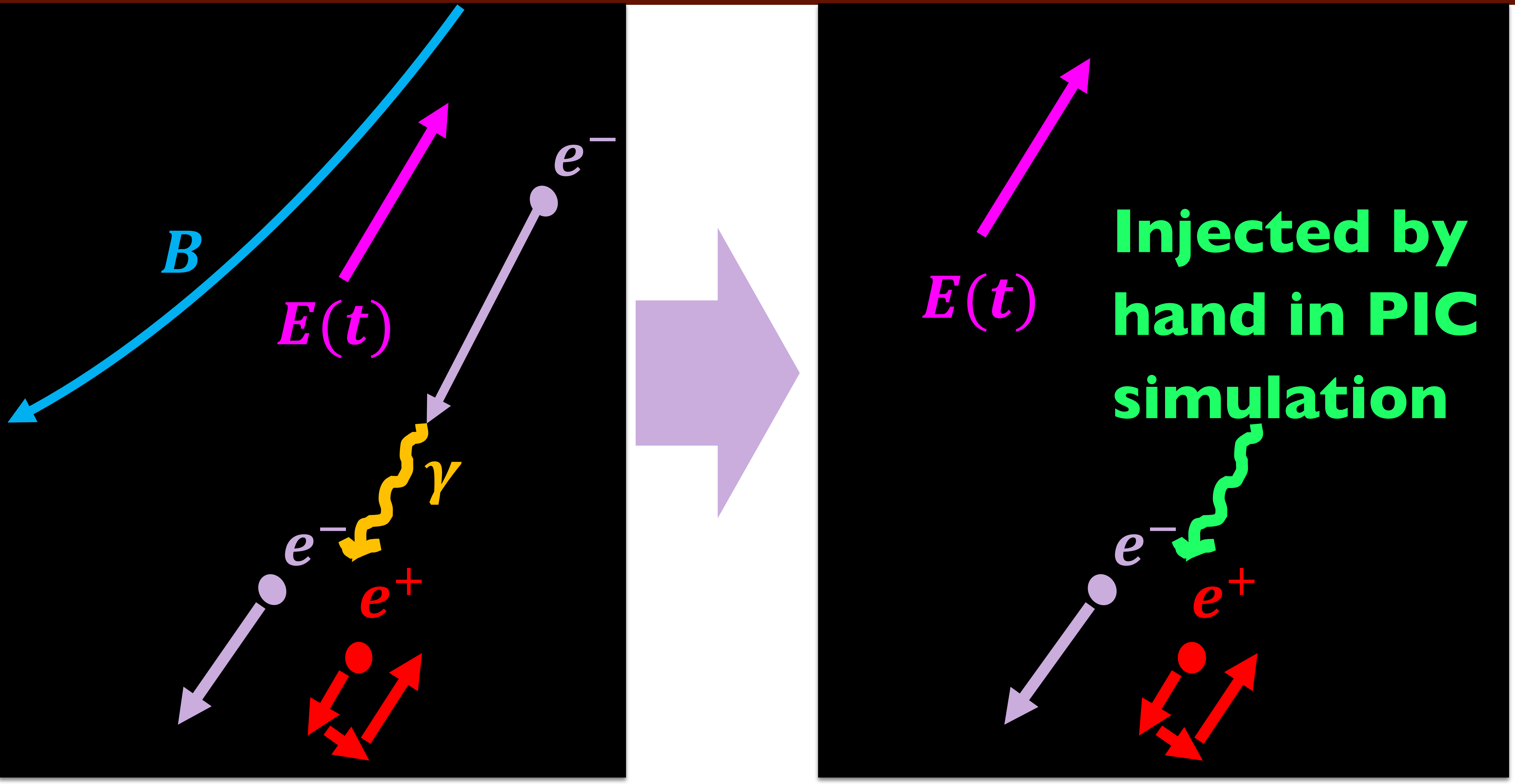
- 2D simulation of polar cap plasma shows EM fluctuations, Poynting flux that escapes plasma as light [Philippov et al. 2020]

- Pairs only move along magnetic field (in x direction), so time evolution of fluctuations is controlled by parallel dynamics:

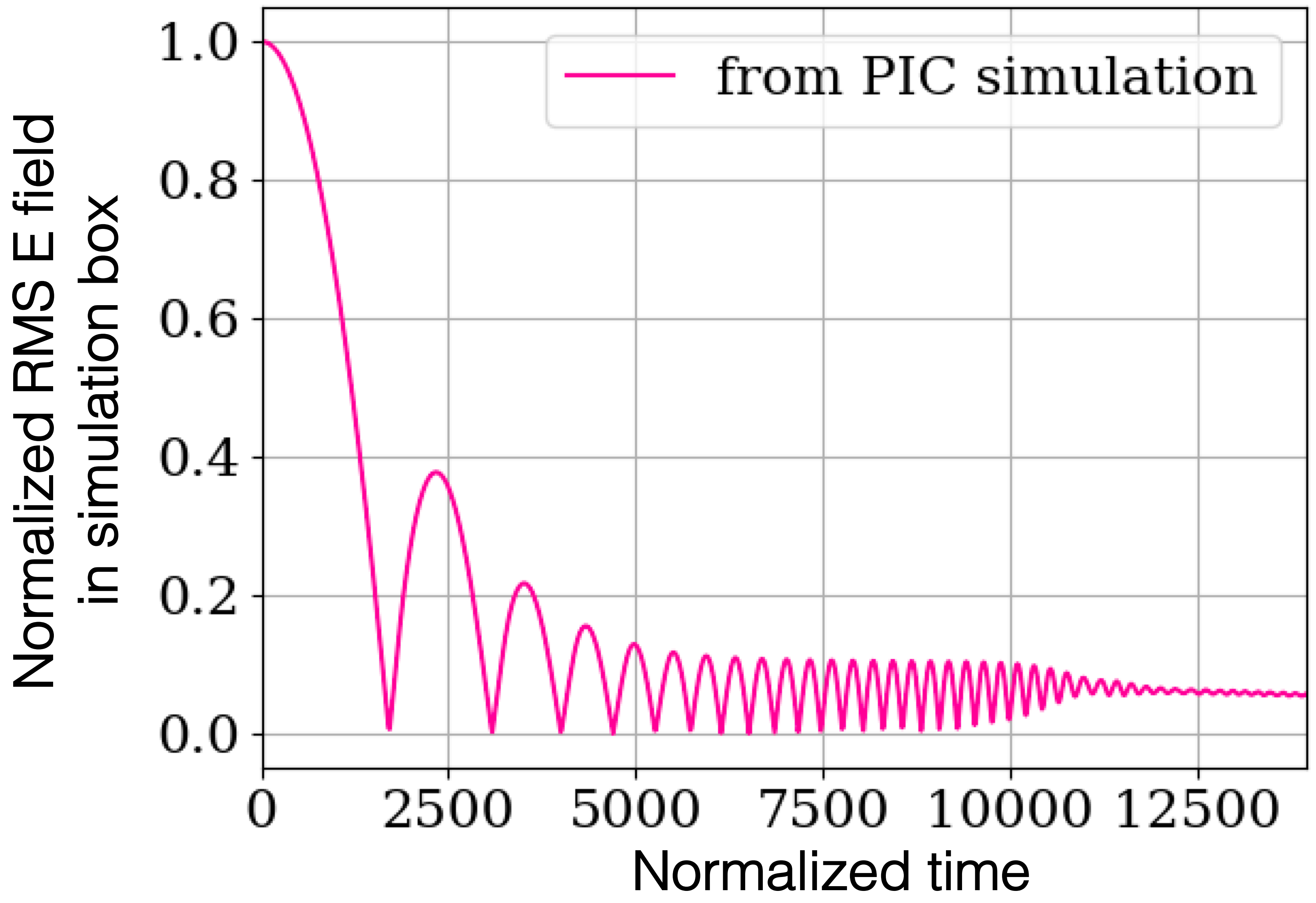
$$\partial_y B_z = \frac{4\pi}{c} j_x + \frac{1}{c} \partial_t E_x$$

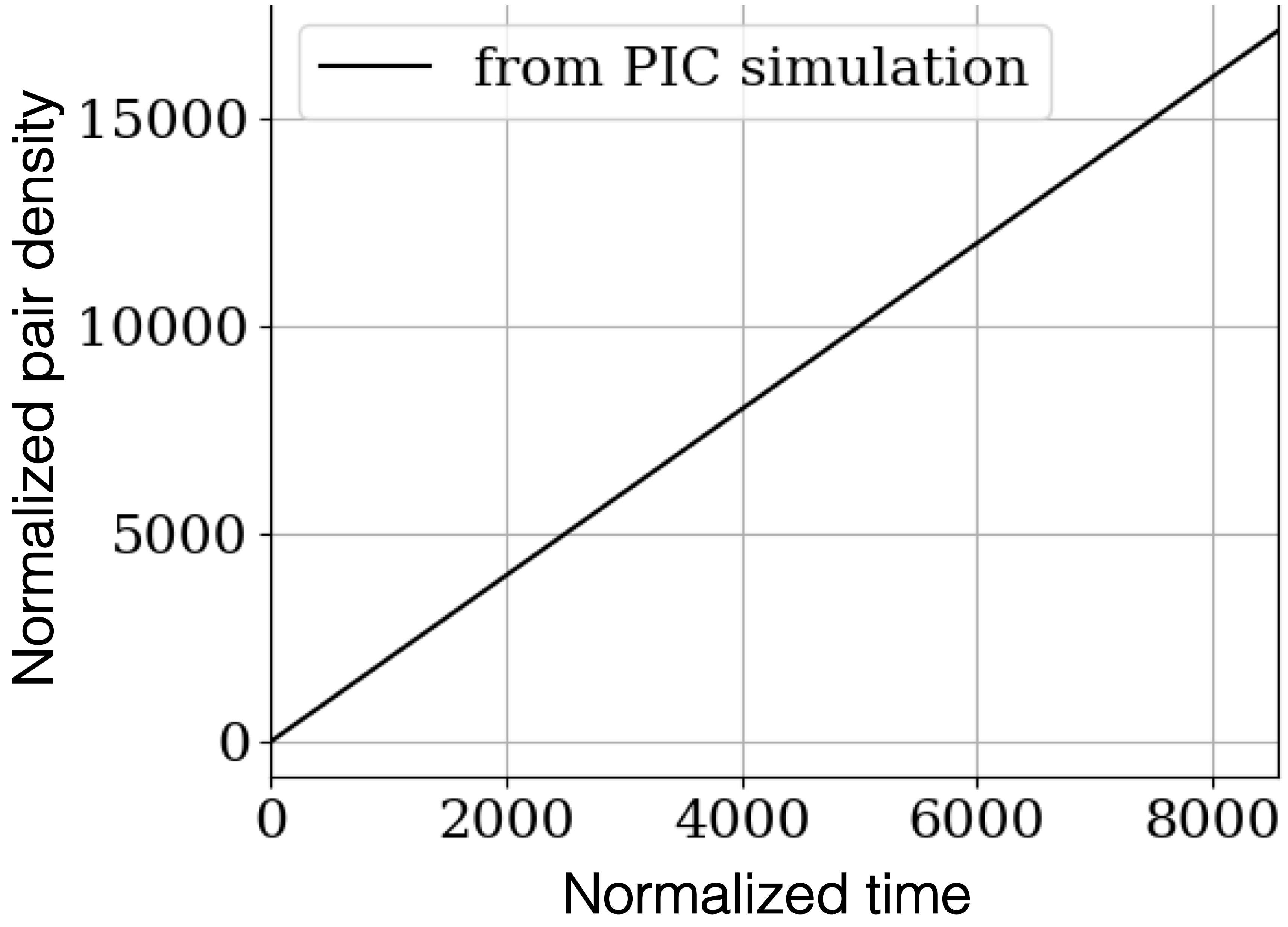
- Evolution of radiation amplitude can be studied in 1D by looking at interaction of parallel E field with newly added pairs

Simplified setup captures essentials of wave evolution



Injected by hand in PIC simulation



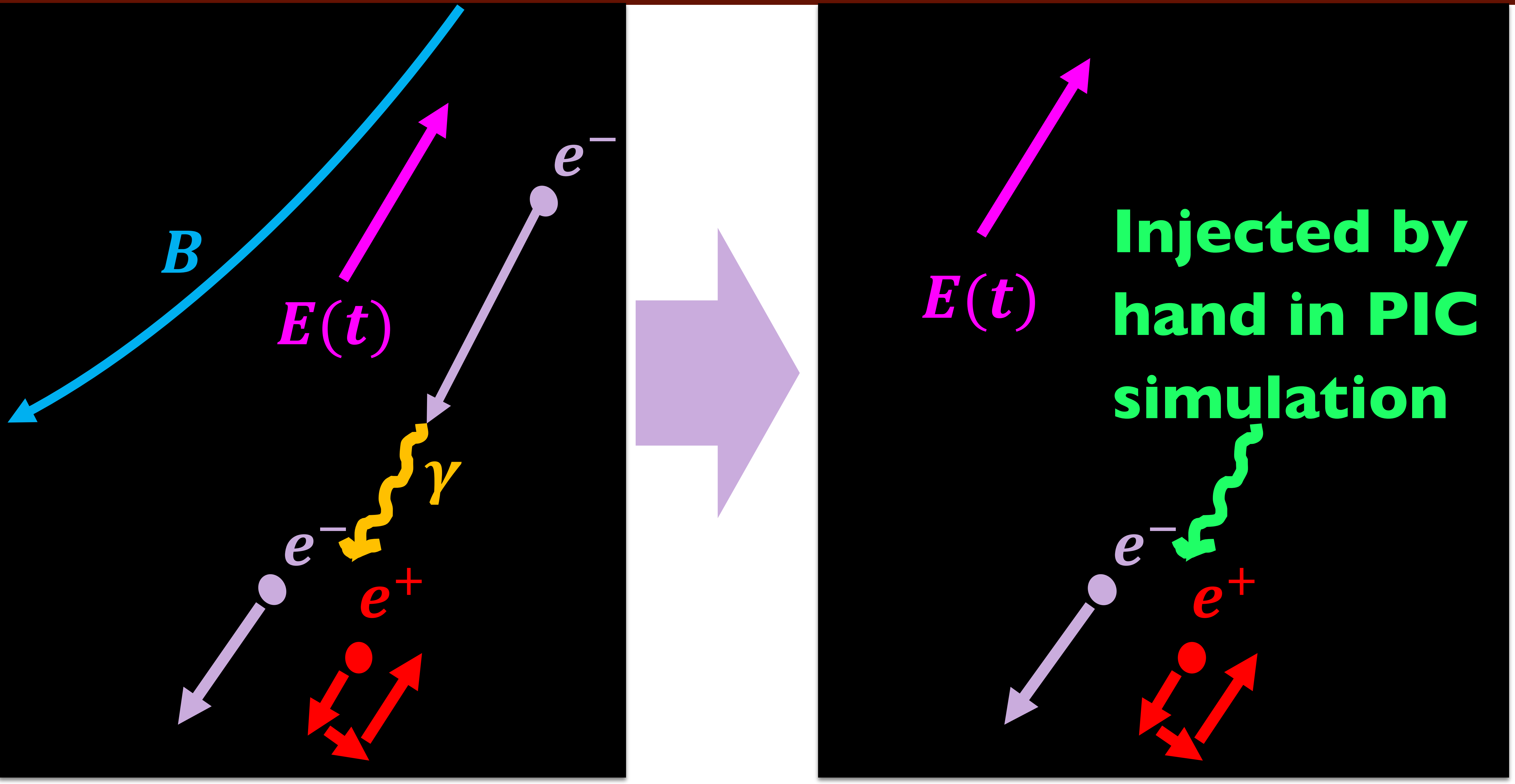


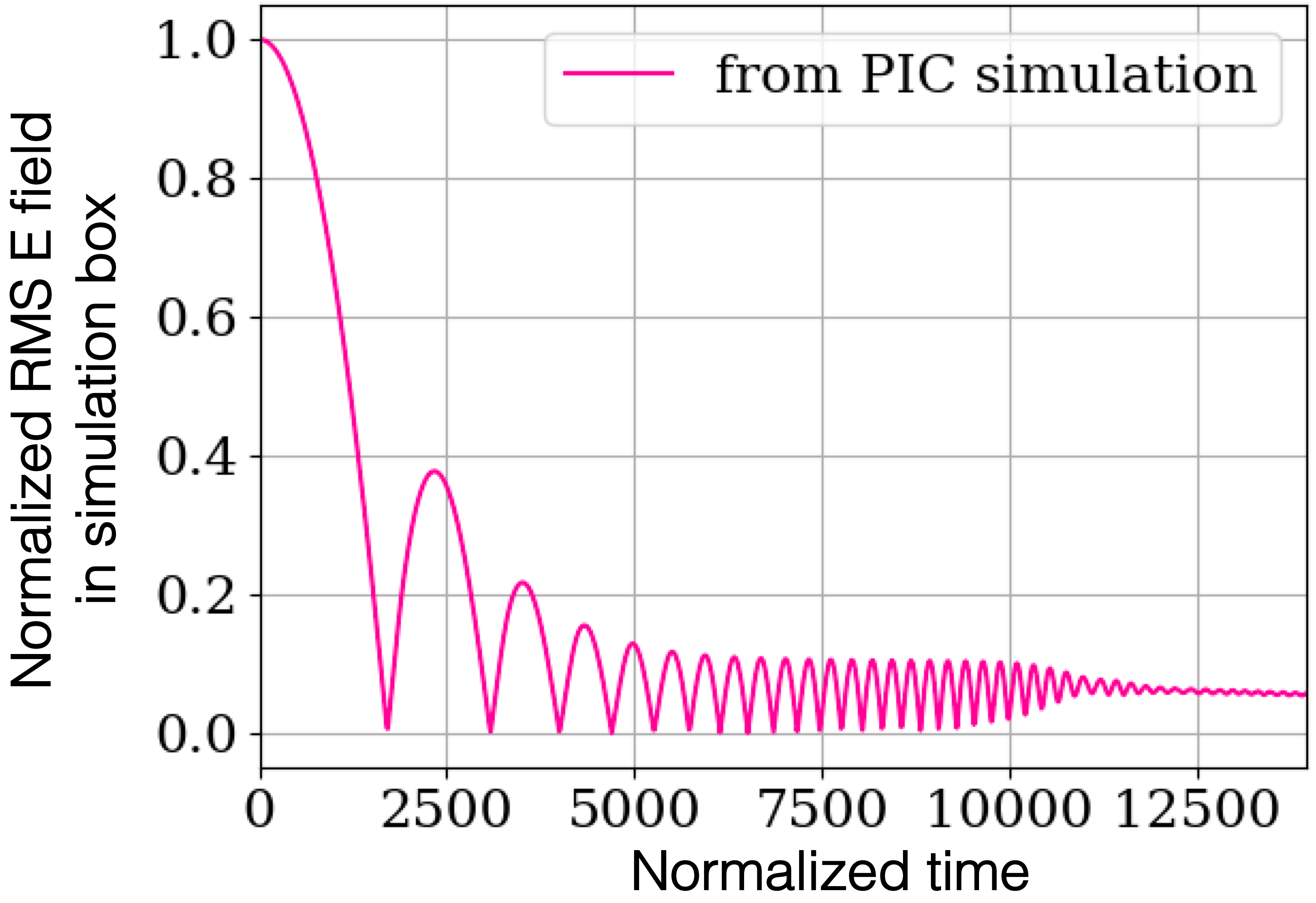
- Discharge evolution modeled by screening, damping of vacuum E field by continuous creation of $\gamma \sim 10$, $T \sim 0.1$ mc^2 pairs

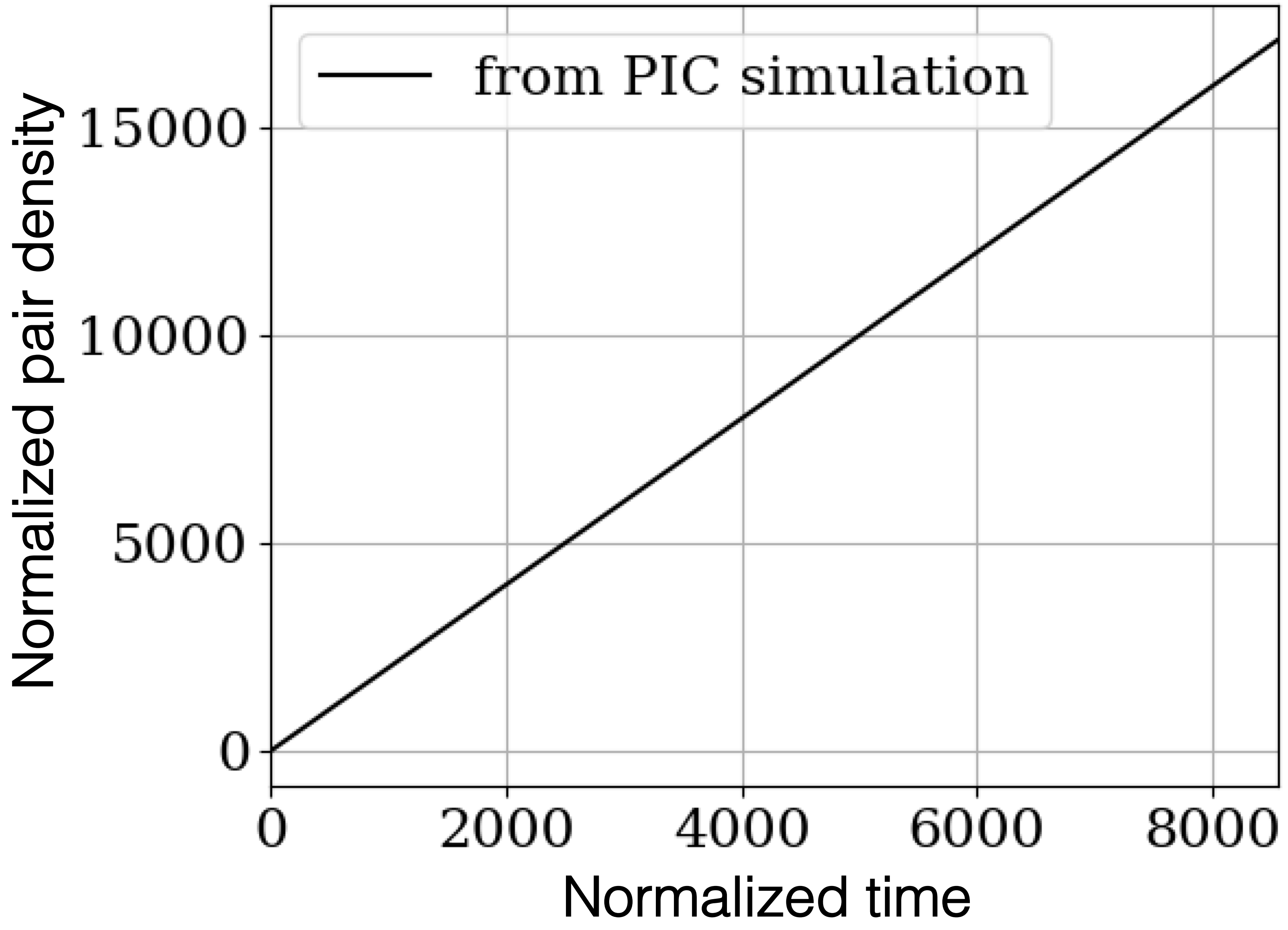
- Seen at right in PIC simulation

- For typical pulsar parameters, time is normalized to 10^{-14} s, electric field to 10^6 G, and density to 10^6 cm^{-3}

We build analytical models of process

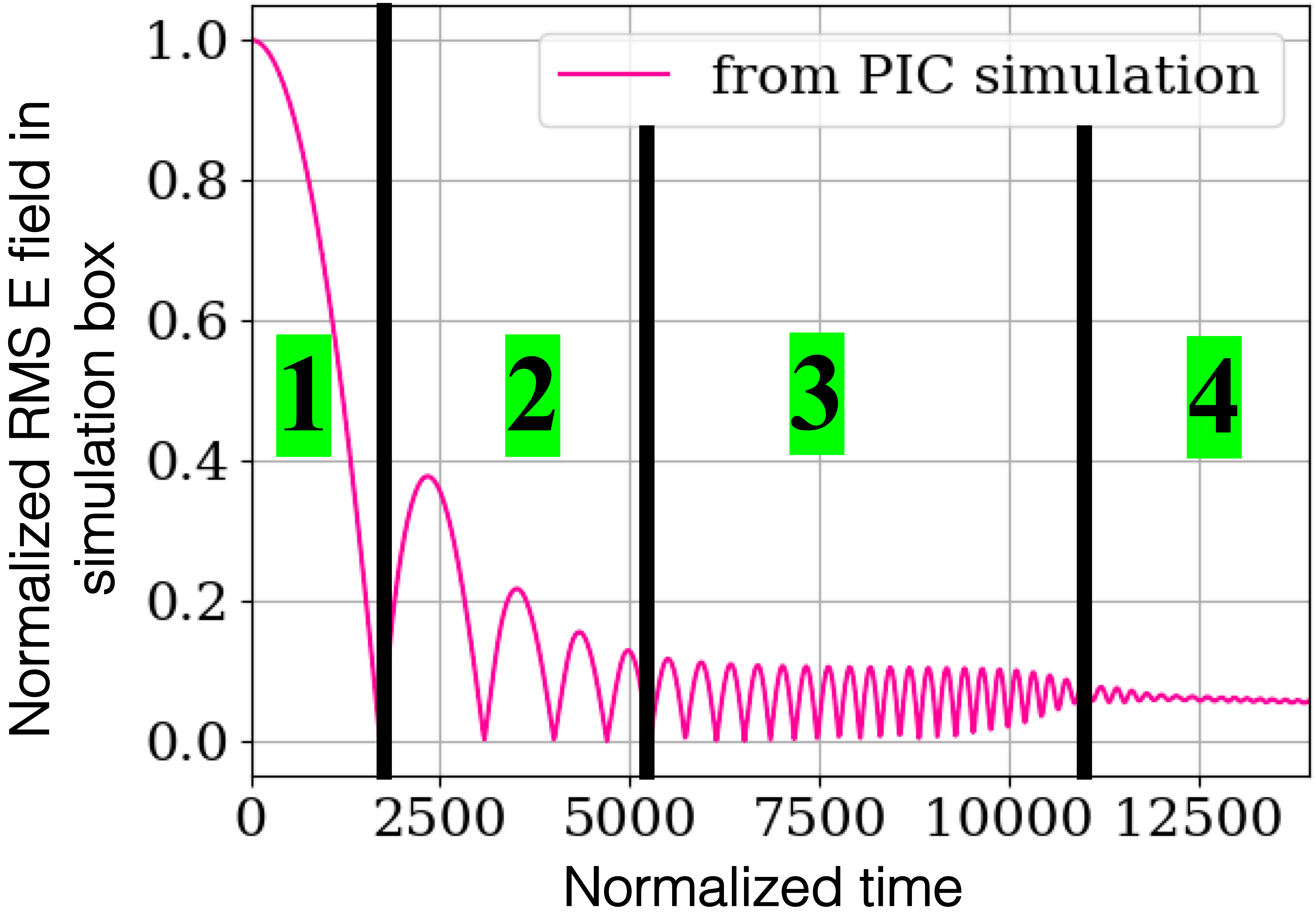






- Our work: analytical models of screening and damping process, compared to PIC
  - explain luminosity
  - give some insight into spectrum
- Inspiration from analogous, but different, physics of plasma heating by waves in tokamaks

E field damping has 4 phases



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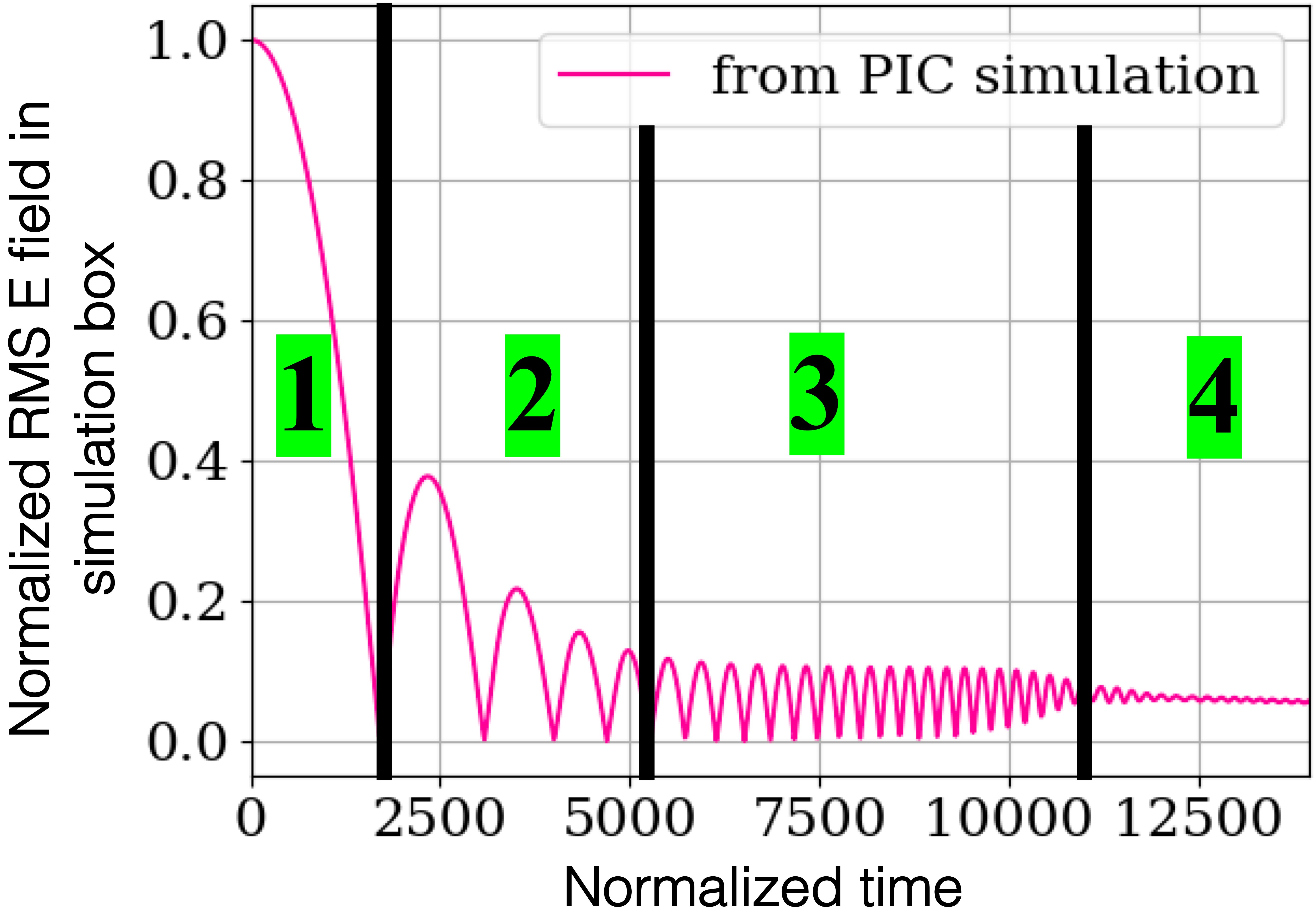
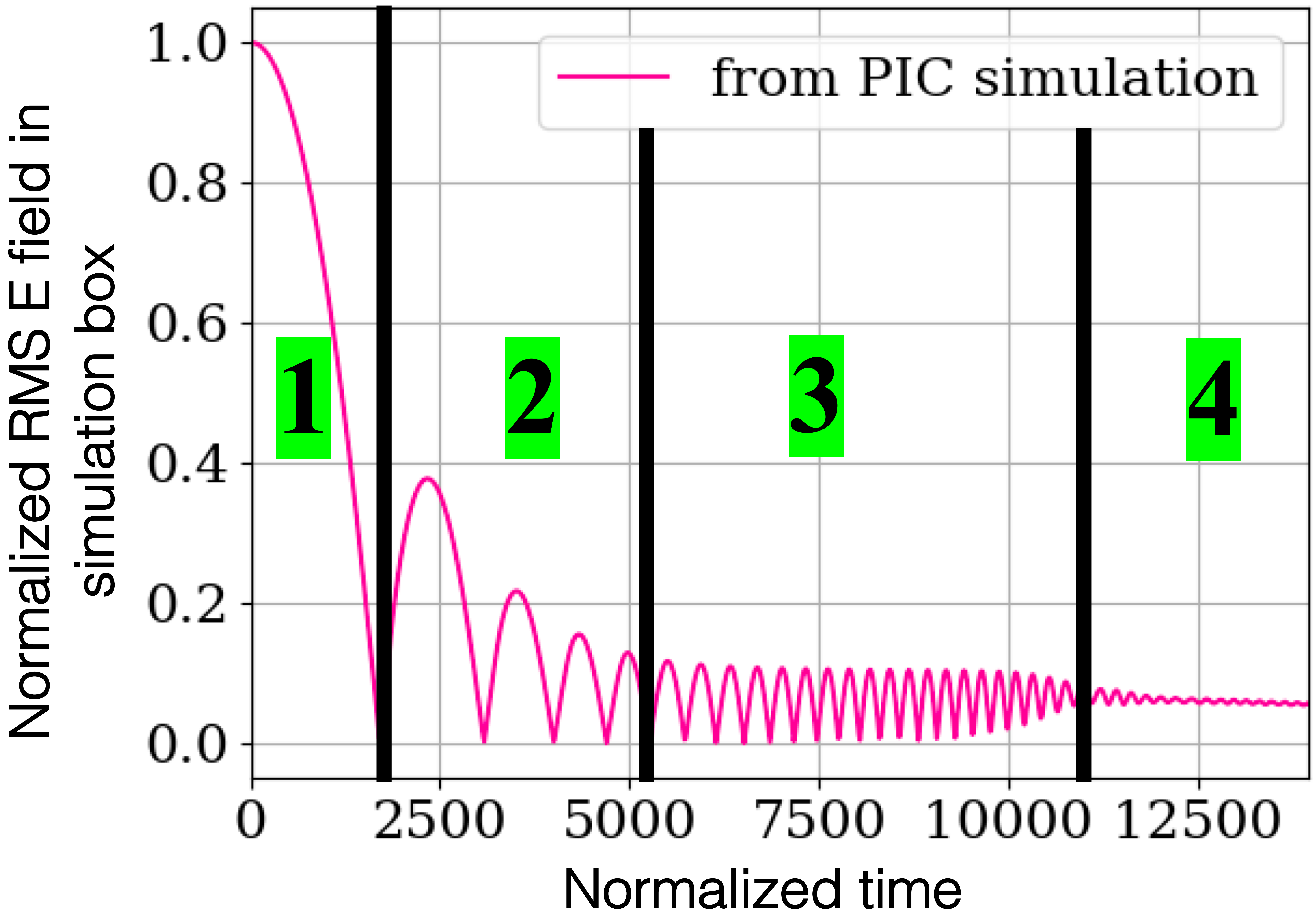
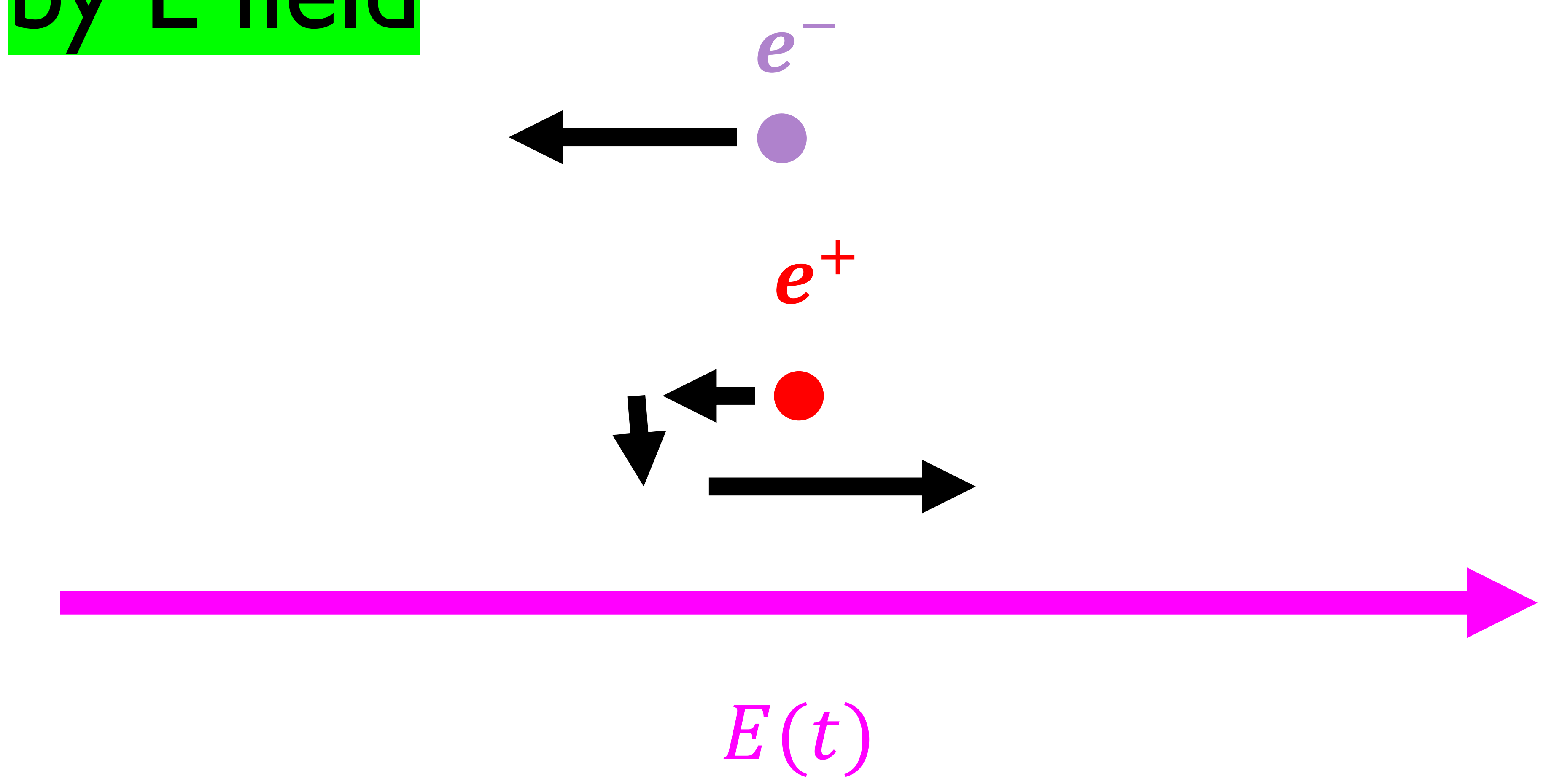


1. **Screening phase**: initial electric field shielded out

E field damping has 4 phases



2. **Nonlinear waves, strong damping**: new pairs fully reversed by E field



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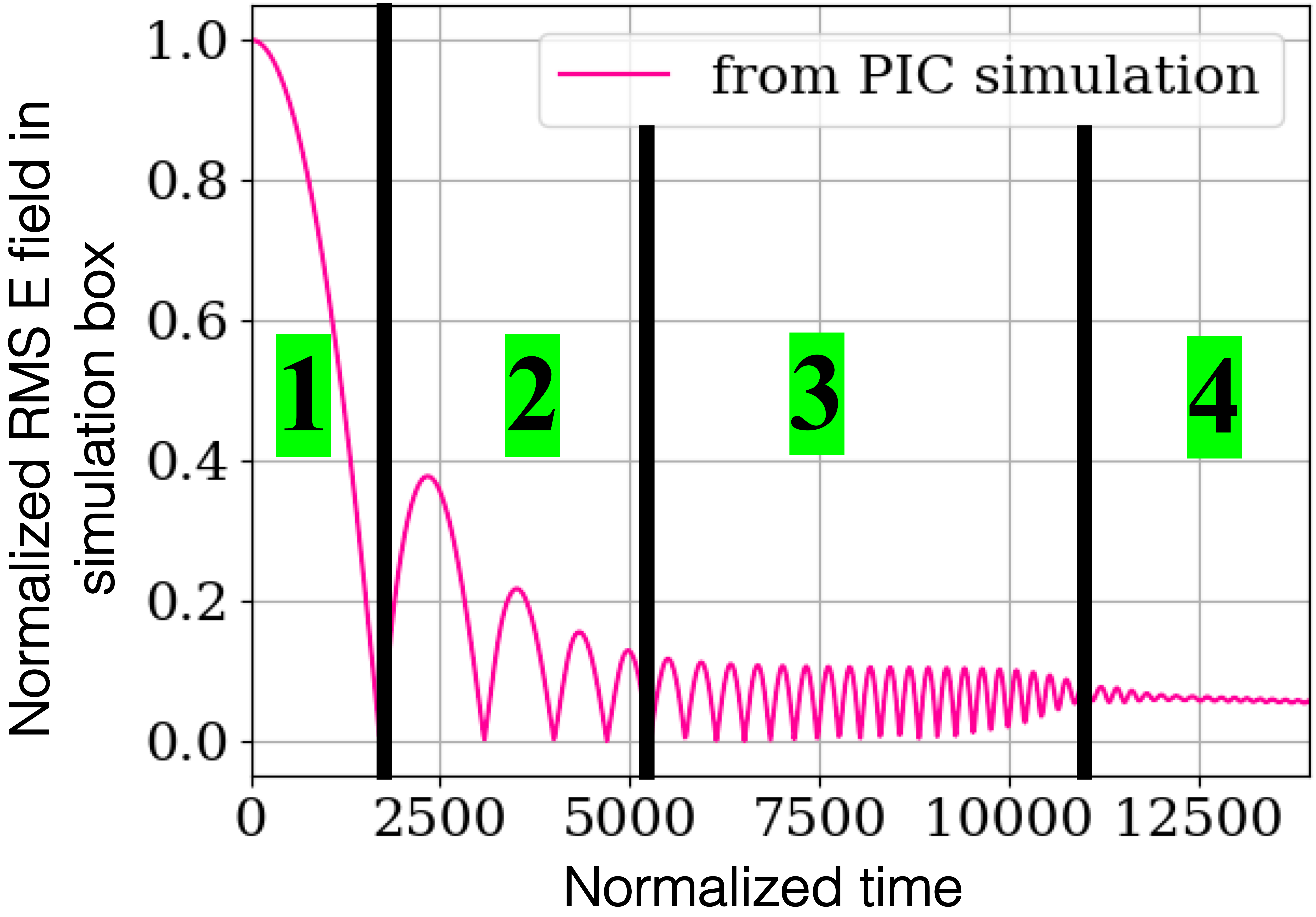
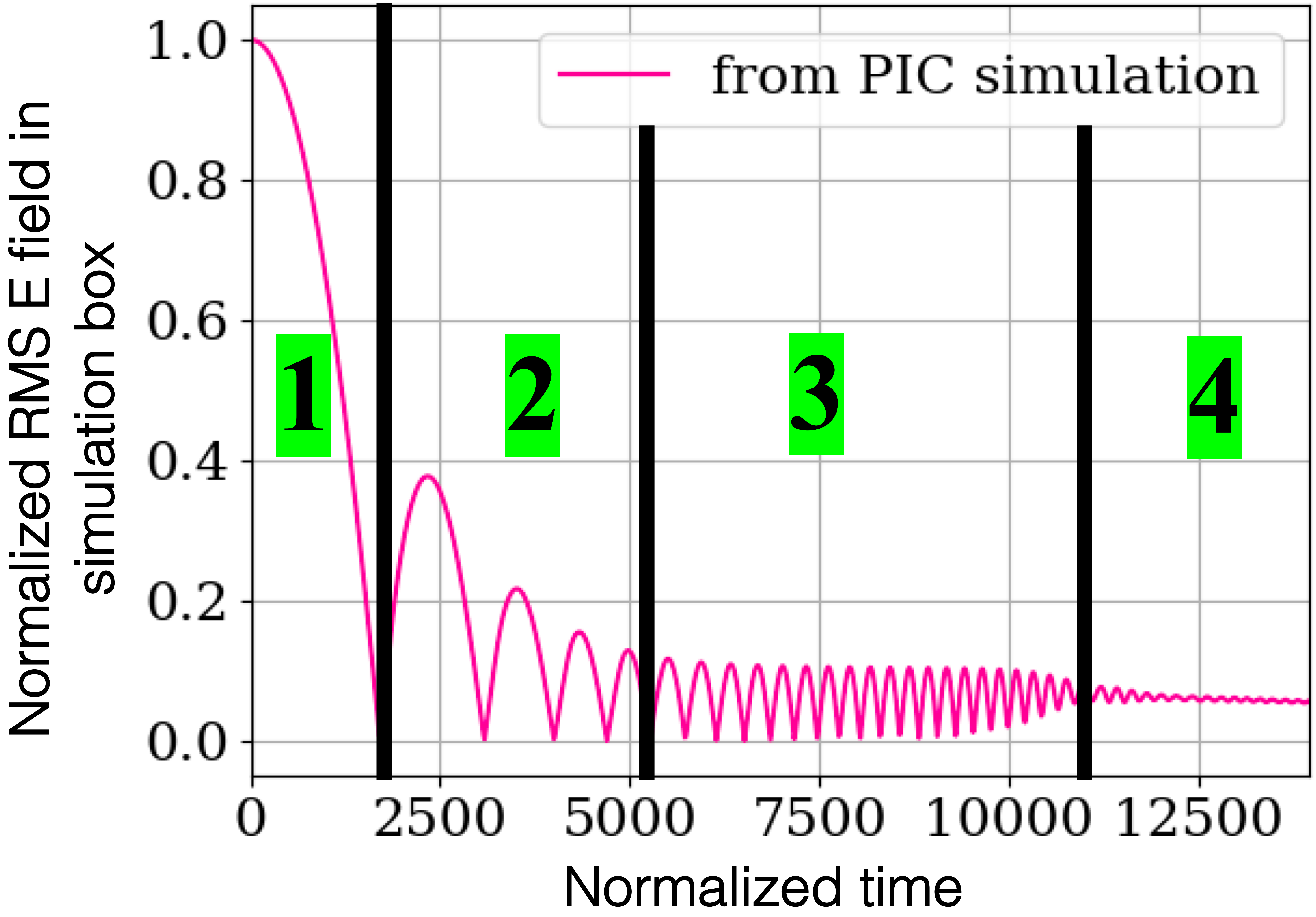


3. **"Frozen" phase**: artifact of simplified setup in simulation; ends when uniform E field breaks up into low-k modes

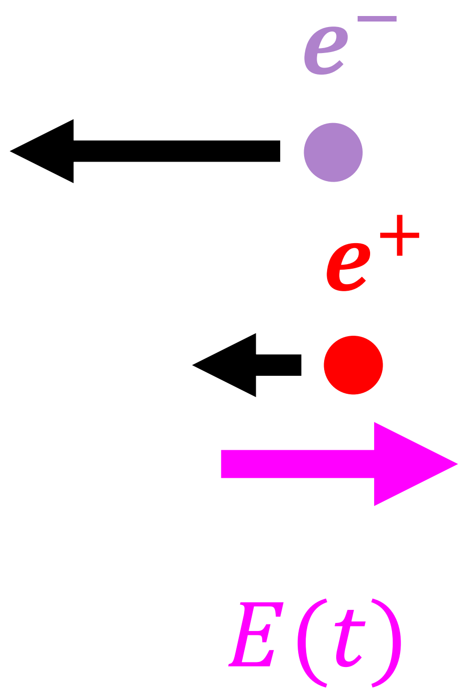
Not expected to occur in 2D setup

Discussed in paper

E field damping has 4 phases



4. **Linear waves, weak damping:** new pairs not reversed by E field



Discharge evolution determined by u_l, ξ

- Under appropriate normalizations (right), equations governing system are:

Vlasov with source: $\partial_{\hat{t}} \hat{f}_s + \frac{q_s}{e} \hat{E}(\hat{t}) \partial_u \hat{f}_s = \delta(u - \boldsymbol{u_l})$

Ampère's law: $\partial_{\hat{t}} \hat{E} + \hat{j} = 0$

Current definition: $\hat{j} \equiv \frac{1}{\xi} \sum_s \frac{q_s}{e} \int_{-\infty}^{\infty} \beta \hat{f}_s \, du$

Normalized Quantity	Description
$\hat{E}$	E field normalized to initial value ($\approx 10^6$ G)
$\hat{t}$	Time normalized to time to change pair momentum by 1 in initial field ($\approx 10^{-14}$ s)
$\hat{f}_s$	Distribution normalized to density injected in time to change pair momentum by 1 in initial field ($\approx 10^6$ cm^{-3})

- Normalized system governed by two parameters:

Parameter	Description	Value in actual pulsar	Value in simulation
$\boldsymbol{u_l}$	momentum of freshly created pairs	$10 - 10^3$	10
ξ	$\frac{\text{energy in initial E field}}{\text{rest mass energy in pairs injected in unit time}}$	$10^{12} \left(\frac{B_0}{10^{12}\ G}\right)^2 \left(\frac{10^5}{\text{multiplicity}}\right) \left(\frac{0.1\ s}{\text{Period}}\right)^{7/2}$	10^6

Discharge evolution determined by u_l, ξ

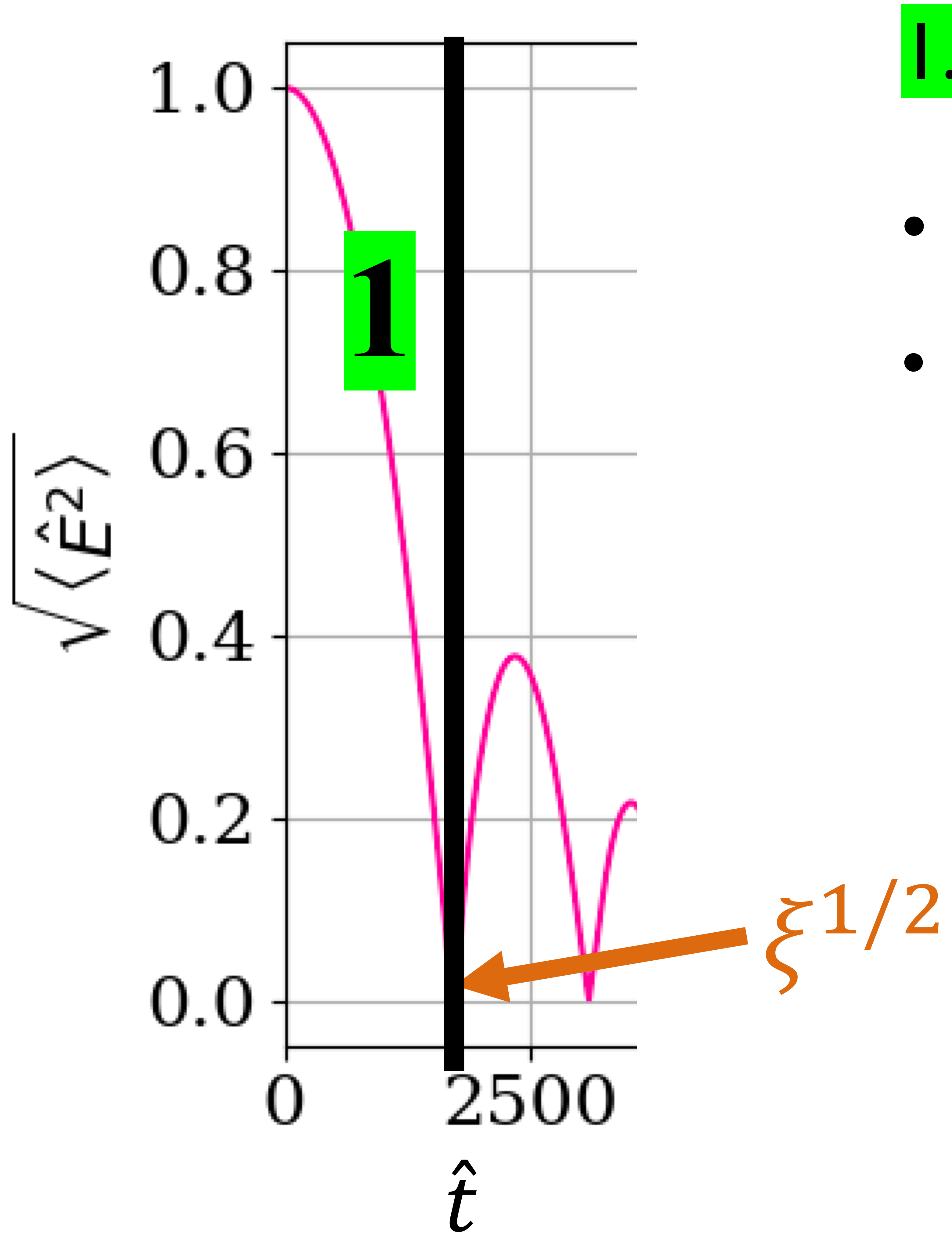
- Can combine previous equations to get evolution of $\hat{E}$ in terms of relativistic plasma frequency

$$\partial_{\hat{t}}^2 \hat{E} + \widehat{\omega}^2(\hat{t})\hat{E}(\hat{t}) = 0$$

$$\widehat{\omega}^2 \equiv \frac{\hat{n}_+}{\xi}\left\langle\frac{1}{\gamma^3}\right\rangle_+ + \frac{\hat{n}_-}{\xi}\left\langle\frac{1}{\gamma^3}\right\rangle_-$$

Normalized quantity	Description
$\widehat{\omega}$	normalized relativistic plasma frequency
$\hat{n}_-, \hat{n}_+$	Normalized e-, e+ density

Screening shields E field



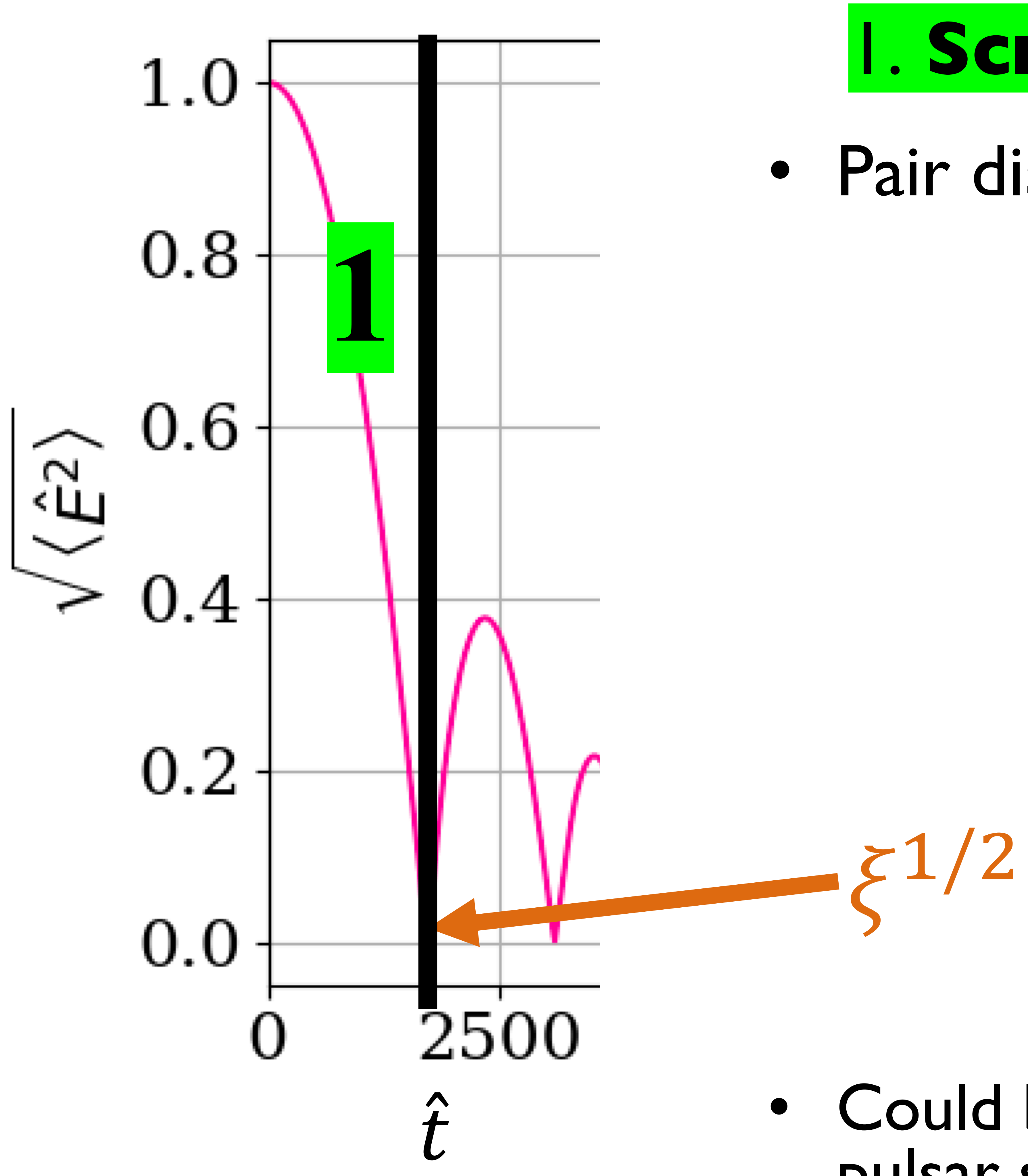
1. **Screening phase**: initial electric field shielded out

- Pairs quickly accelerated to $\pm c$
- E field screened in $\hat{t}_{\text{screen}} = \xi^{1/2}$

$$\xi \sim 10^{12} \left(\frac{B_0}{10^{12}\ G}\right)^2 \left(\frac{10^5}{\text{multiplicity}}\right) \left(\frac{0.1\ s}{\text{Period}}\right)^{7/2}$$

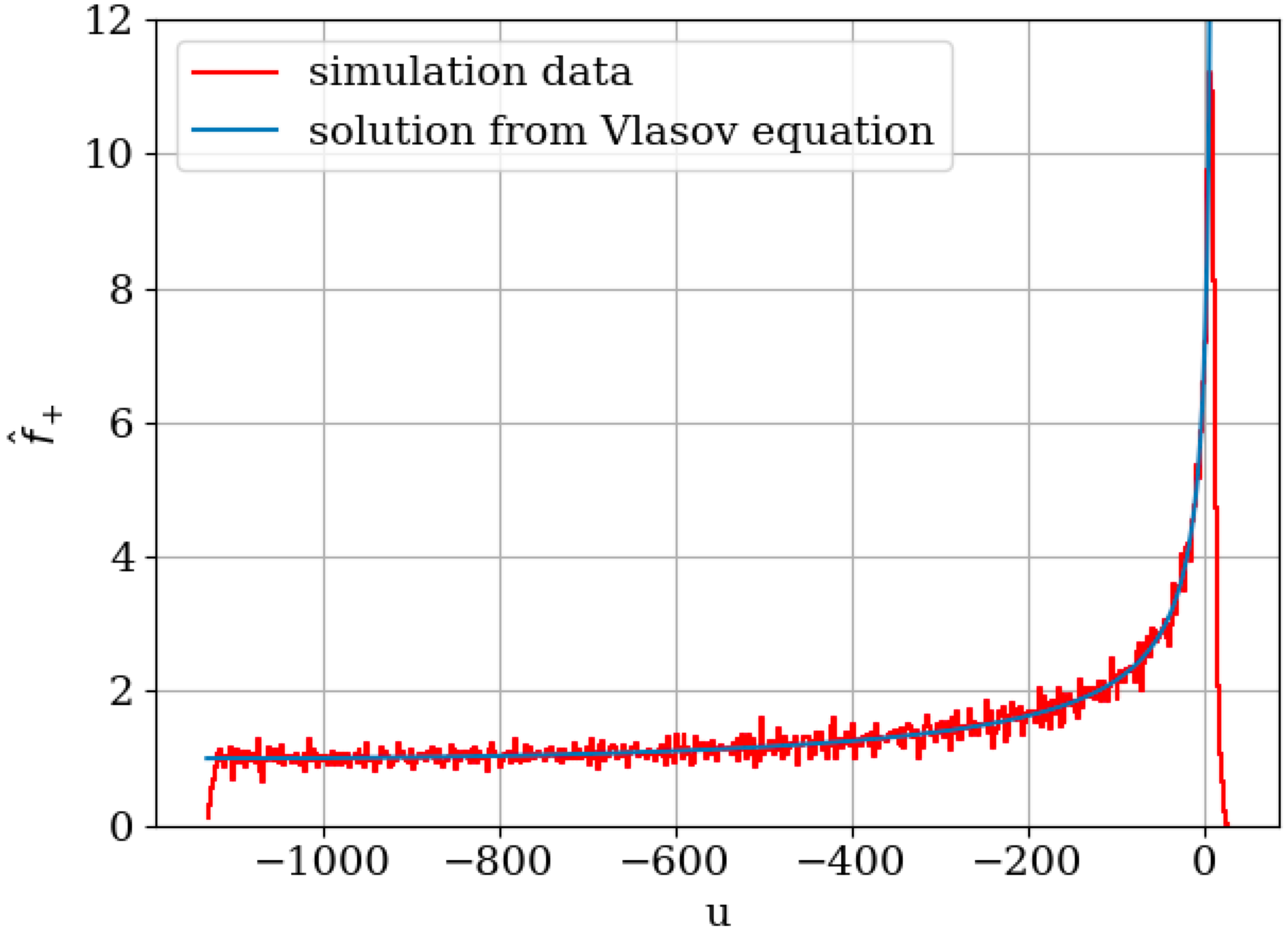
- Longer for higher B_0: more energy in initial E field
- Shorter for higher multiplicity: pairs injected more quickly
- Shorter for higher period: less energy in initial E field

Screening accelerates pairs



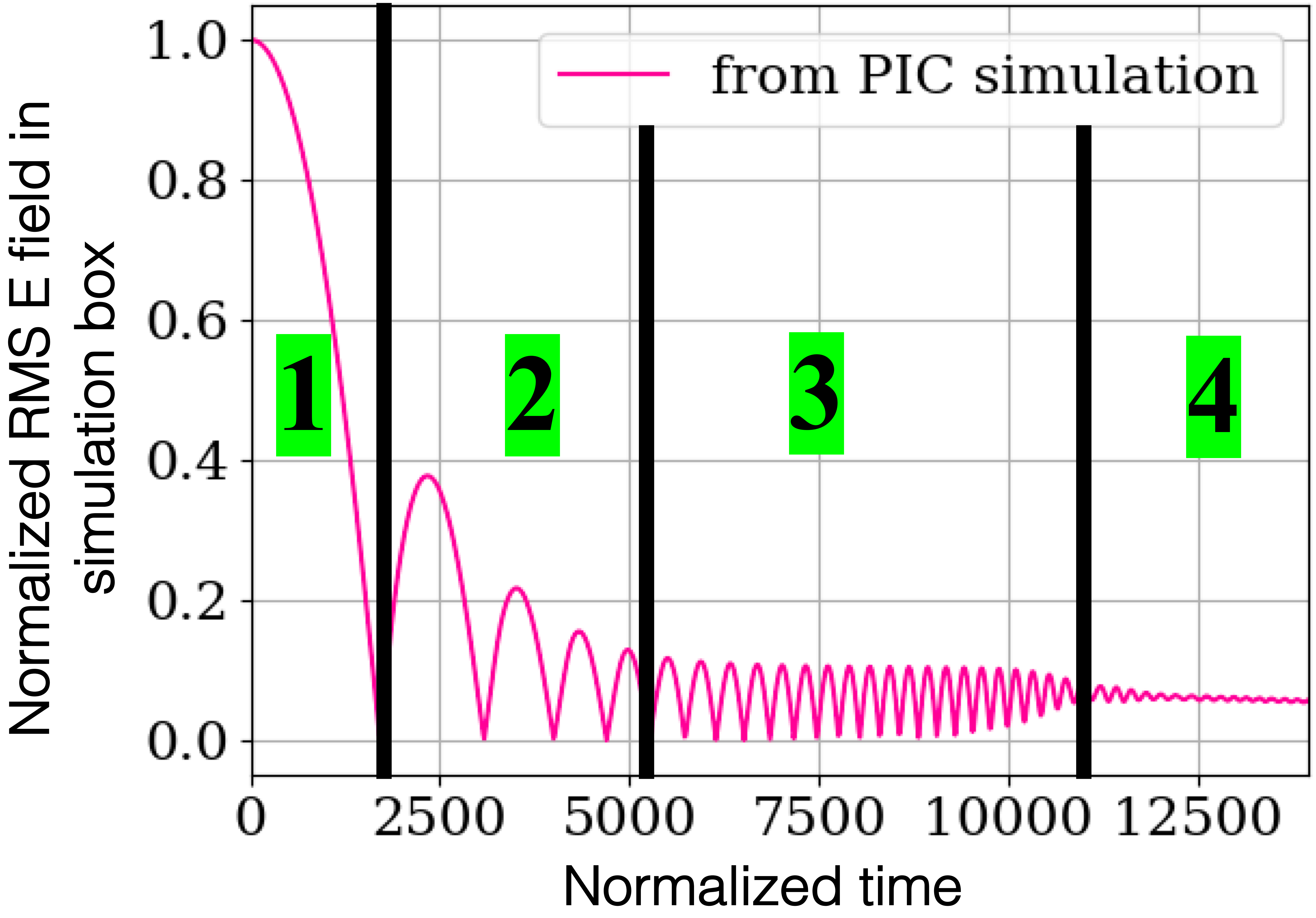
1. Screening phase: initial electric field shielded out

- Pair distribution function analytically solvable from Vlasov

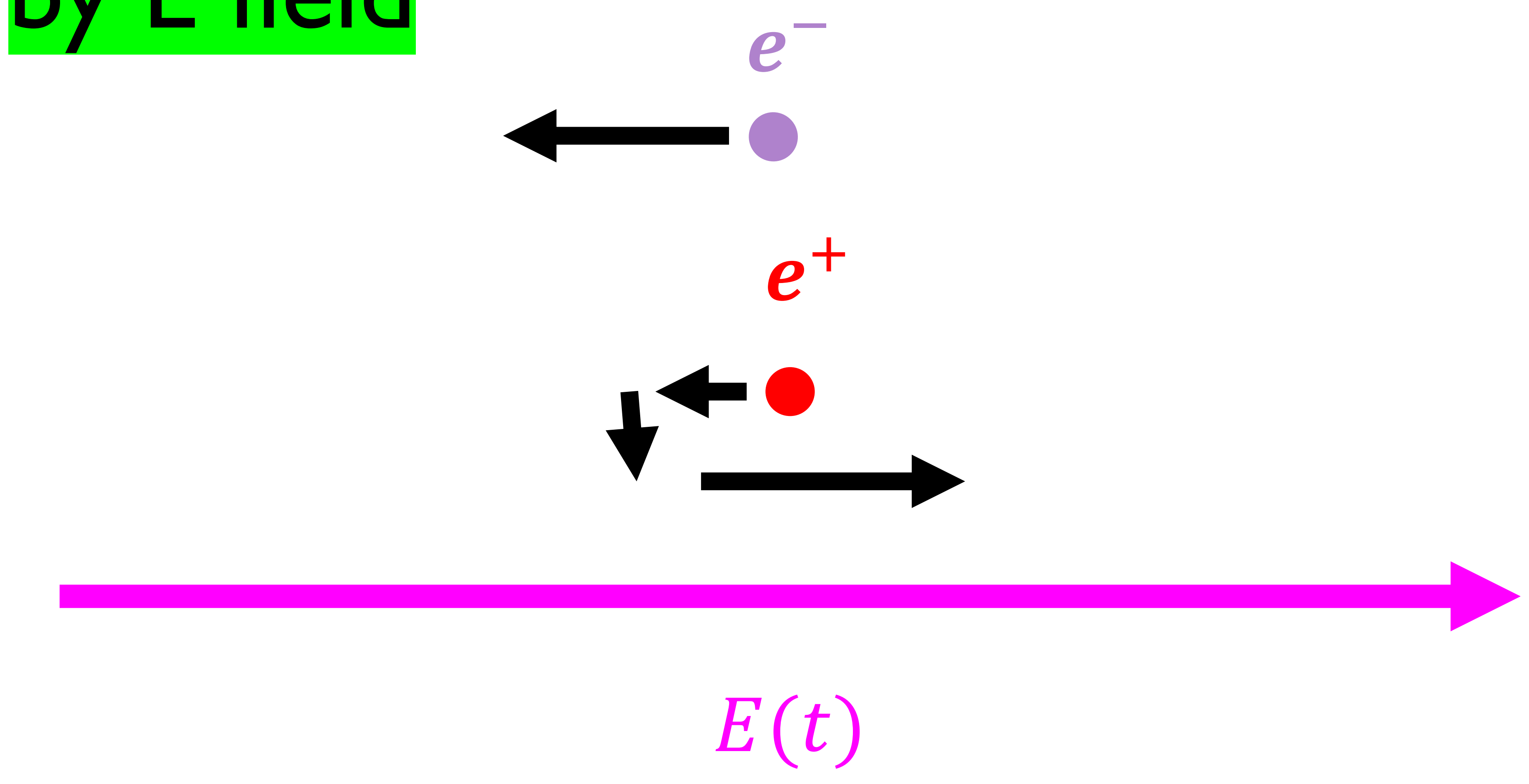


- Could be used to roughly model spectrum of particles backflowing to pulsar surface in studies of x-ray hotspots observed by NICER

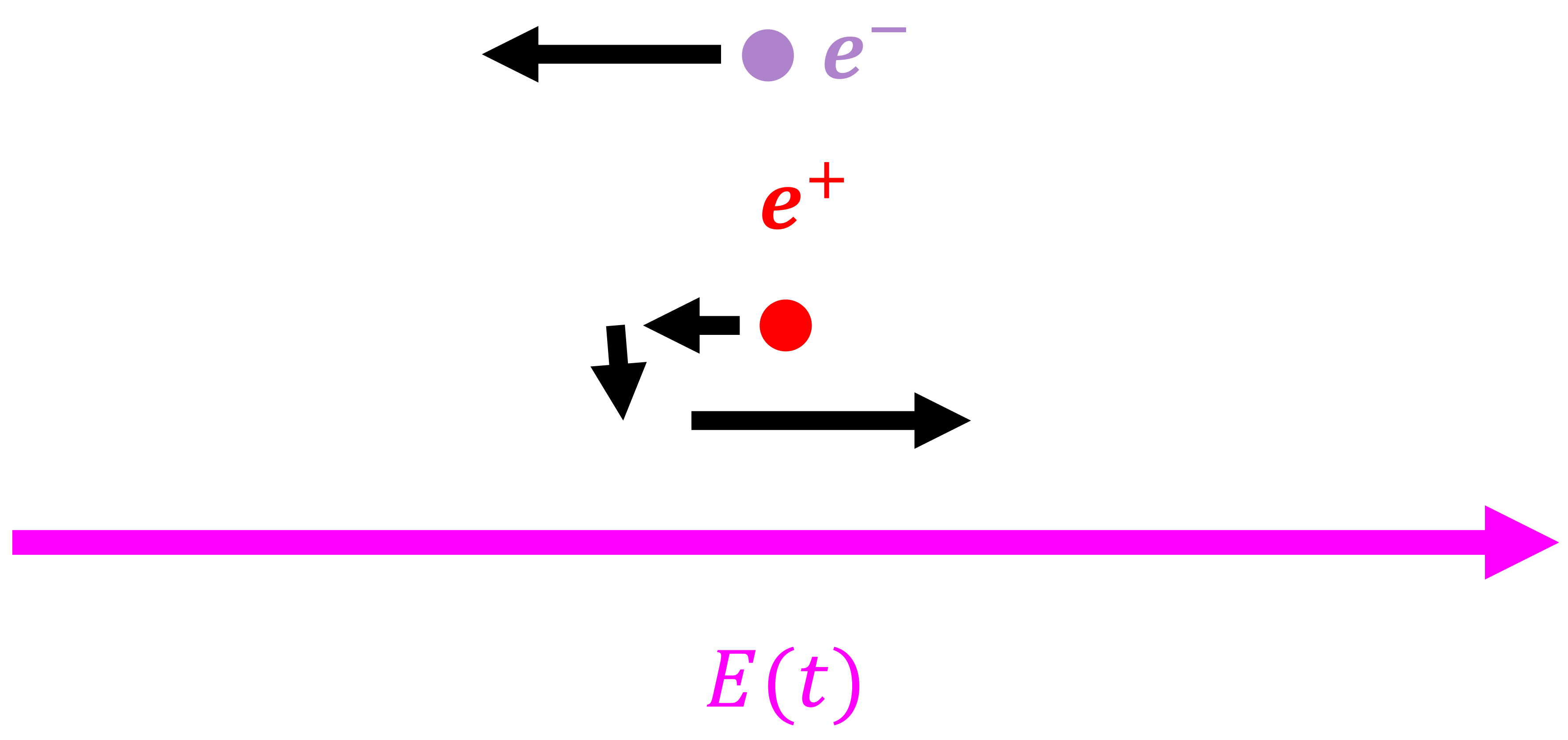
Nonlinear stage marked by strong damping, spiking frequency

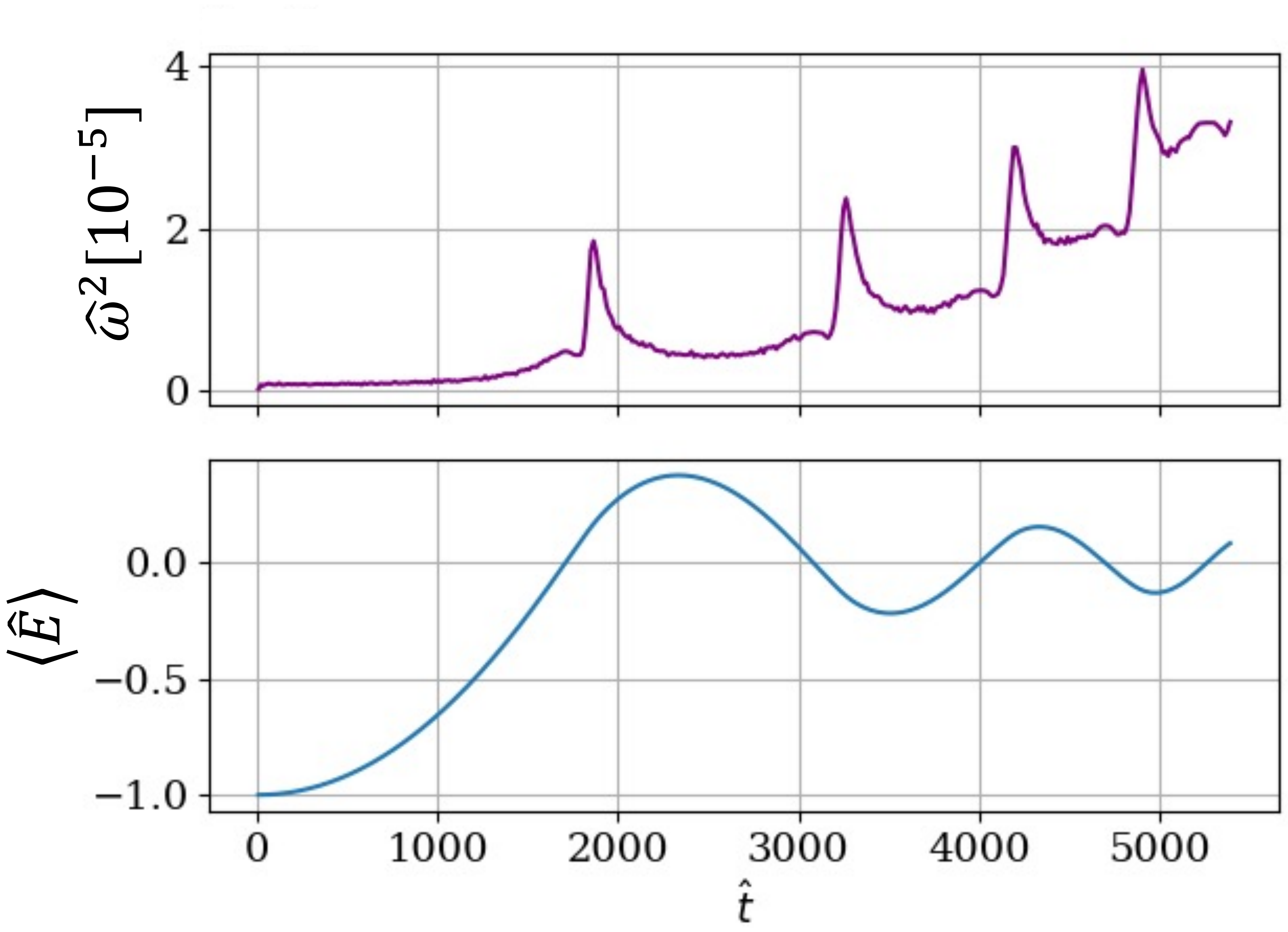


2. **Nonlinear waves, strong damping**: new pairs fully reversed by E field



Nonlinear stage marked by strong damping, spiking frequency





2. Nonlinear waves, strong damping

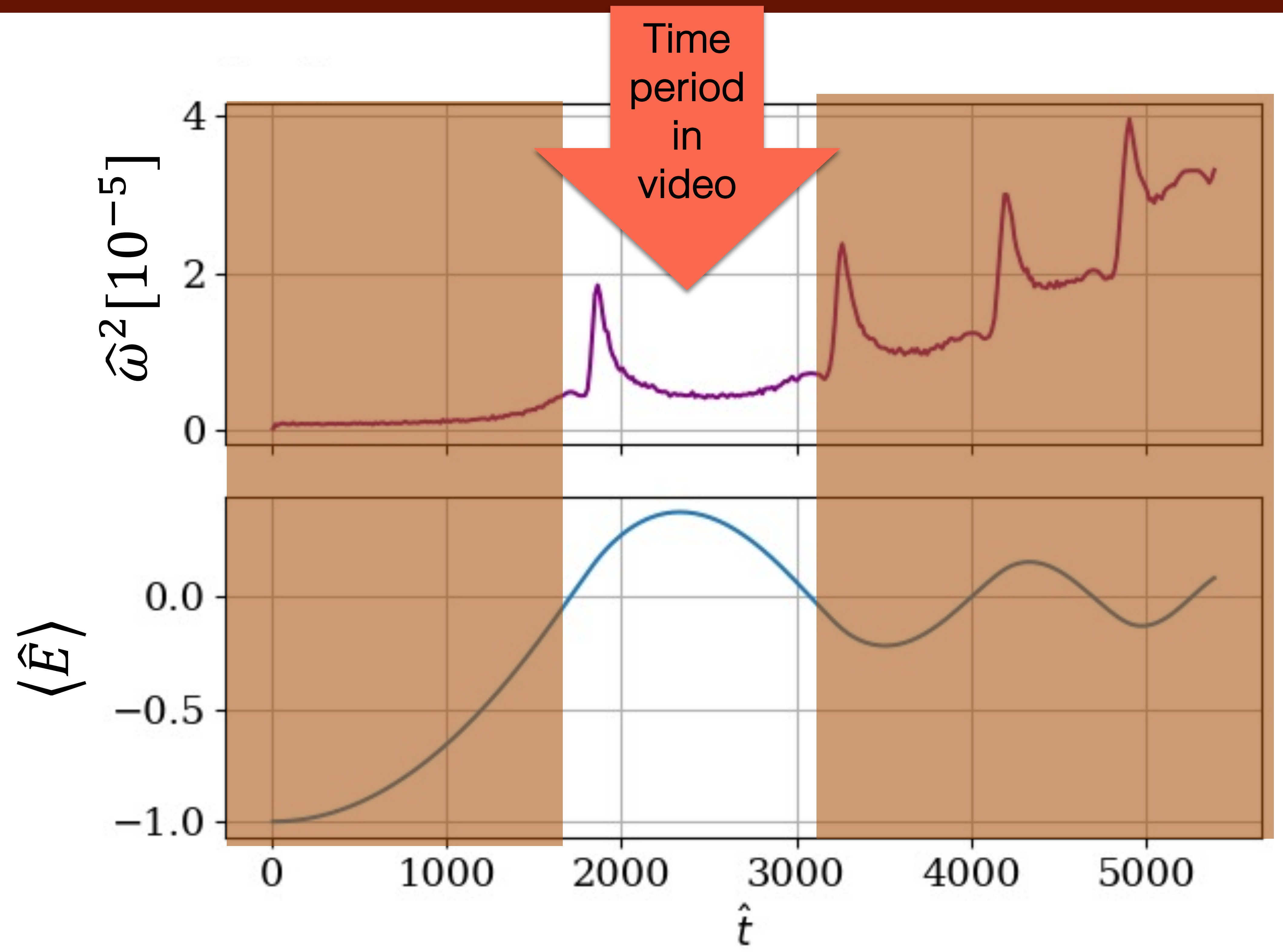
- Nonlinear stage marked by strong damping, governed by:

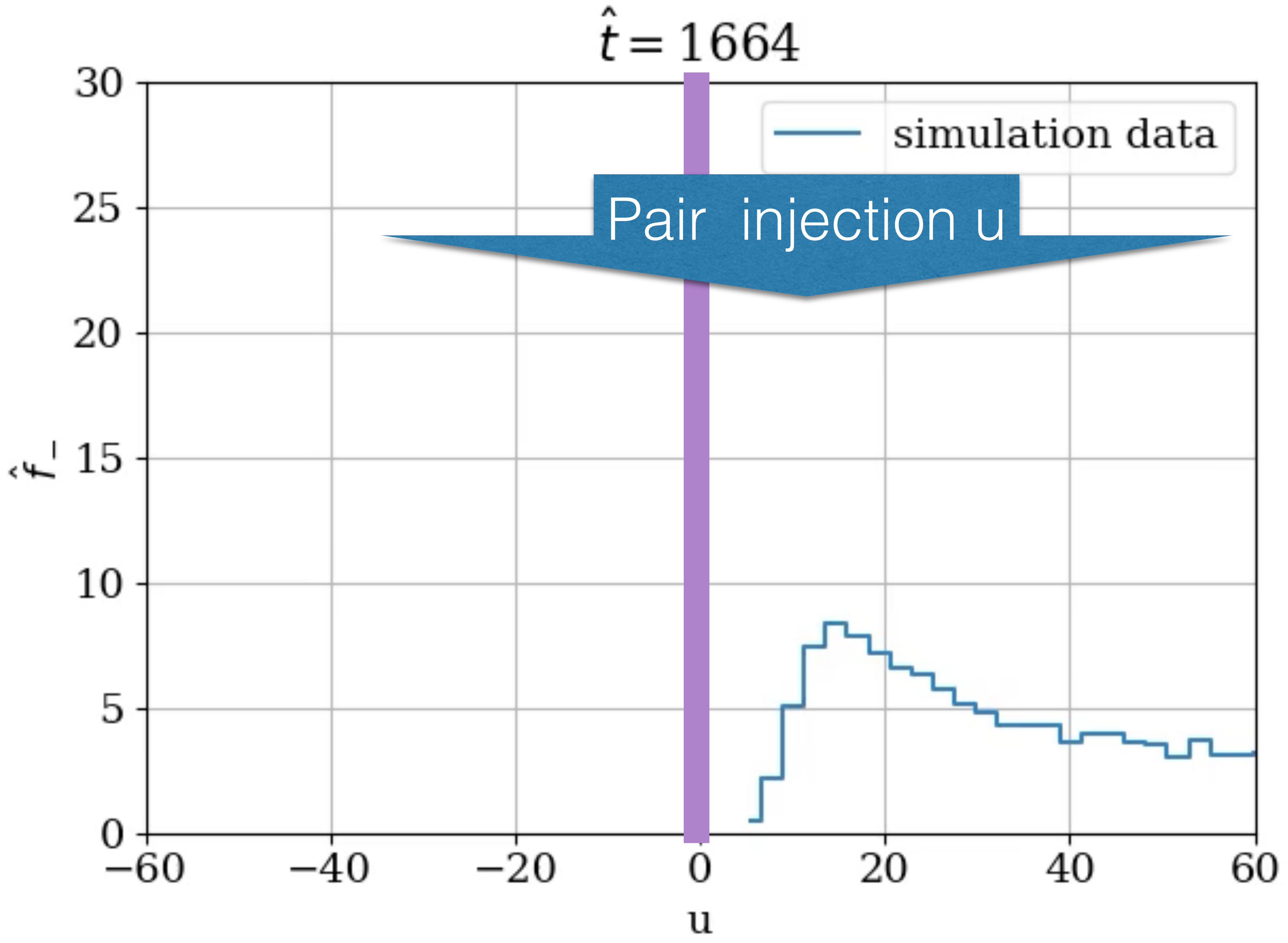
$$\partial_{\hat{t}}^2 \hat{E} + \hat{\omega}^2 \hat{E} = 0$$

$$\hat{\omega}^2 \equiv \frac{\hat{n}_+}{\xi}\left\langle \frac{1}{\gamma^3} \right\rangle_+ + \frac{\hat{n}_-}{\xi}\left\langle \frac{1}{\gamma^3} \right\rangle_-$$

- Frequency exhibits spiking behavior

Nonlinear stage marked by spiking frequency





- Frequency $\hat{\omega}^2 \propto \left\langle \frac{1}{\gamma^3} \right\rangle$ exhibits spiking behavior
  - Pairs added near E=0 build up at moderate γ, are later dragged through $u = 0, \gamma = 1$

$\left\langle \frac{1}{\gamma^3} \right\rangle$: Gets larger when average γ is smaller

Overall bounce distribution function has wave foam shape

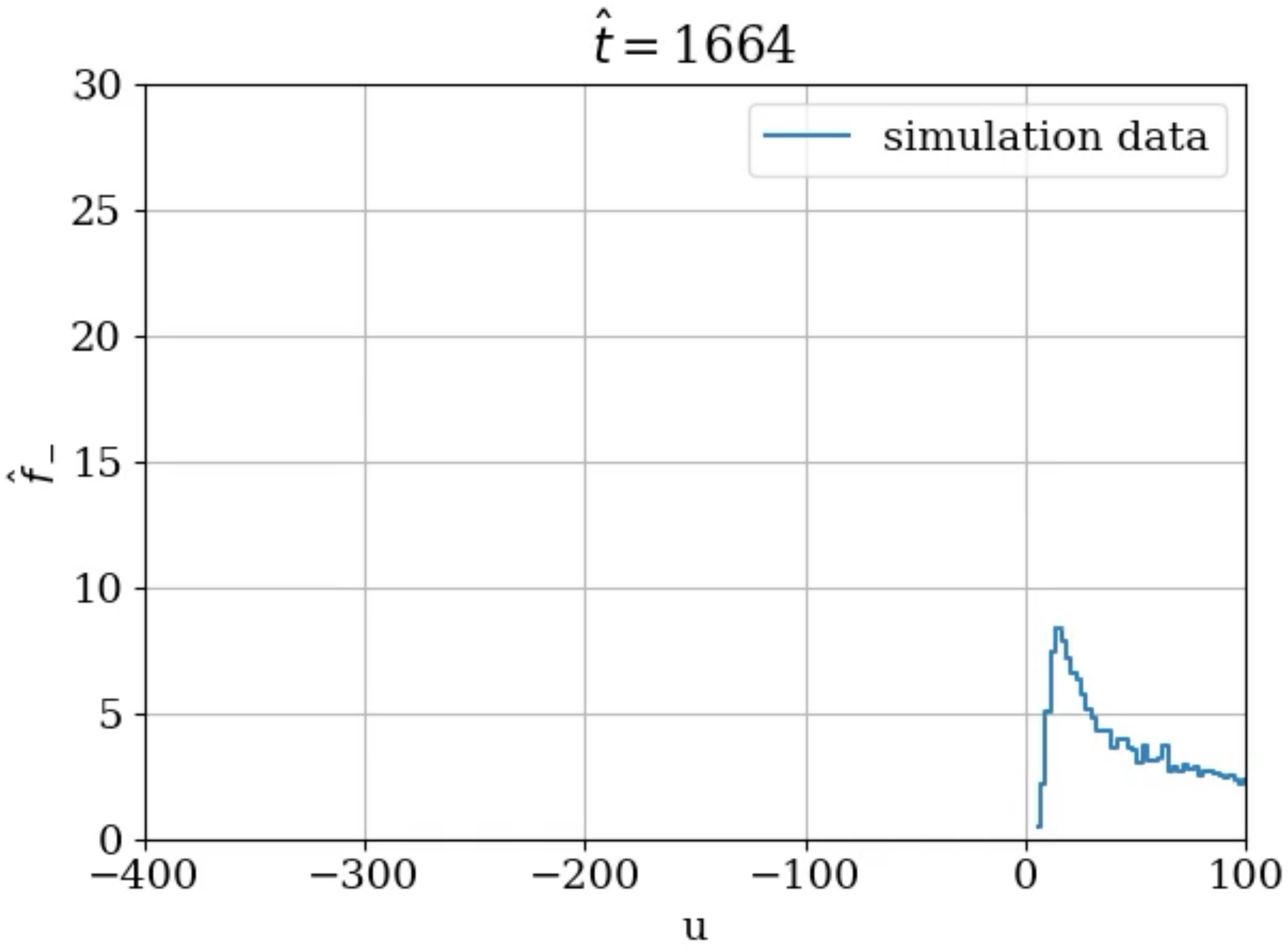
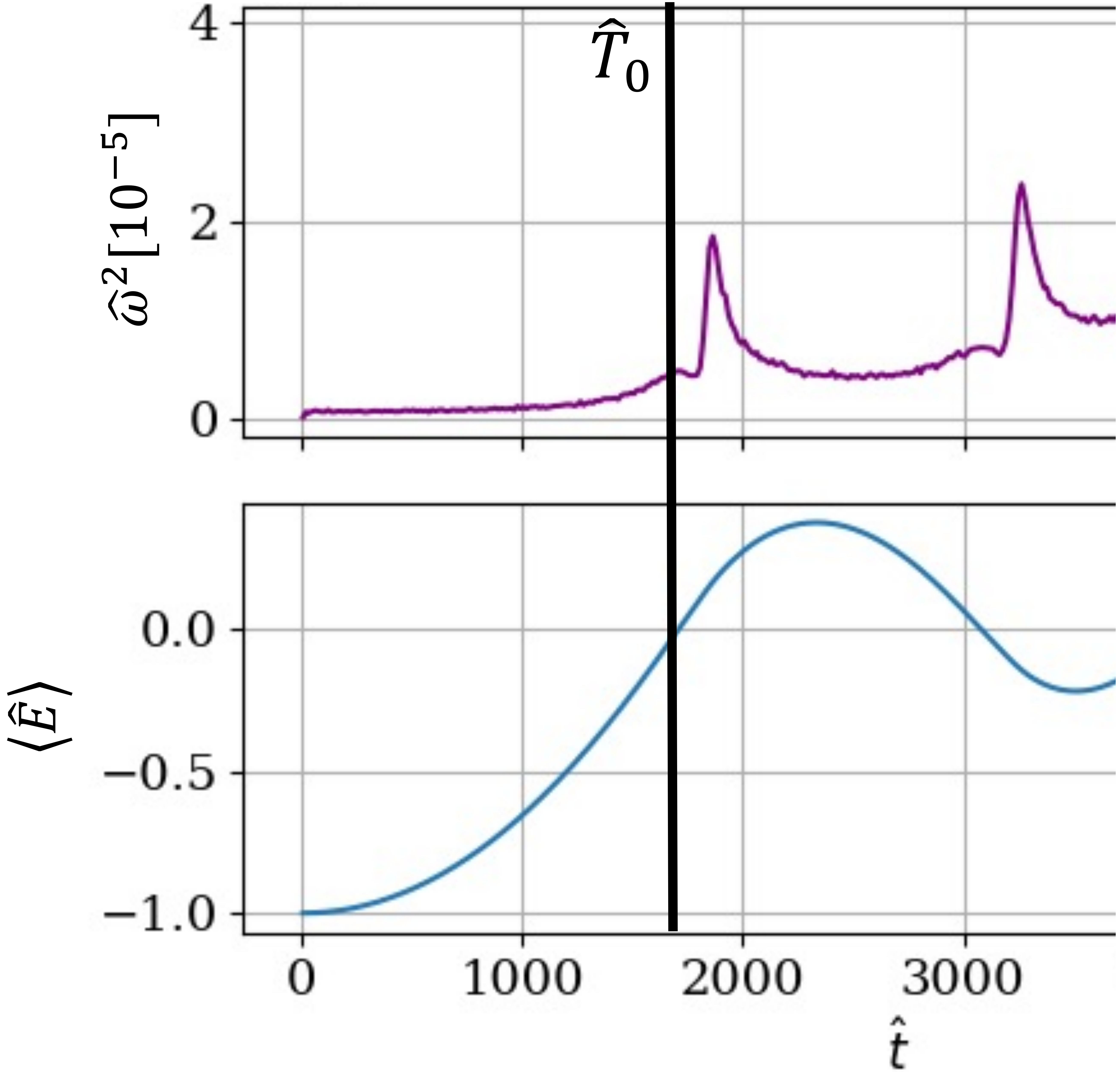


Image credit: https://www.theguardian.com/artanddesign/2017/may/19/hokusai-japanese-artist-late-blossoming-great-wave-mount-fuji

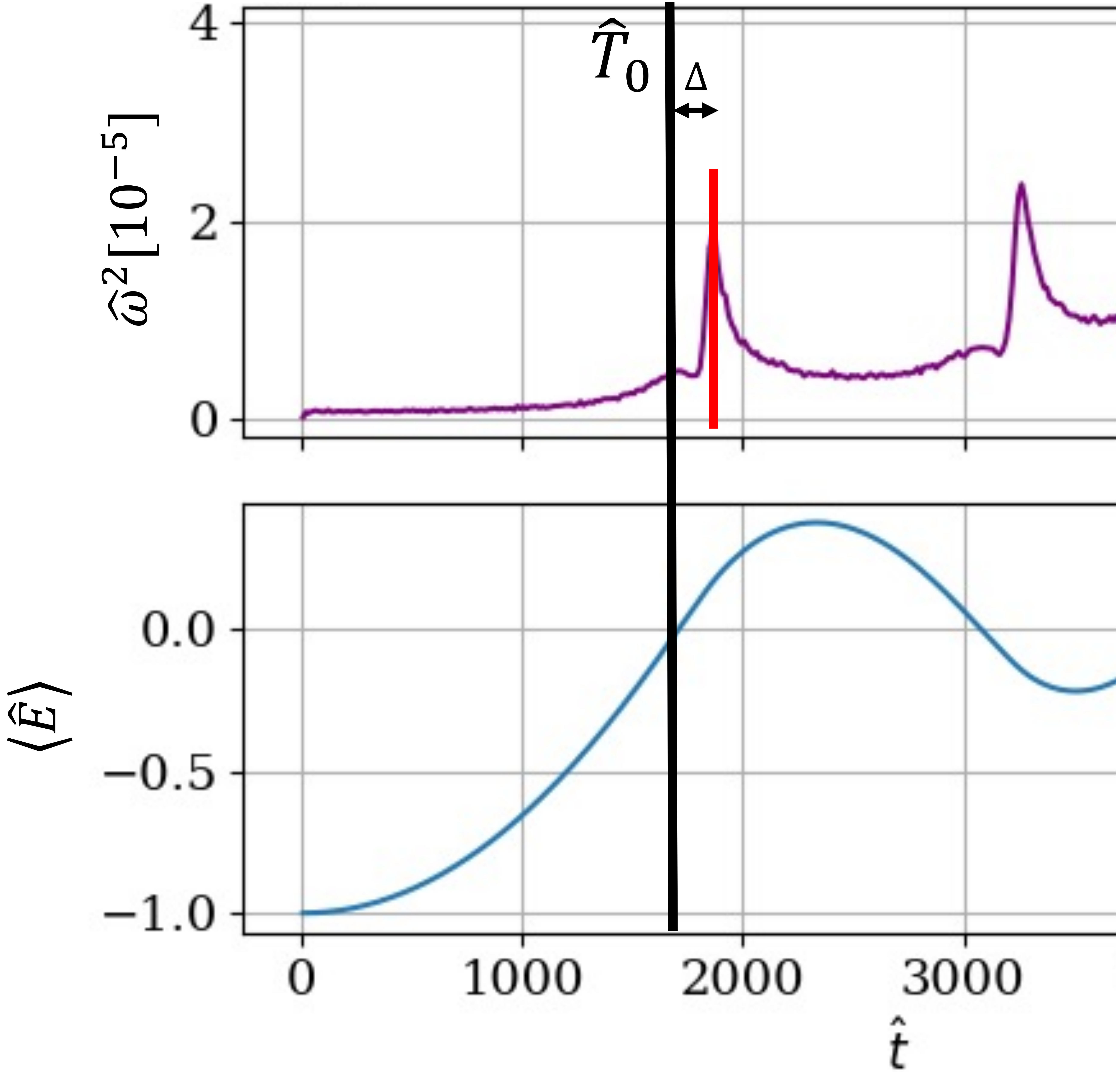
- Description credit: Hayk Hakobyan, developer of Tristan-MP v2 PIC code we use

Spiked frequency damps electric field



$$\partial_{\hat{t}}^2 \hat{E} + \hat{\omega}^2(\hat{t})\hat{E}(\hat{t}) = 0$$

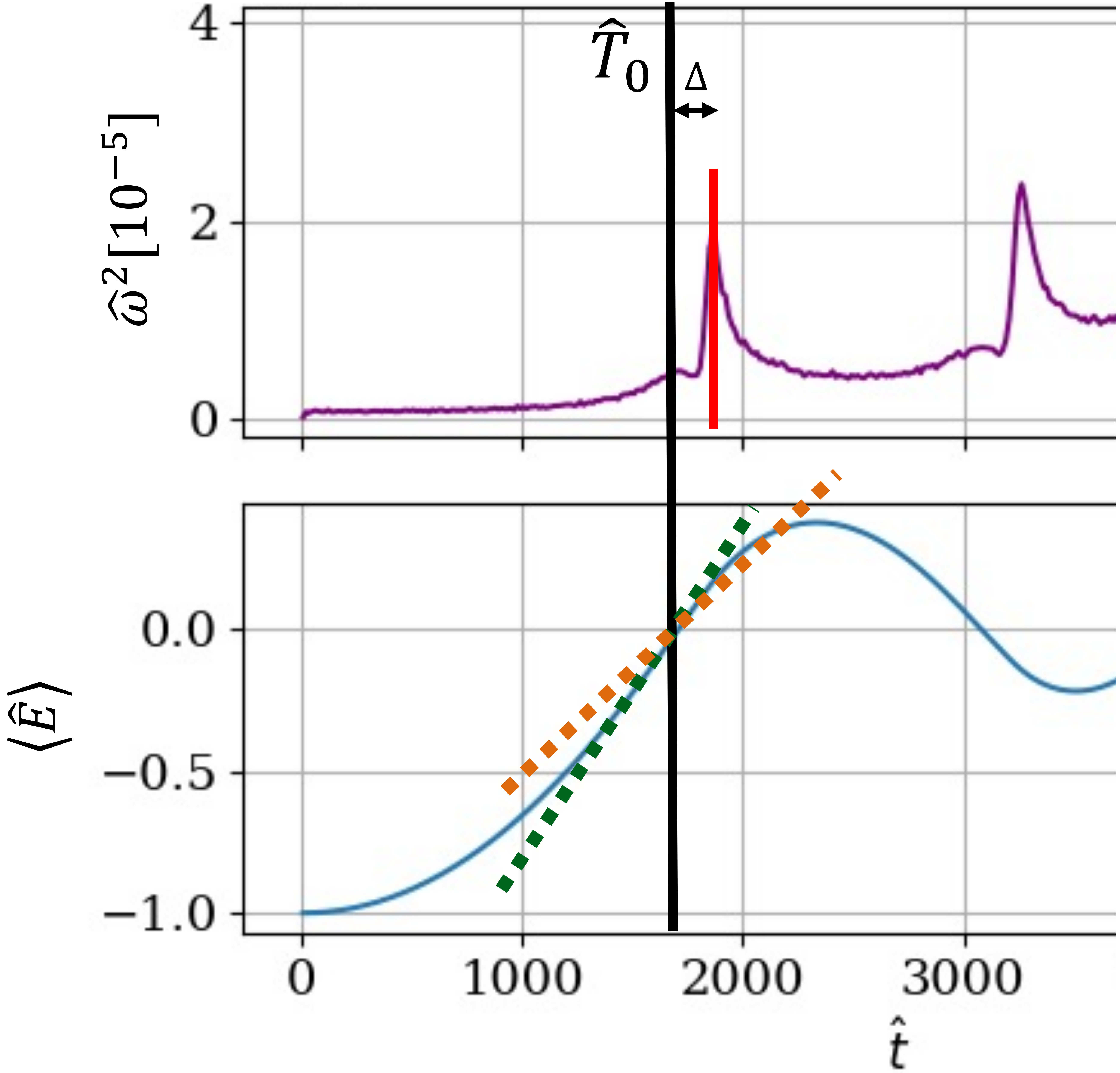
Spiked frequency damps electric field



$$\partial_{\hat{t}}^2 \hat{E} + \delta(\hat{t} - [\widehat{T_0} + \Delta])\hat{E}(\hat{t}) = 0$$

Integrate

Spiked frequency damps electric field

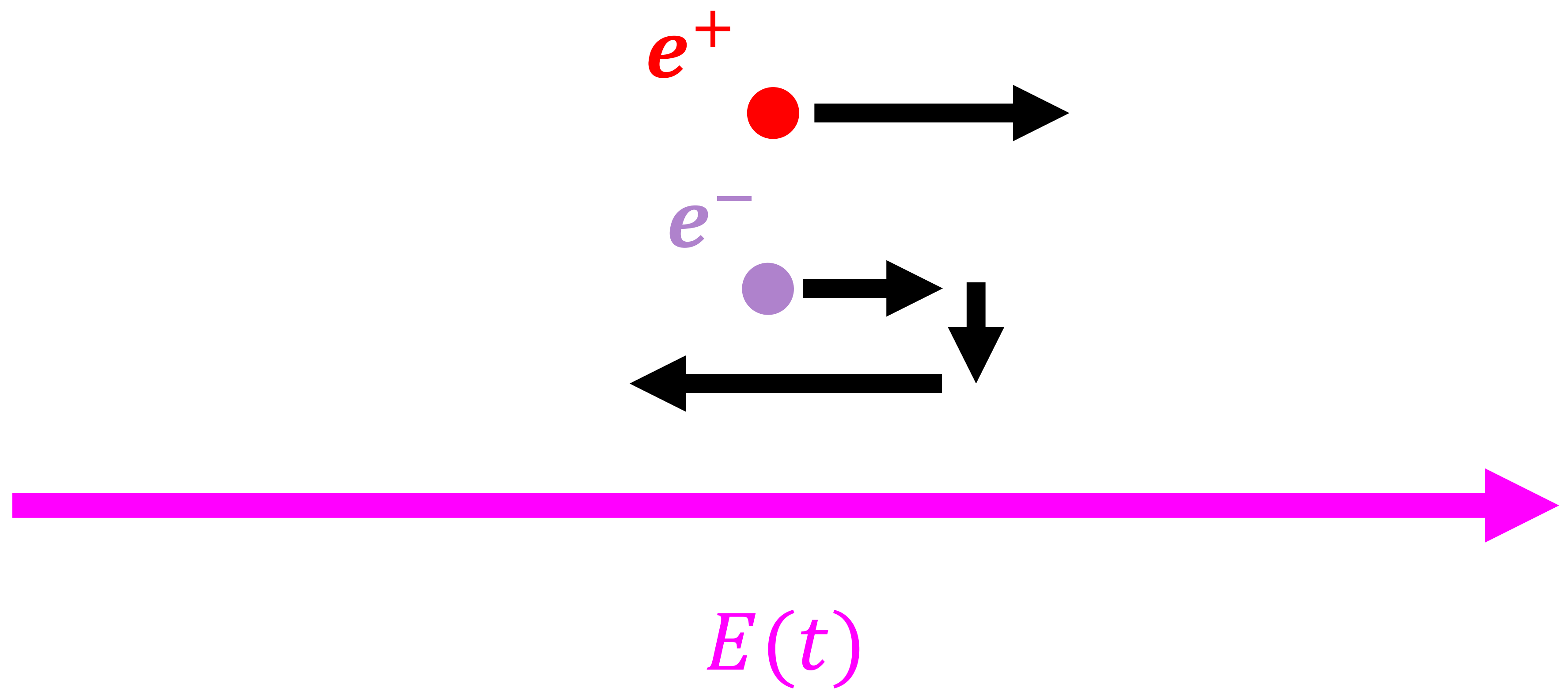


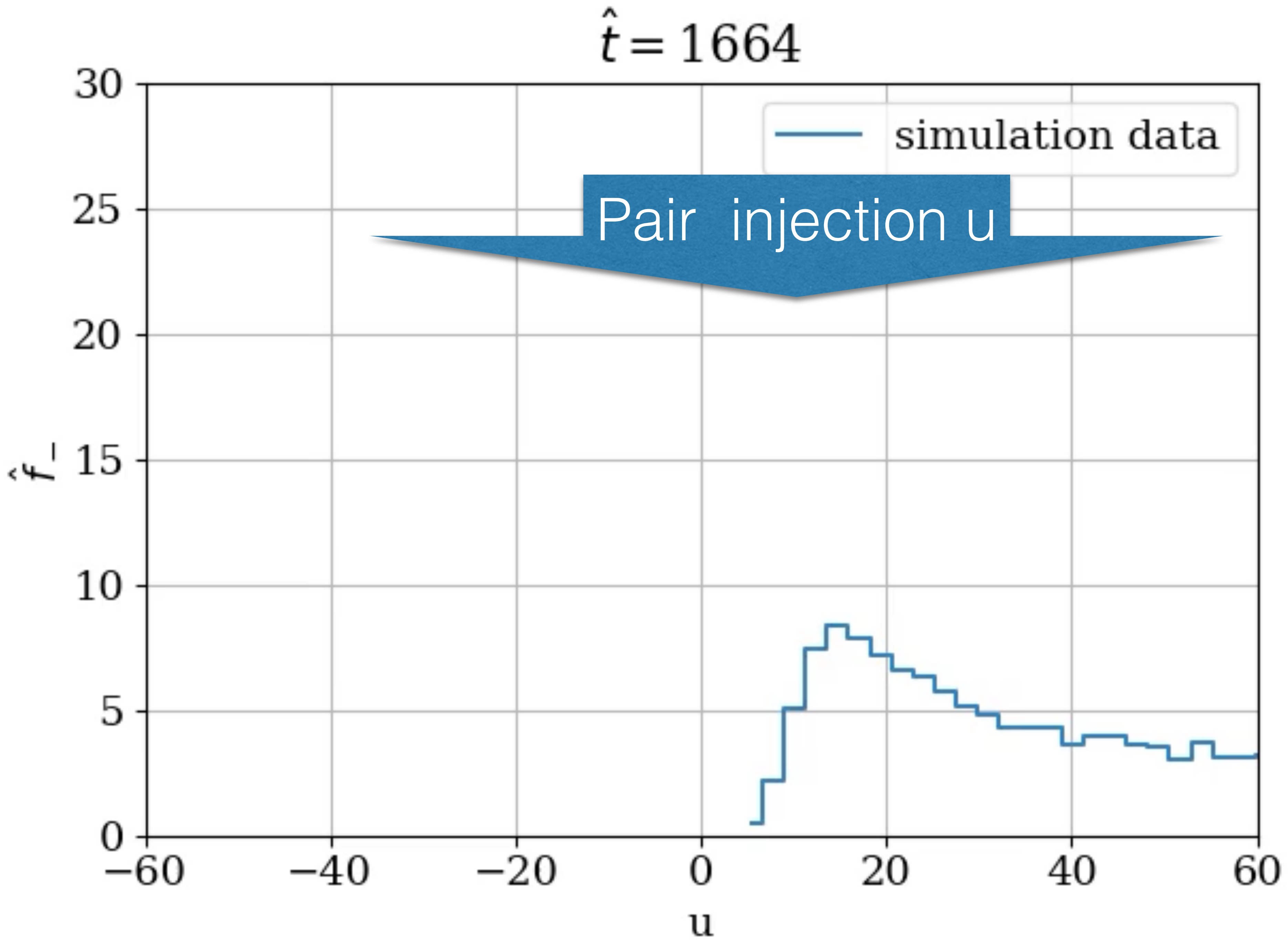
$$\partial_{\hat{t}}^2 \hat{E} + \delta(\hat{t} - [\widehat{T_0} + \Delta])\hat{E}(\hat{t}) = 0$$

Integrate

$$\partial_{\hat{t}}\hat{E}(\hat{t} > \widehat{T_0} + \Delta) = \partial_{\hat{t}}\hat{E}(\hat{t} < \widehat{T_0} + \Delta) - \hat{E}(\widehat{T_0} + \Delta)$$

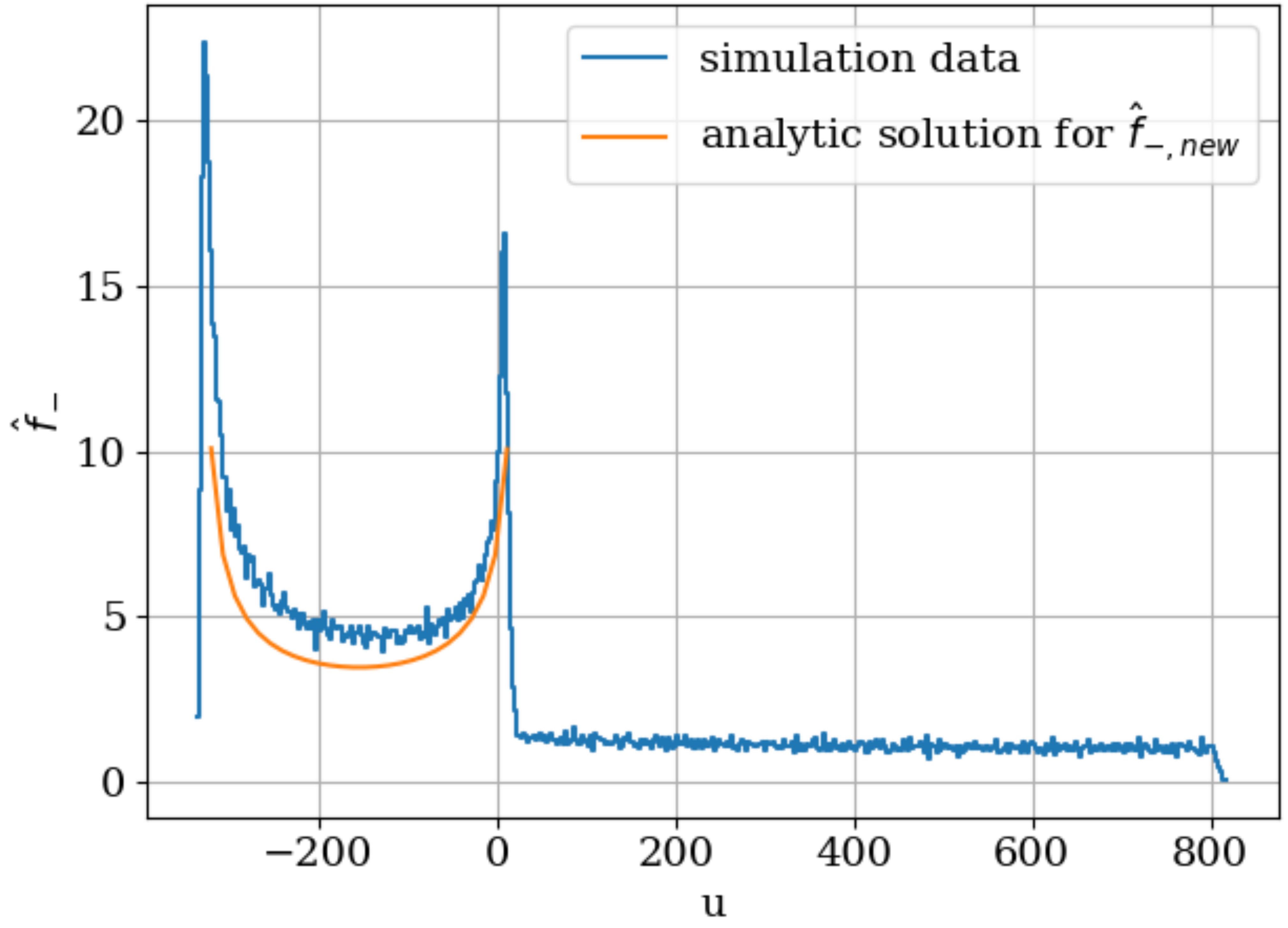
Physical mechanism is reversal of newly added pairs





- Lots of energy extracted from E field to reverse pairs

Analytic estimate of effect is possible

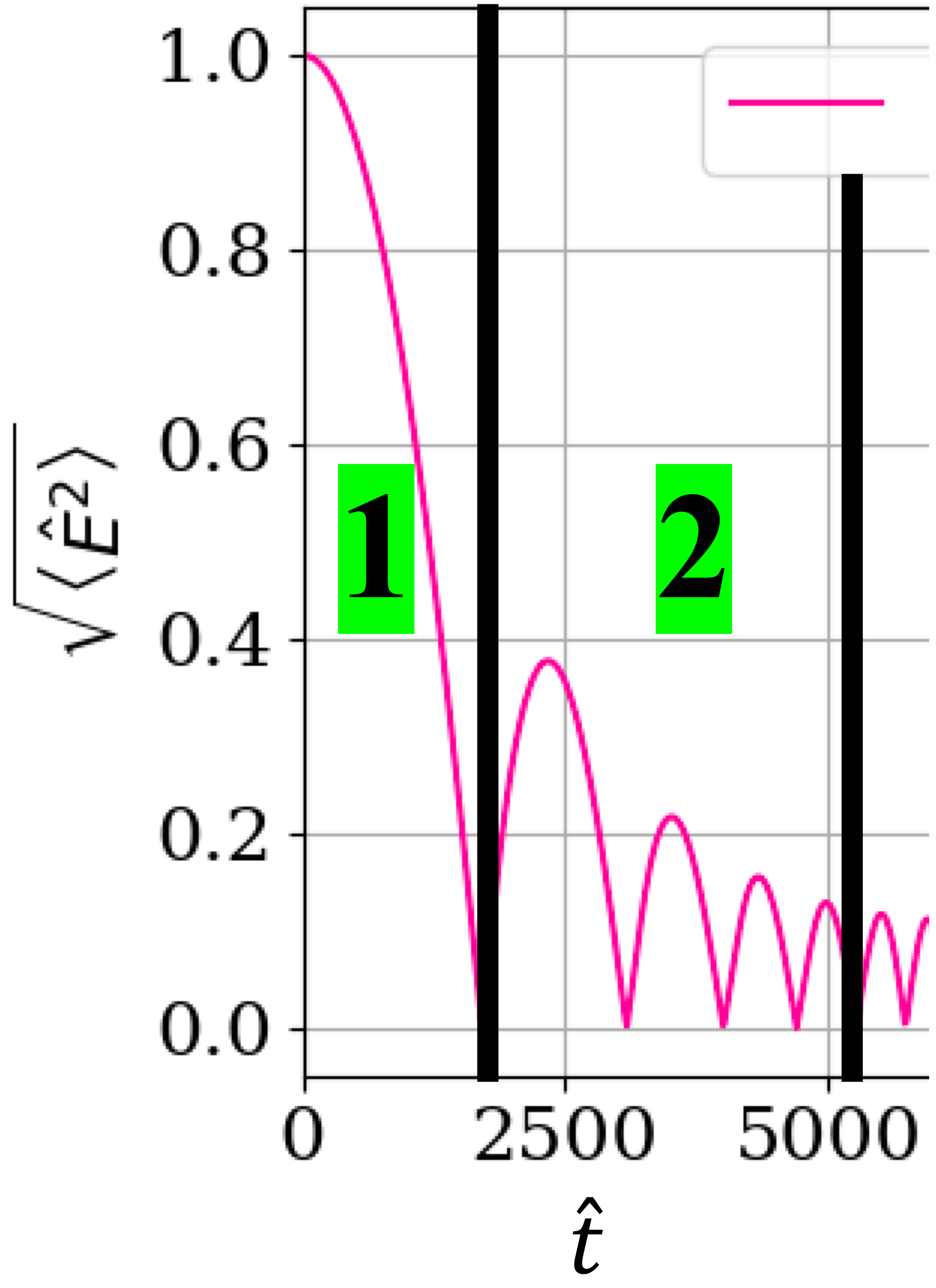


- Damping can be analytically modeled in more detail:
  1. Analytically solve Vlasov equation for sinusoidal field to get form of $\hat{f}_-$
  2. Numerically solve for $\left\langle \frac{1}{\gamma^3} \right\rangle$
  3. Evaluate damping strength:

$$\left|\hat{E}\right| \propto e^{-\xi^{-0.5}\hat{t}}$$

- Agrees with observed damping in simulation

Analytic estimate of effect is possible

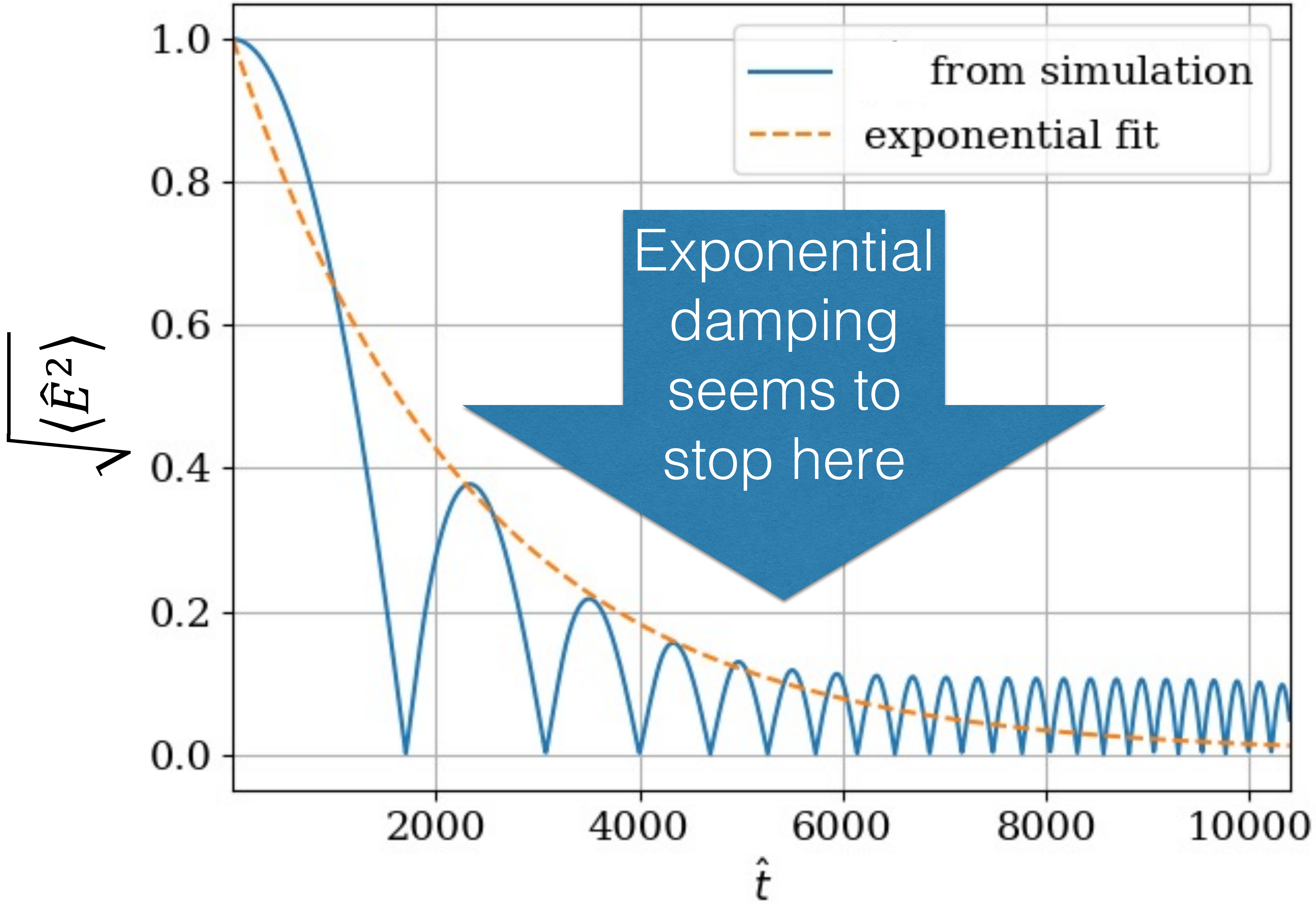


$$\left|\hat{E}\right| \propto e^{-\xi^{-0.5}\hat{t}}$$

$$\xi \sim 10^{12}\left(\frac{B_0}{10^{12}\,G}\right)^2\left(\frac{10^5}{\text{multiplicity}}\right)\left(\frac{0.1\,s}{\text{Period}}\right)^{7/2}$$

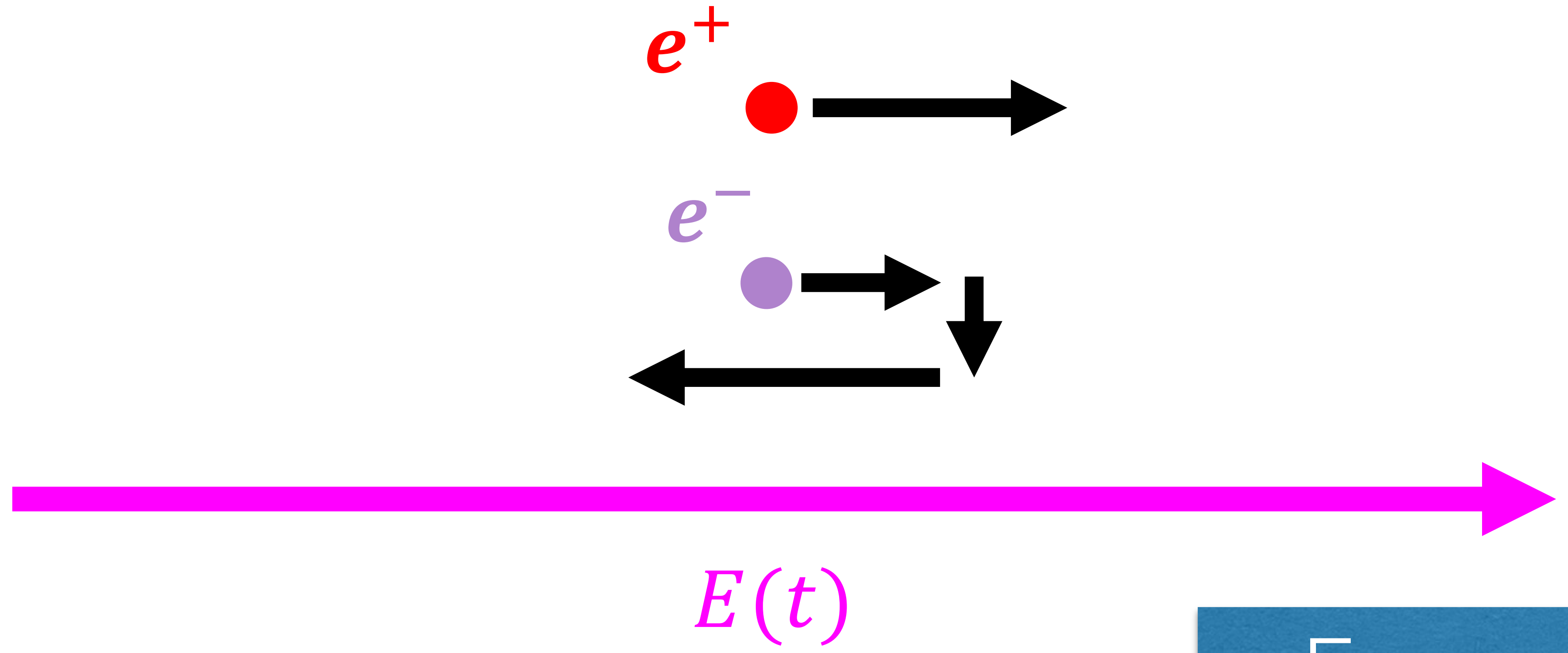
- Nonlinear damping is:
  - Weaker for higher B_0: more energy in initial E field
  - Stronger for higher multiplicity: more pairs injected
  - Stronger for higher period: less energy in initial E field
  - Very strong overall
- Nonlinear stage is expected to complete in all pulsars

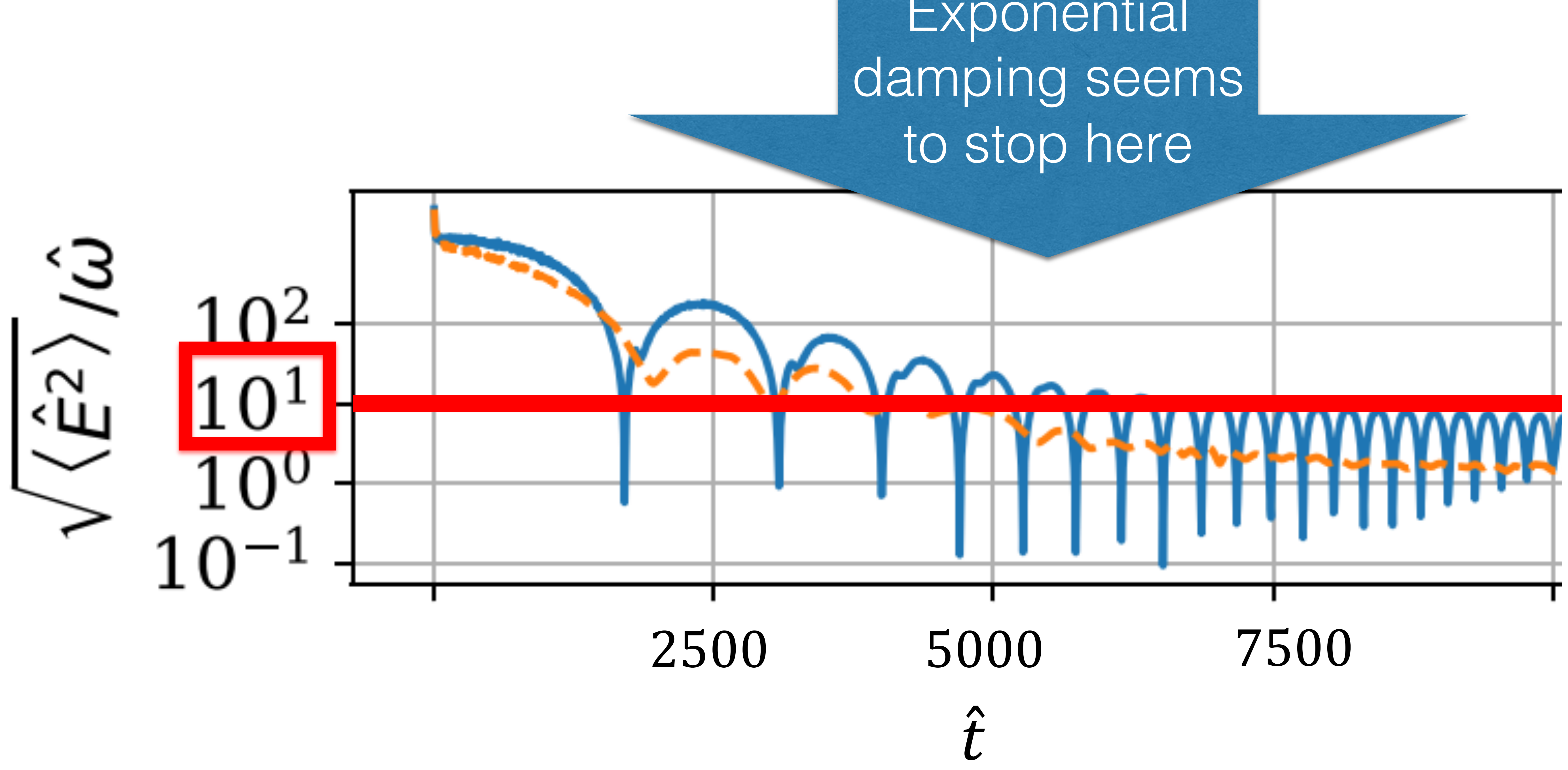
Exponential damping only lasts so long: why?



Exponential damping seems to stop here

Exponential damping only lasts so long: why?




Exponential damping seems to stop here

In this plot, the orange line refers to a discussion in the paper which we have not considered in this presentation

- Damping of E caused by E reversing pairs injected at u_l=10 and pulling them through $u = 0, \gamma = 1$
- Amount of momentum imparted to pair by field of amplitude $\hat{E}_\star$ is $\hat{E}_\star/\hat{\omega}$
- When $\hat{E}_\star/\hat{\omega}$ reaches u_l, pairs cannot be reversed
- End of strong damping

Transition from strong to weak damping gives luminosity

- Rewriting condition in un-normalized units, with realistic polar cap u_l gives

change in
pair γ from wave

Injected pair γ

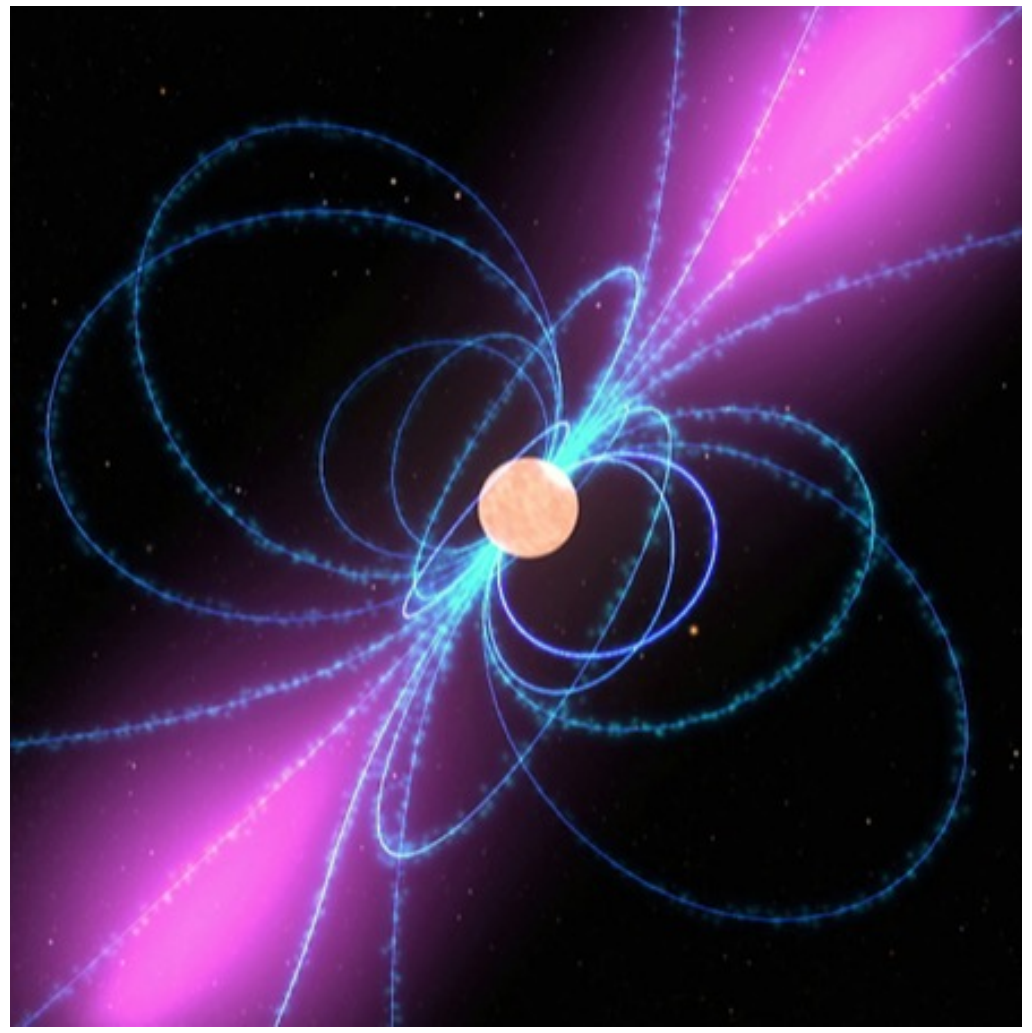
$$\frac{e\,E}{m\,\omega c} \sim 10^2$$

$\omega \sim 10^9 s^{-1}$in radio

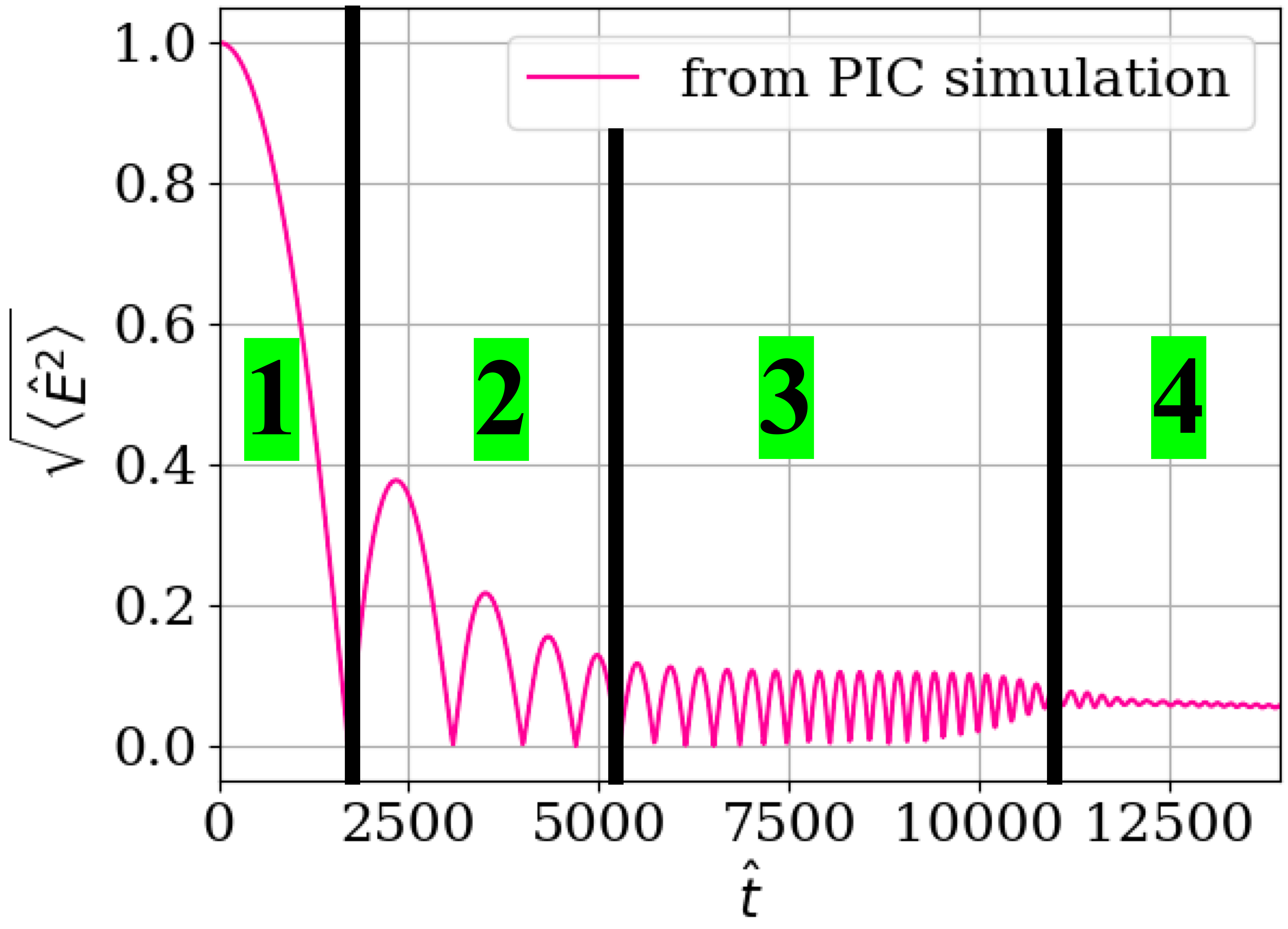
$$E \sim 10^4\ \mathrm{G}$$

- Consider emission at this amplitude across polar cap:

$cE^2\pi r_{pc}^2 \approx 10^{28}\ \mathrm{erg\,s^{-1}}$: consistent with observed radio luminosity

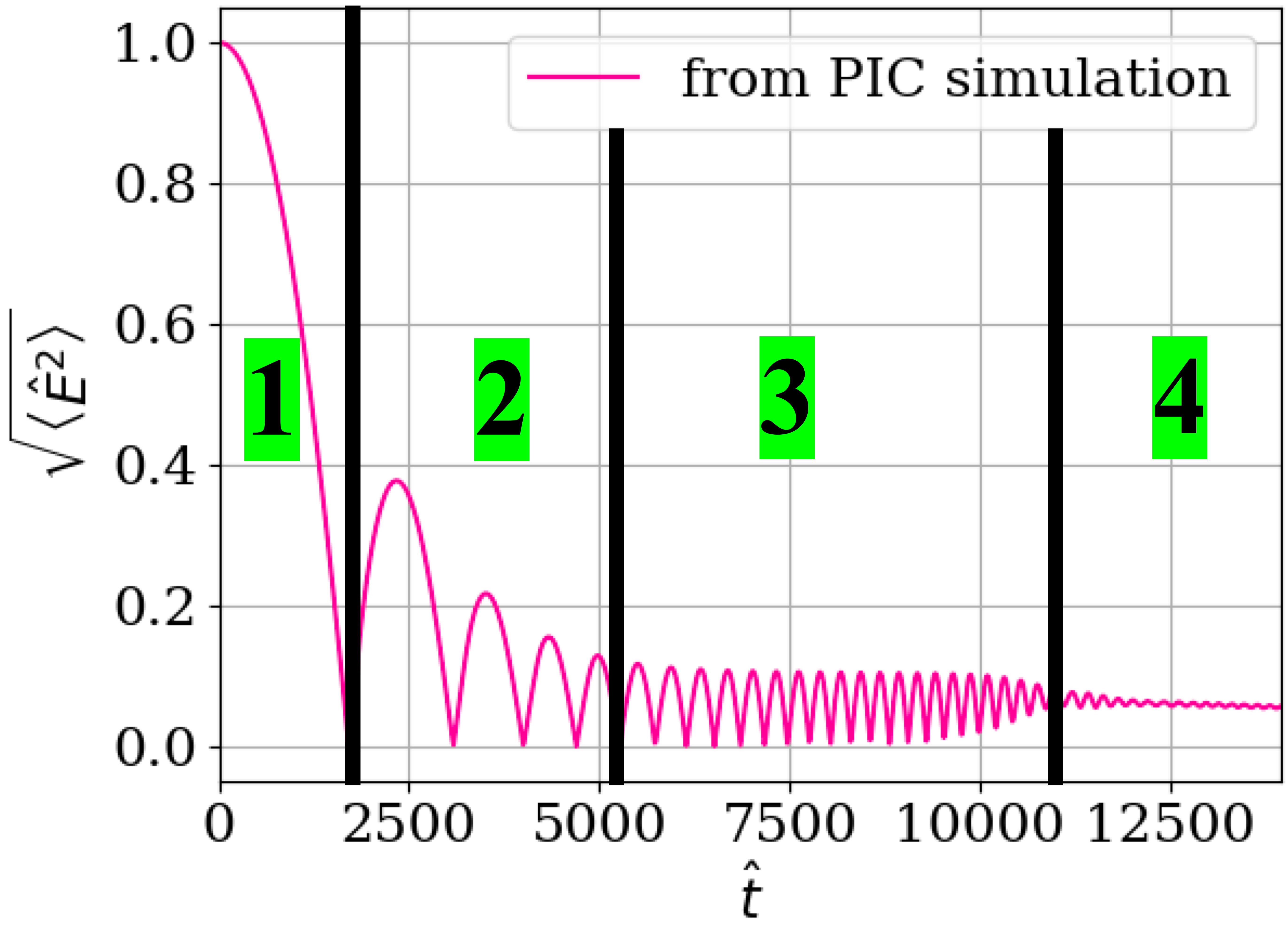


Linear stage has weak, slow damping



- After transition, system experiences "frozen" phase **3**
  - Artifact of uniform E field in simulation
  - Ends with fragmentation into low-k modes
- After transition, system becomes linear, phase **4**
  - Change in newly added pair u is much less than u_l

Linear stage has weak, slow damping



- In linear phase, wave amplitude and frequency continue to evolve due to pair creation
- This physics will occur across polar cap for realistic EM waves
- At this stage, some waves start to escape plasma as radio emission
- Across polar cap, emission escapes at **different** points in damping with **different** $\hat{E}$, $\hat{\omega}$

Frequency change small in linear stage

- Relationship between $\hat{E}$, $\widehat{\omega}$ gives spectrum
- $\hat{E}$, $\widehat{\omega}$ governed by $\partial_{\hat{t}}^2 \hat{E} + \widehat{\omega}^2 \hat{E} = 0$
- Change in $\widehat{\omega}$ is slow compared to $\widehat{\omega}$
- No sharp spikes in $\widehat{\omega}$ from pair spikes getting dragged through $\gamma = 1$

WKB analysis suggests explanation for spectrum

$$\partial_{\hat{t}}^2 \hat{E} + \hat{\omega}^2 \hat{E} = 0$$

- Applying WKB gives

$$\hat{E}^2 \sim S_{\hat{\omega}} \sim \hat{\omega}^{-1}$$

- Agrees with simulation
- May help to contribute to observed spectrum

$$S_\omega \sim \omega^{-1.4 \pm 1.0}$$

Conclusions

- Pulsar radio emission may be created by electric field screening in polar cap
- Radio luminosity can be understood as transition from nonlinear to linear physics
- Radio spectrum has contributions from linear damping

Based on *E.A. Tolman, A.A. Philippov, and A.N. Timokhin, Electric field screening in pair discharges and generation of pulsar radio emission, ApJL 933 L37 (2022)*

Slides available at elizabethtolman.com

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