Drift kinetic theory of alpha particle transport by tokamak perturbations

JPP Colloquium
January 14th, 2021

Based on paper accepted to JPP: "Drift kinetic theory of alpha transport by tokamak perturbations," Tolman and Catto. Available as arXiv:2011.04920, should appear shortly in JPP

Slides available later today at elizabethtolman.com

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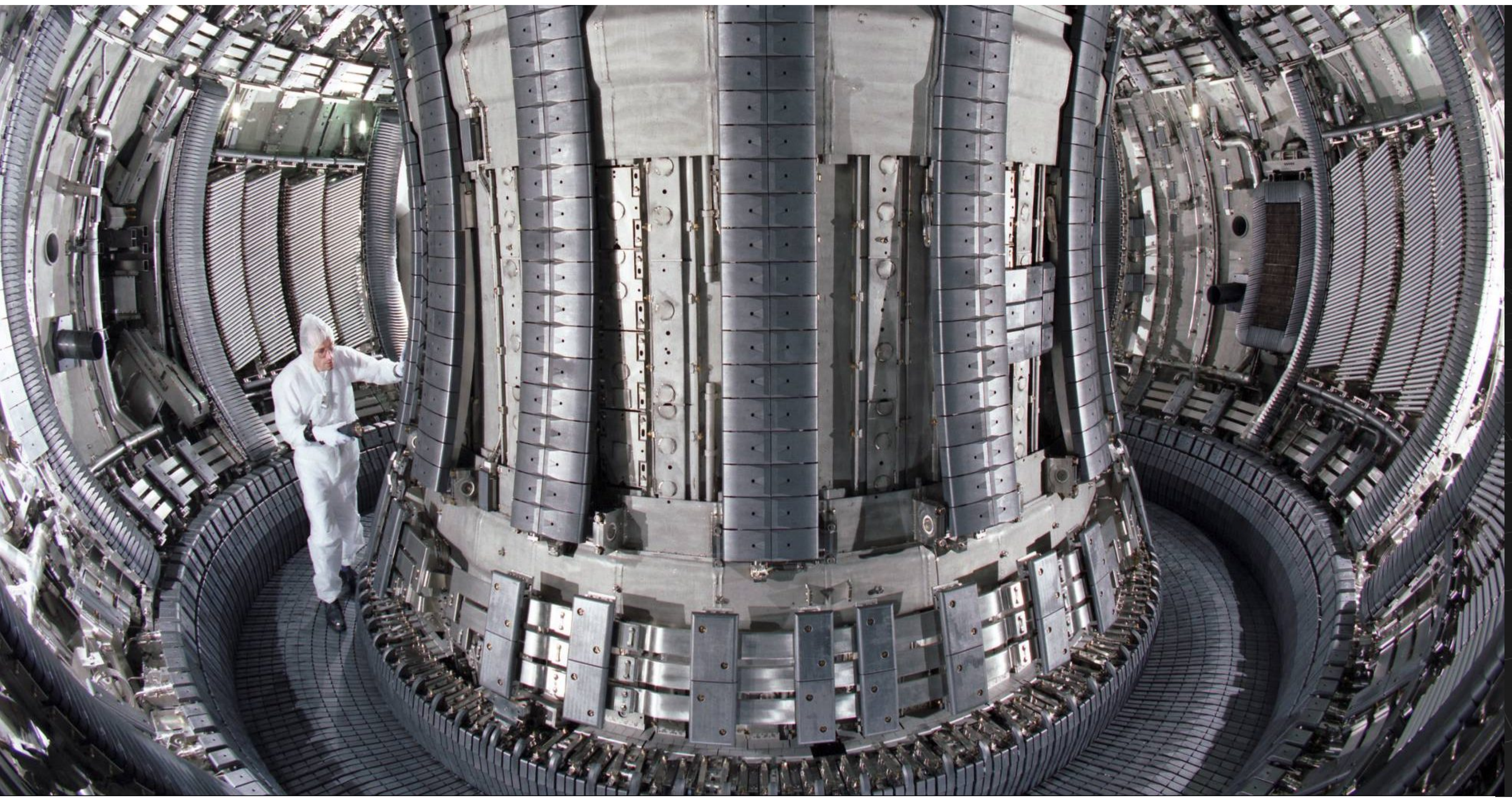
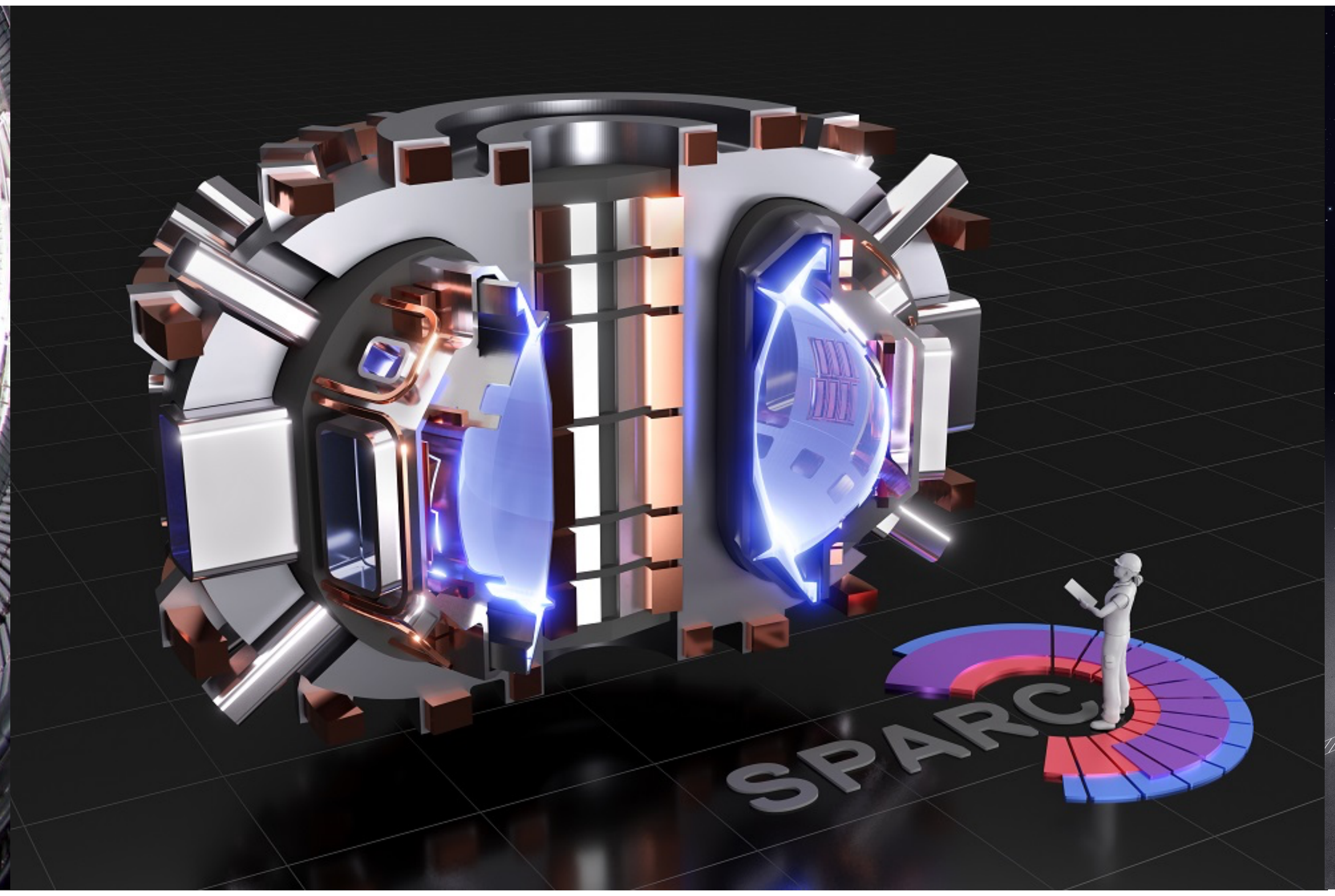
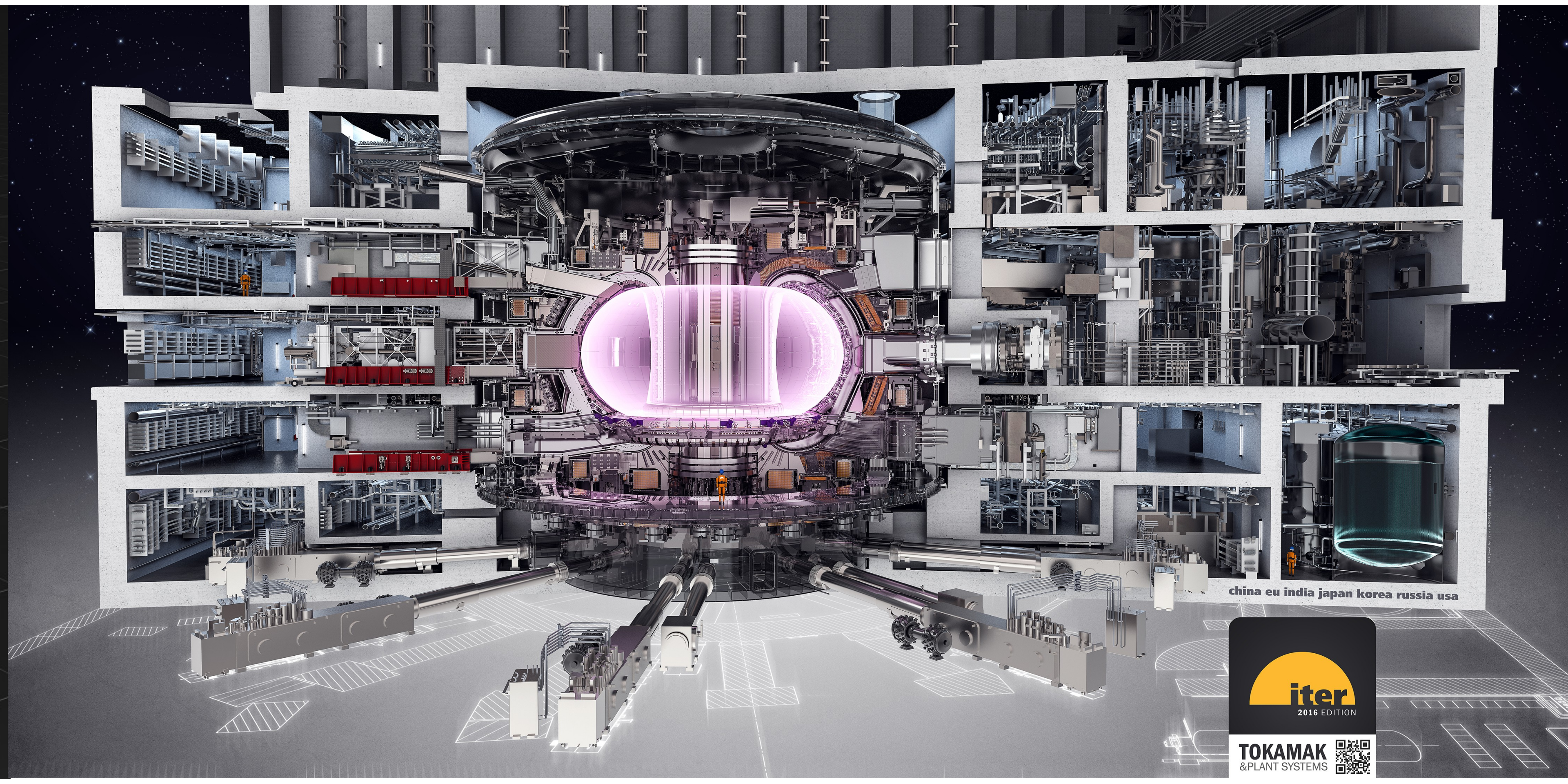
Multiple upcoming tokamak experiments will run DT plasmas



The Joint European Torus (JET) chamber
Source: CCFE



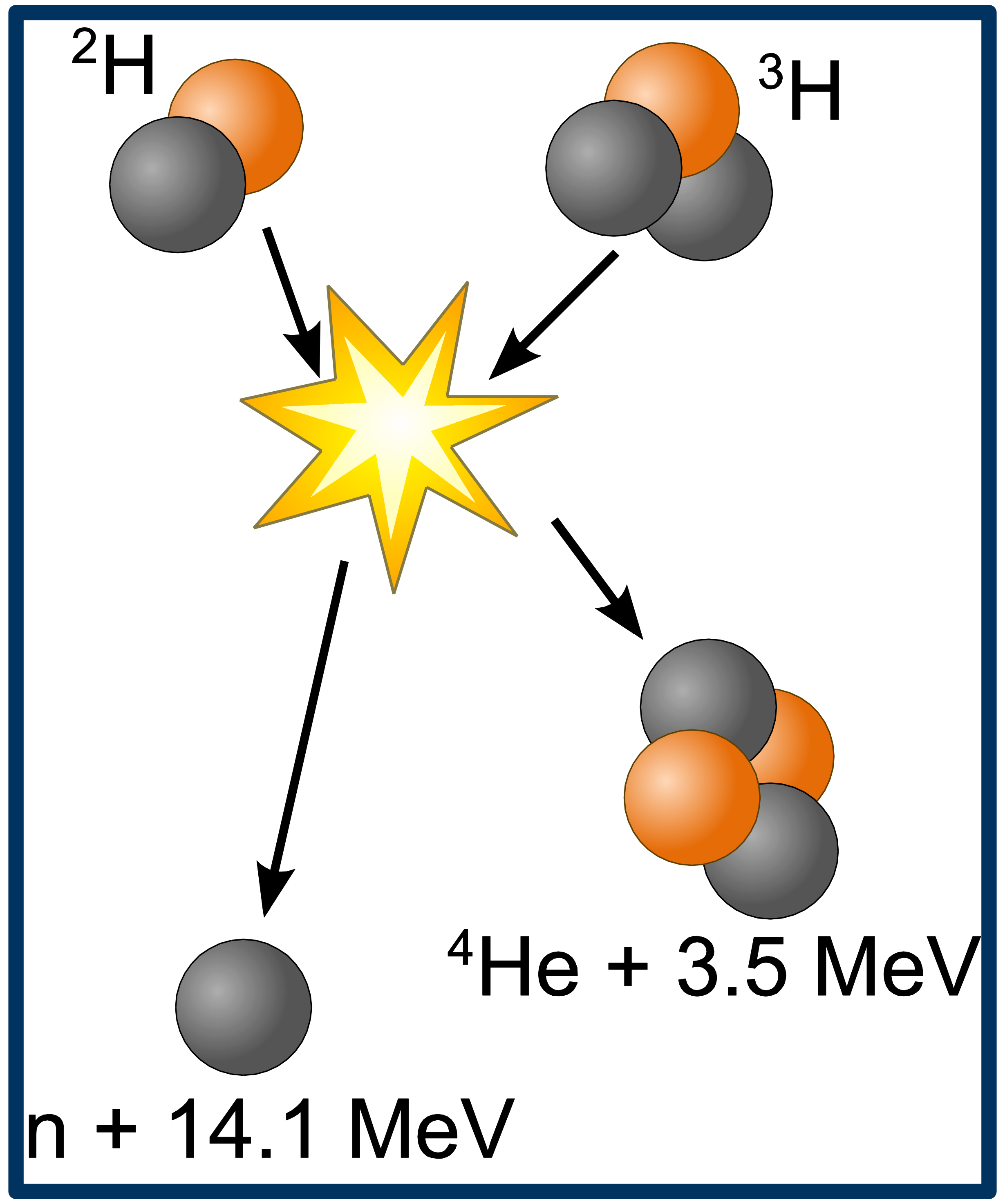
Rendering of the SPARC tokamak
Source: CFS/MIT-PSFC - CAD Rendering by T. Henderson



Rendering of the ITER tokamak
Source: ITER Organization, http://www.iter.org/

- A series of upcoming tokamak experiments plan to run with DT fuel
  - JET DT campaign
  - SPARC
  - ITER
- First DT experiments since the 1990's
  - (with exception of trace tritium experiments)
- Exciting plasma physics motivates new attention to relevant theory

Alpha physics is novel part of next-generation DT tokamaks



- One novel, important part of DT tokamak physics is alpha particle behavior

- Alphas heat bulk plasma, help maintain its temperature
  - For SPARC primary reference plasma: $P_\alpha \approx 28\ \mathrm{MW}$, $P_{rf,coupled} + P_{ohmic} \approx 12.8\ \mathrm{MW}$ [Creely et al. APS DPP 2020]

- Alphas can interact with perturbations to tokamak electric and magnetic fields, causing transport
  - Transport can modify heat deposition
  - Loss can degrade performance, damage device

Outline

- Unperturbed alpha distribution and the perturbations that affect it
- Drift kinetic equation governing transport
- Evaluation of transport
- Strength of TAE transport

Unperturbed alpha distribution and the perturbations that affect it

Unperturbed alpha distribution is peaked in core

- Unperturbed alpha population given by slowing down distribution:

$$f_0(\mathrm{v},\psi) = \frac{S_{fus}(\psi)\tau_s(\psi)H(\mathrm{v}-\mathrm{v}_0)}{4\,\pi[\mathrm{v}^3 + \mathrm{v}_c^3(\psi)]}$$

$$S_{fus}\tau_s = n_D n_T \langle \sigma \mathrm{v} \rangle\, \tau_s \propto nT^{7/2}$$

Parameter definitions

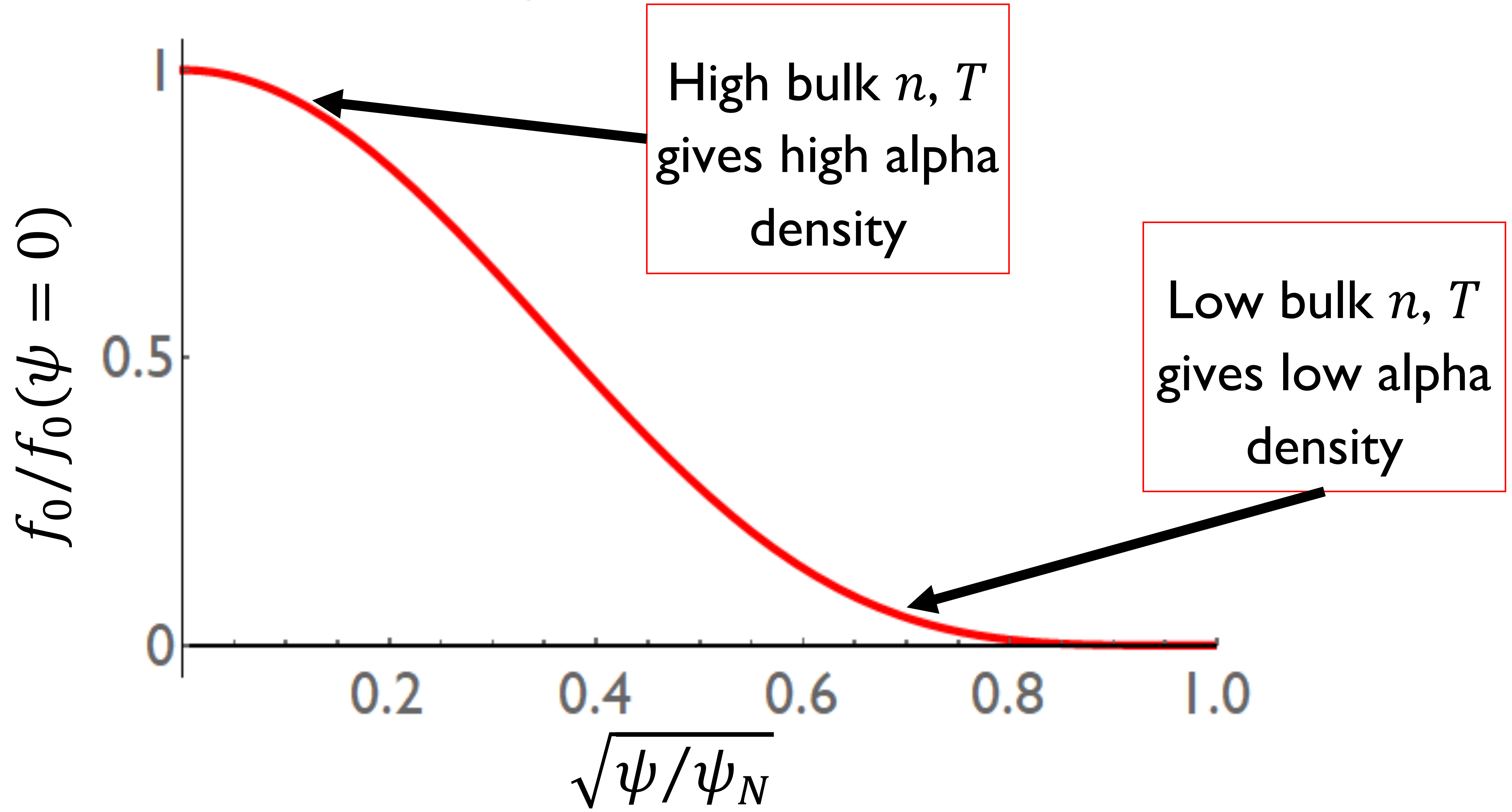
S_{fus}: fusion rate

$H(\mathrm{v}-\mathrm{v}_0)$: Heaviside step function

v_c : critical speed

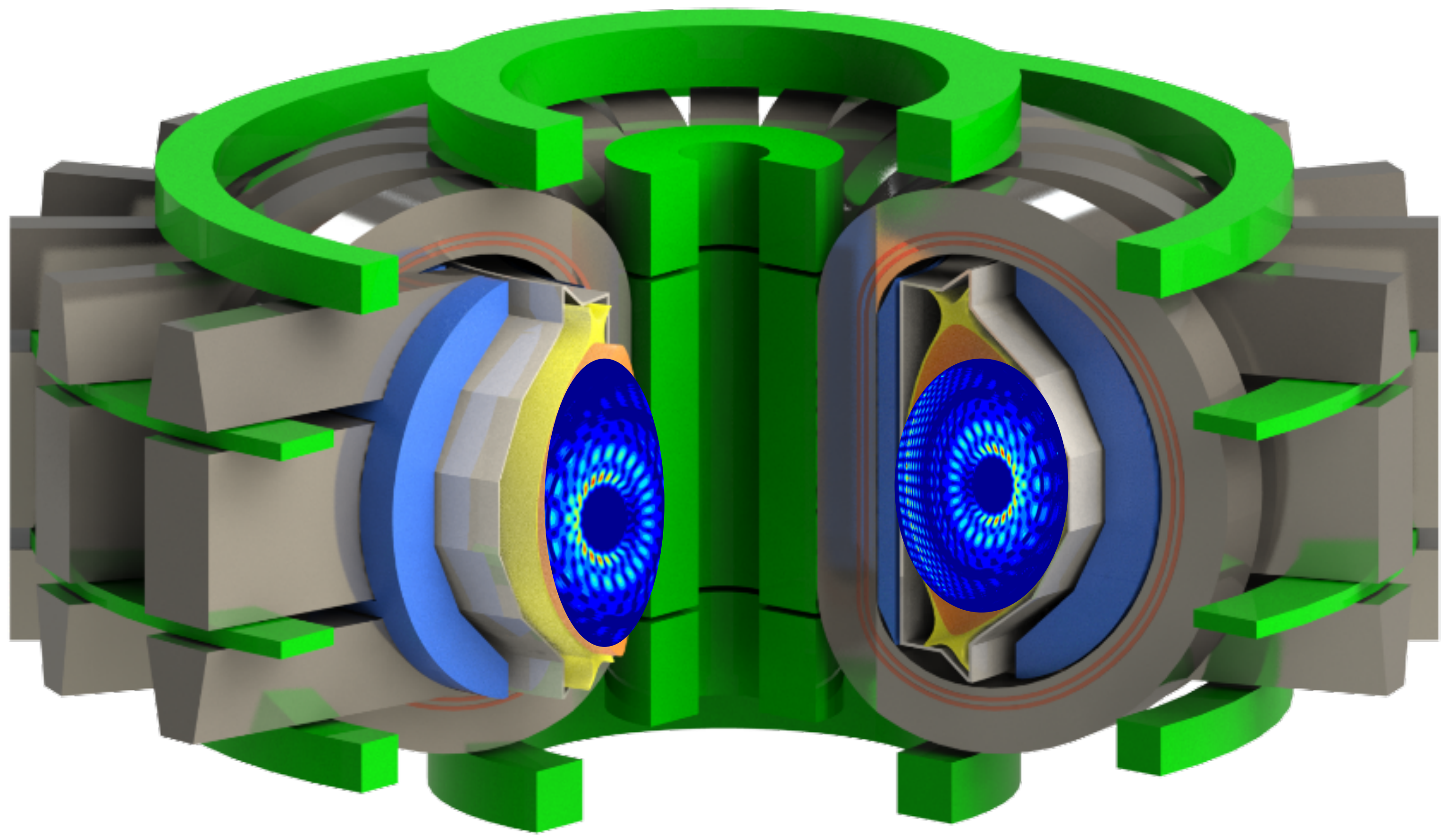
$\tau_s \propto T_e^{\frac{3}{2}}/n_e$: slowing down time

- Many alphas toward core; few towards edge

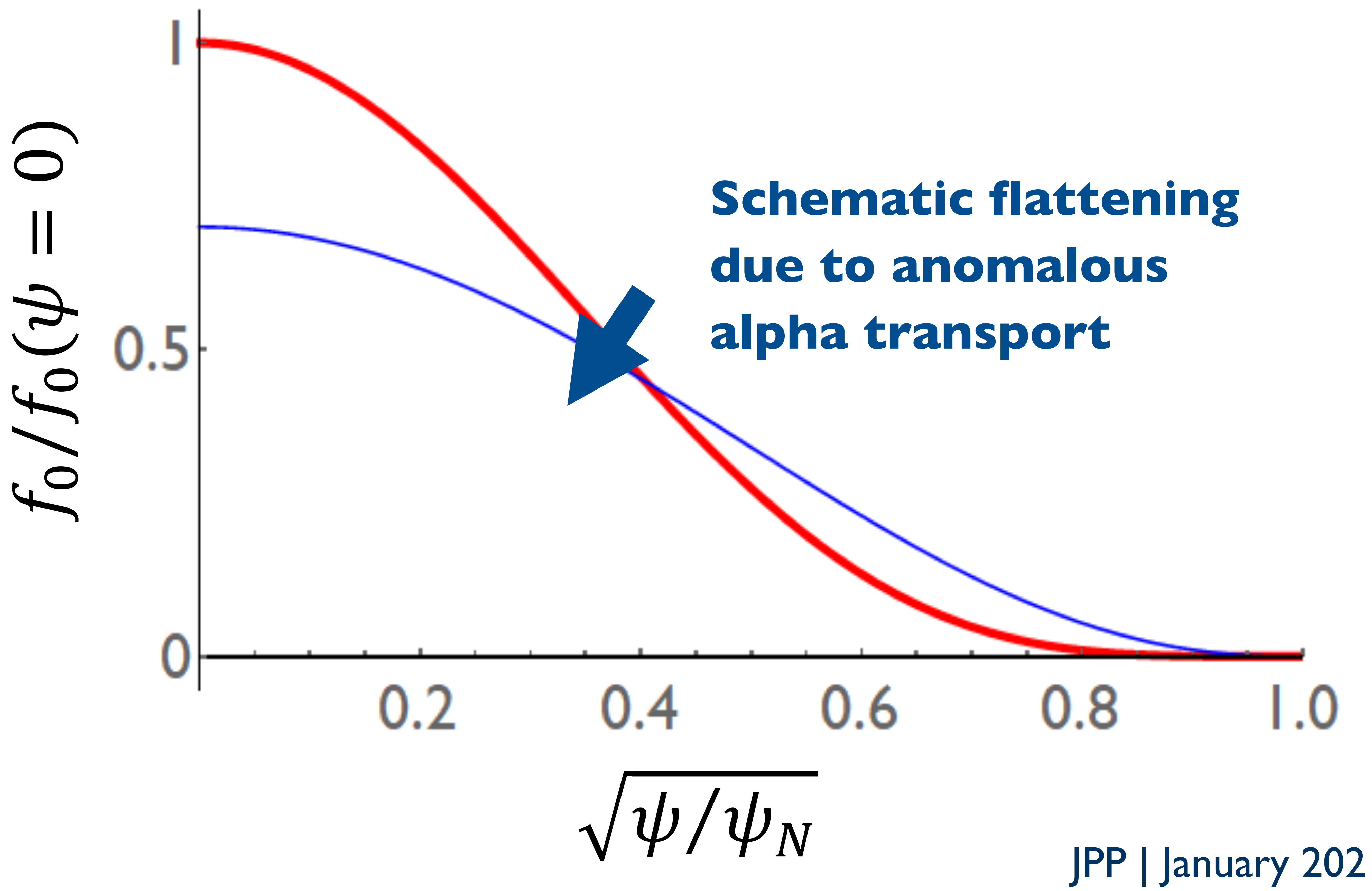


Interaction of alphas and perturbations leads to transport

- Tokamak fields experience a variety of perturbations
  - Ripple
  - MHD modes (Alfvén eigenmodes, NTMs, etc.)
  - RMP coils
- These perturbations create perturbed:
  - Alpha distribution: f_1
  - Alpha radial velocity: v_r
- Leads to transport of alphas from core to edge
  - Excessive transport could lead to loss of necessary alpha heat or to damage to device



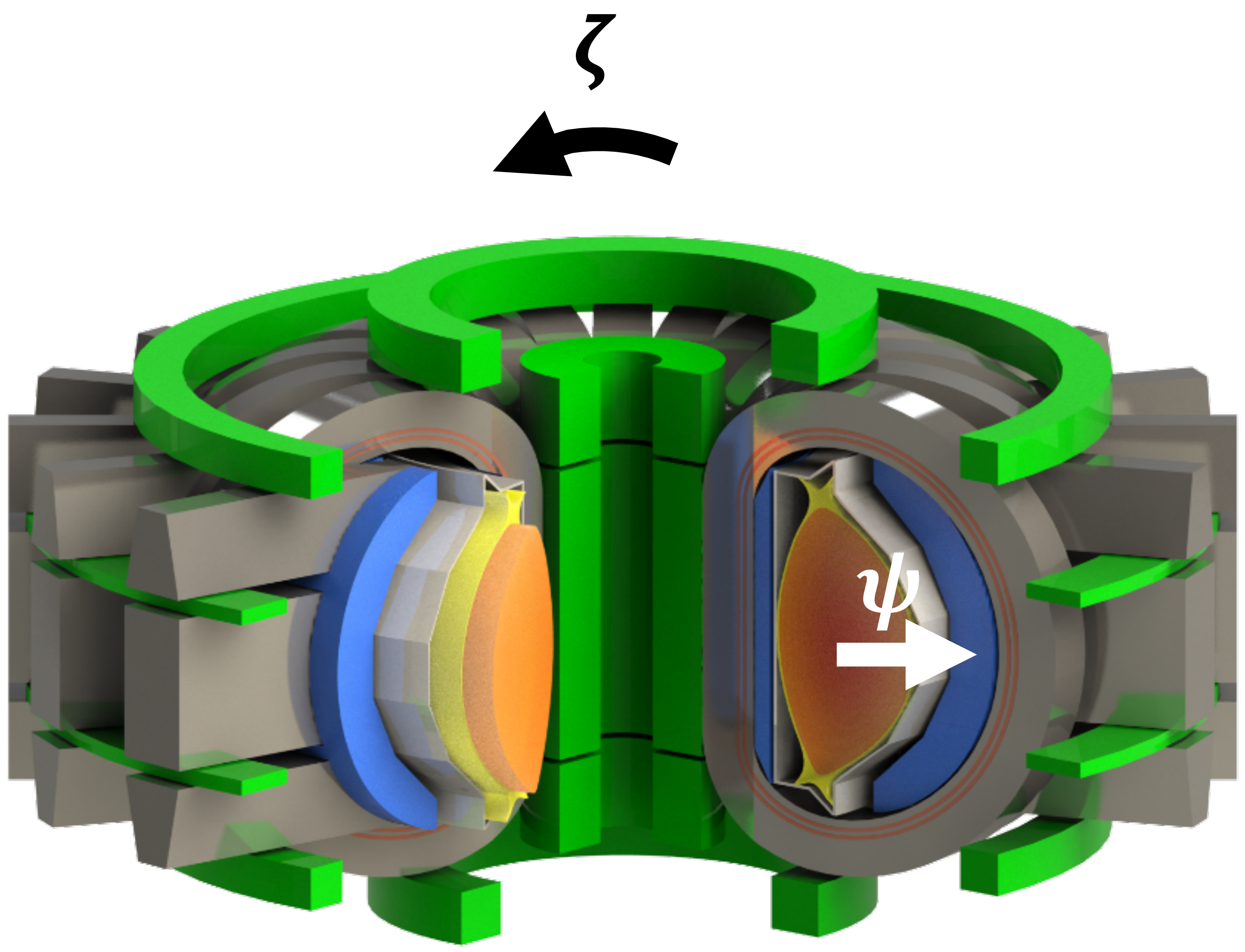
[Image sources: Mumgaard APS DPP 2018 + Snicker et al. NF 2013]



This presentation uses ripple as one example

- Tokamaks have discrete toroidal field coils (≈ 18)
- Discrete coils yield a small, stationary perturbation to tokamak magnetic field

$$B_1 \approx B_n(\psi)\cos(n\zeta)$$



[Image source: Mumgaard APS DPP 2018]

The toroidal Alfvén eigenmode is another example

- In a tokamak, Alfvén waves can exist as eigenmodes (AEs)

$$B_1 = \sum_m B_{mn\omega}(\psi)\cos(n\zeta - m\vartheta - \omega t)$$

$$E_1 = \sum_m E_{mn\omega}(\psi)\cos(n\zeta - m\vartheta - \omega t)$$

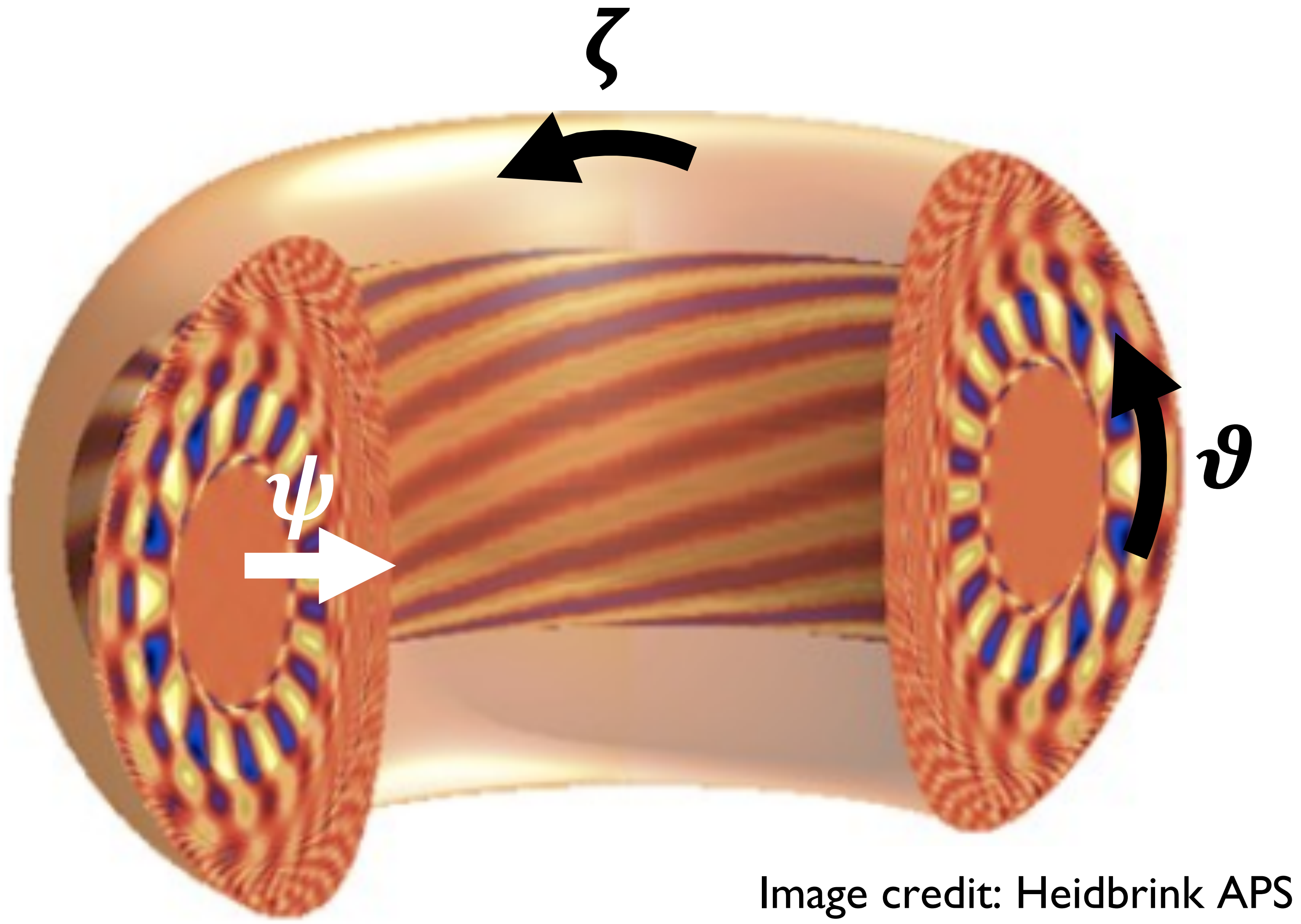
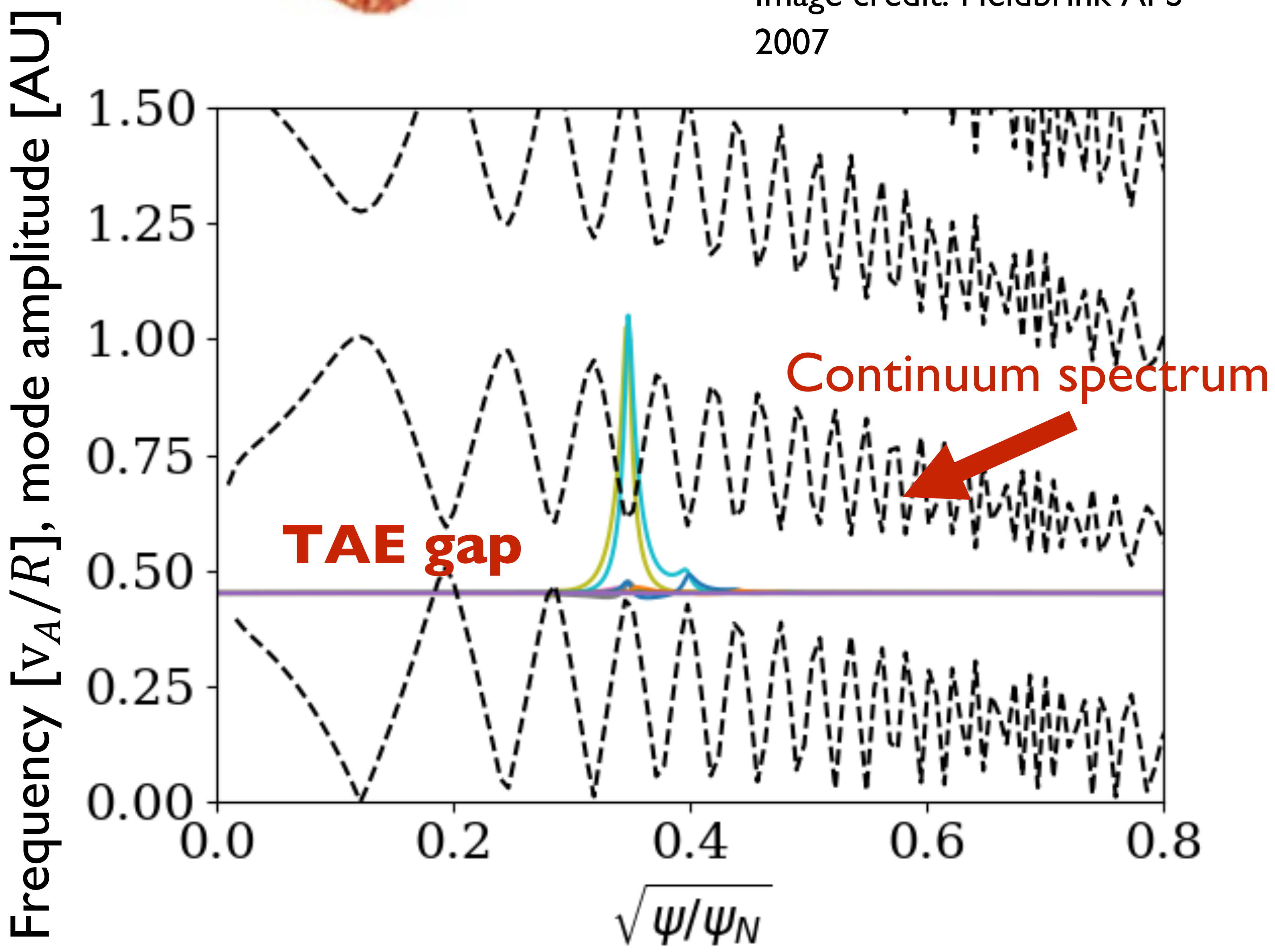


Image credit: Heidbrink APS 2007

- AEs exist at a set of discrete frequencies
  - The TAE exists at $\omega_{TAE} = \frac{v_A}{2qR}$

- AEs driven by spatial gradient of alpha population through resonance with alpha orbits
  - Resonant speed depends on bounce harmonic and particle pitch angle
  - Increases as Alfvén speed increases



We develop drift kinetic theory of transport

- Theory of alpha transport by perturbations focuses on single alpha trajectories
  - Codes used to study alpha distribution

*We develop a drift kinetic theory for **D**, the alpha diffusivity caused by a tokamak perturbation, as a function of perturbation characteristics*

•Apply theory to ripple (not very interesting) and TAE (interesting)

Drift kinetic equation governing transport

Perturbations have E and B field perturbations

- Ripple has a magnetic field perturbation
- TAE includes magnetic field and electric field perturbations

Magnetic perturbation is given by:

$$\vec{B}_1 = \vec{B}_{mn\omega} e^{i(n\,\zeta - m\vartheta - \omega t)} e^{i\int d\psi \frac{k_\psi}{RB_p}}$$

Amplitude | Wave phase | Radial variation

Electric potential perturbation is given by:

$$\Phi_1 = \Phi_{mn\omega}(B_{mn\omega}) e^{i(n\,\zeta - m\vartheta - \omega t)} e^{i\int d\psi \frac{k_\psi}{RB_p}}$$

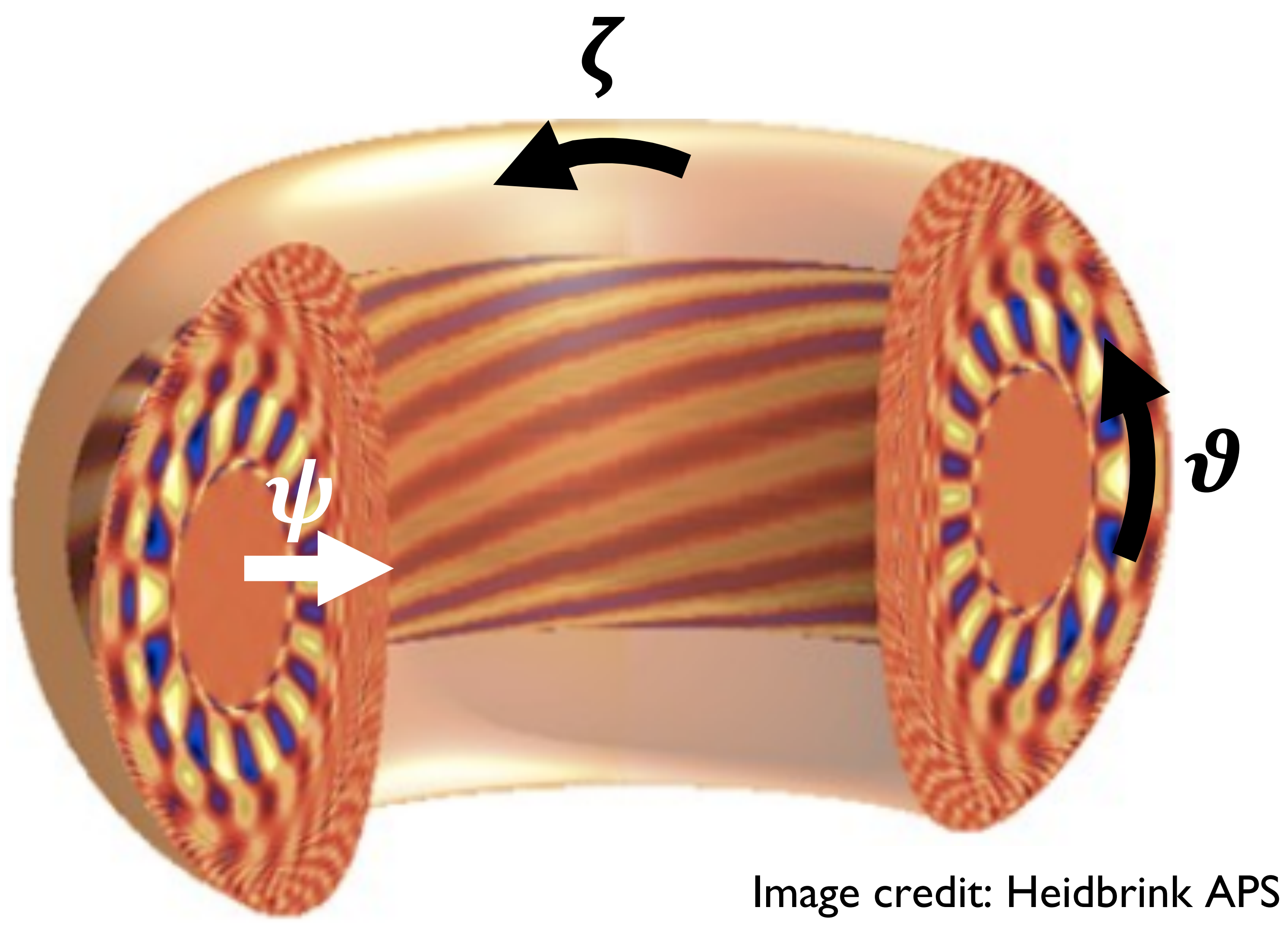


Image credit: Heidbrink APS 2007

Field perturbations perturb alpha distribution, velocity

Magnetic perturbation is given by:

$$\vec{B}_1 = \vec{B}_{mn\omega} e^{i(n\zeta - m\vartheta - \omega t)} e^{i\int d\psi \frac{k_\psi}{RB_p}}$$

Electric potential perturbation is given by:

$$\Phi_1 = \Phi_{mn\omega}(B_{mn\omega}) e^{i(n\zeta - m\vartheta - \omega t)} e^{i\int d\psi \frac{k_\psi}{RB_p}}$$

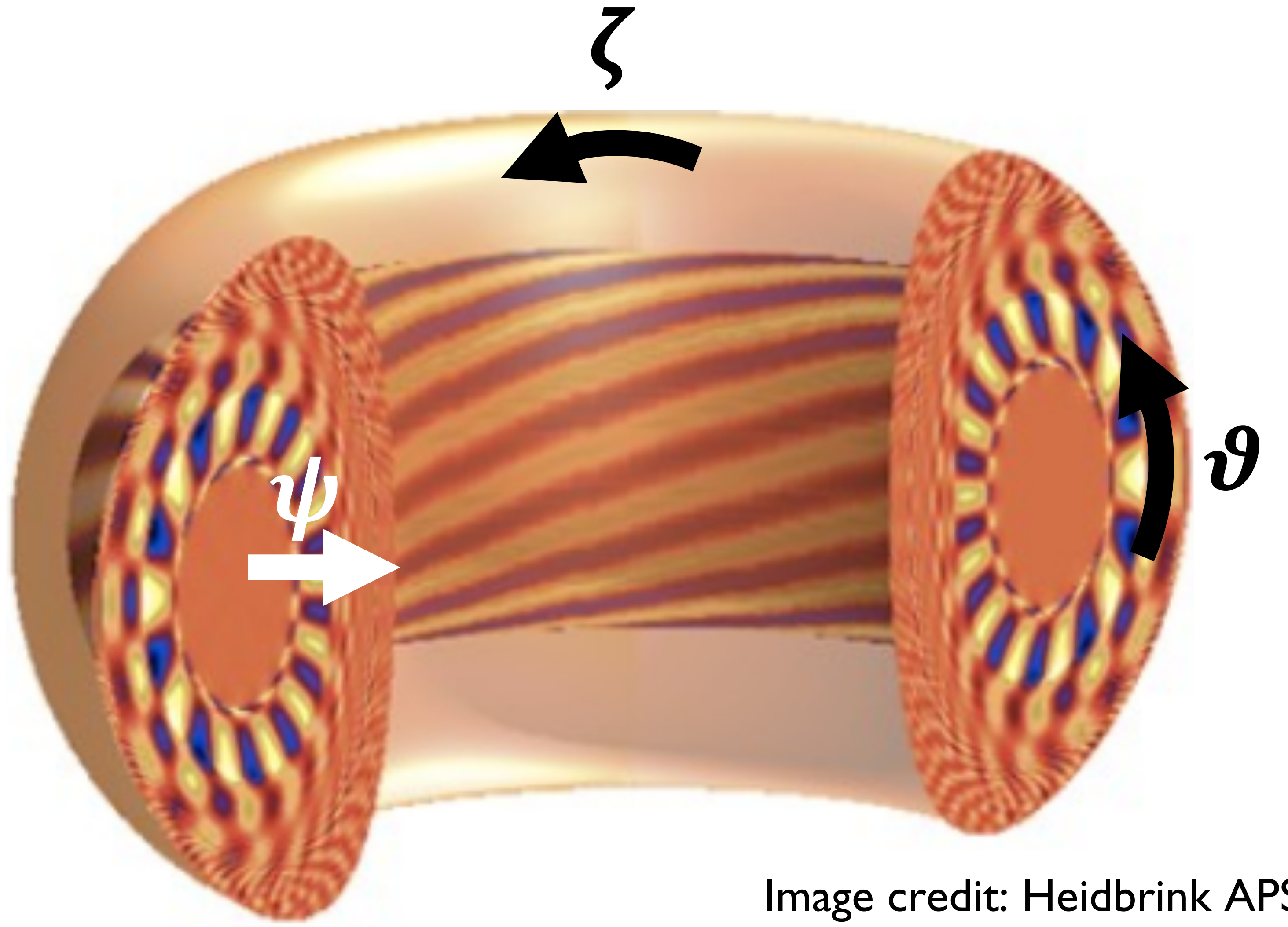


Image credit: Heidbrink APS 2007

- Corresponding perturbation to alpha distribution function is created

$$f_1 = \frac{Ze\,\Phi_1}{M}\frac{\partial f_0}{\partial \mathcal{E}} + h(\vartheta) e^{i[n(\zeta - q\vartheta) - \omega t]} e^{i\int d\psi \frac{k_\psi}{RB_p}}$$

Adiabatic response ($\mathcal{E}$ is energy) Response that causes transport

- A radial perturbation to the alpha particle velocity, v_r, is created
  - Results from drifts and changed B field direction

- Transport determined by product of h and v_r

Multiple perturbations work together to cause transport

- In realistic tokamak, multiple perturbations and multiple poloidal harmonics per perturbation
- Perturbations at different radial locations work together to cause transport across cross section
- For perturbations with significant radial overlap:

$n \neq n'$	$n = n'$
• h from one perturbation does not couple to v_r from other • Diffusion is superimposed	• Transport couples for similar m • See discussion in paper

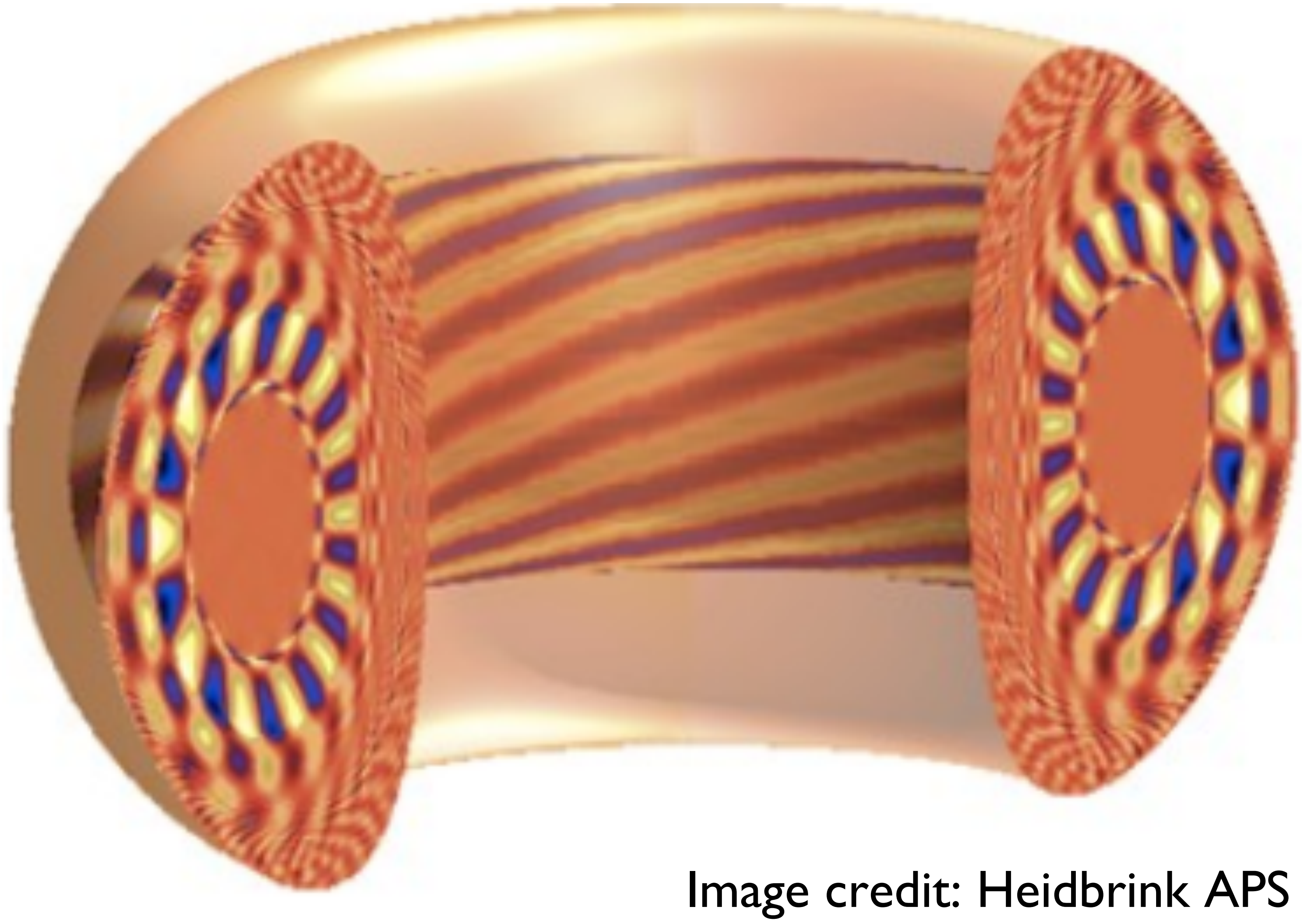


Image credit: Heidbrink APS 2007

Drift kinetic equation models transport, gives *h*

The perturbed drift kinetic equation used to find h is:

$$\boxed{v_{\parallel}\hat{b}\cdot\nabla\vartheta\,\frac{\partial h}{\partial\vartheta} - i\,[\omega - n\omega_{\alpha}]h + i v_{r}\frac{\partial f_0}{\partial r}G(\vartheta) = \nu_{pas}\frac{\partial^2 h}{\partial\lambda^2}}$$

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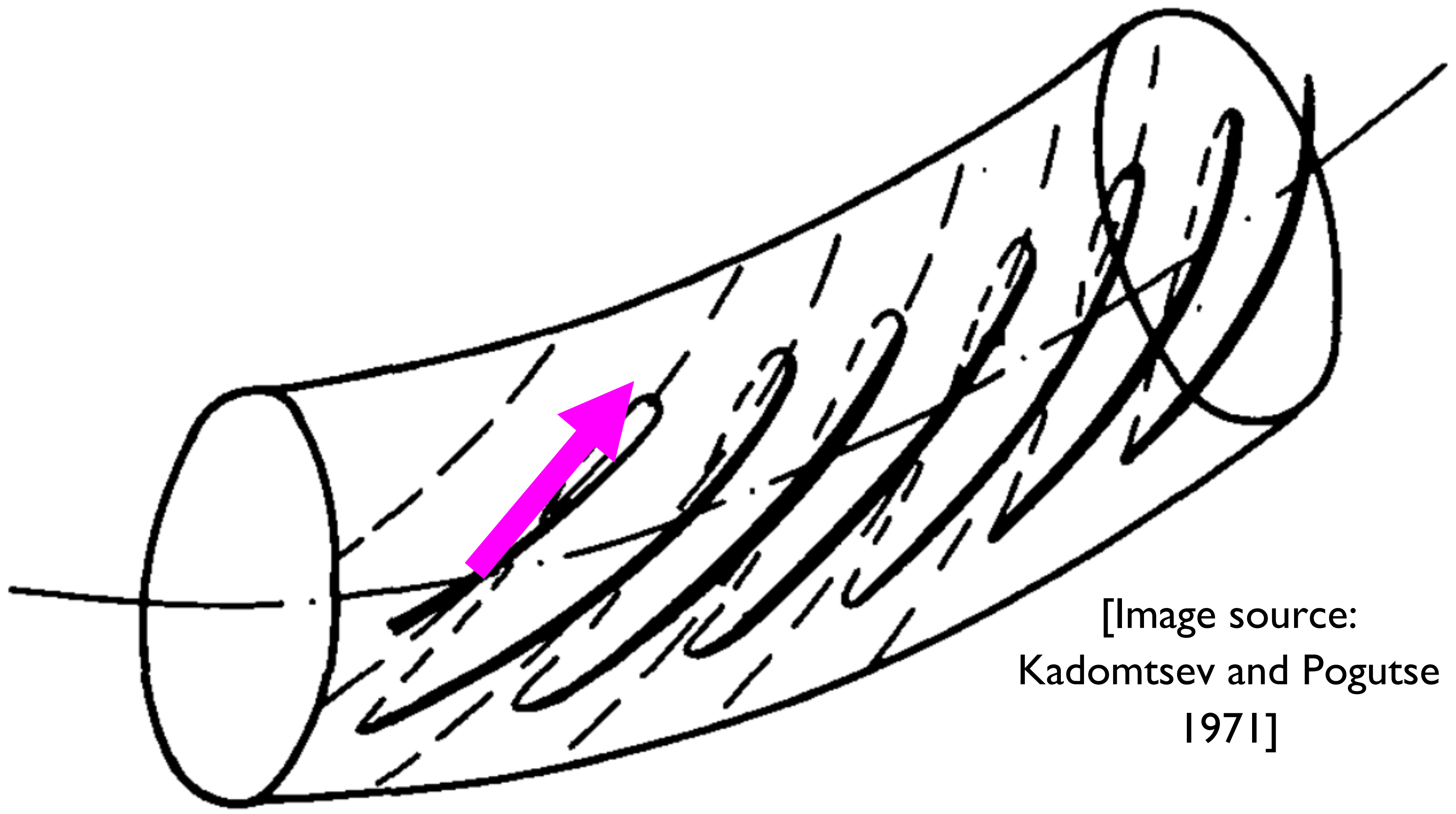
Streaming of unperturbed alpha orbit along magnetic field



[Image source: Kadomtsev and Pogutse 1971]

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Perturbation frequency

Drift kinetic equation models transport, gives *h*

The perturbed drift kinetic equation used to find h is:

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Toroidal mode number

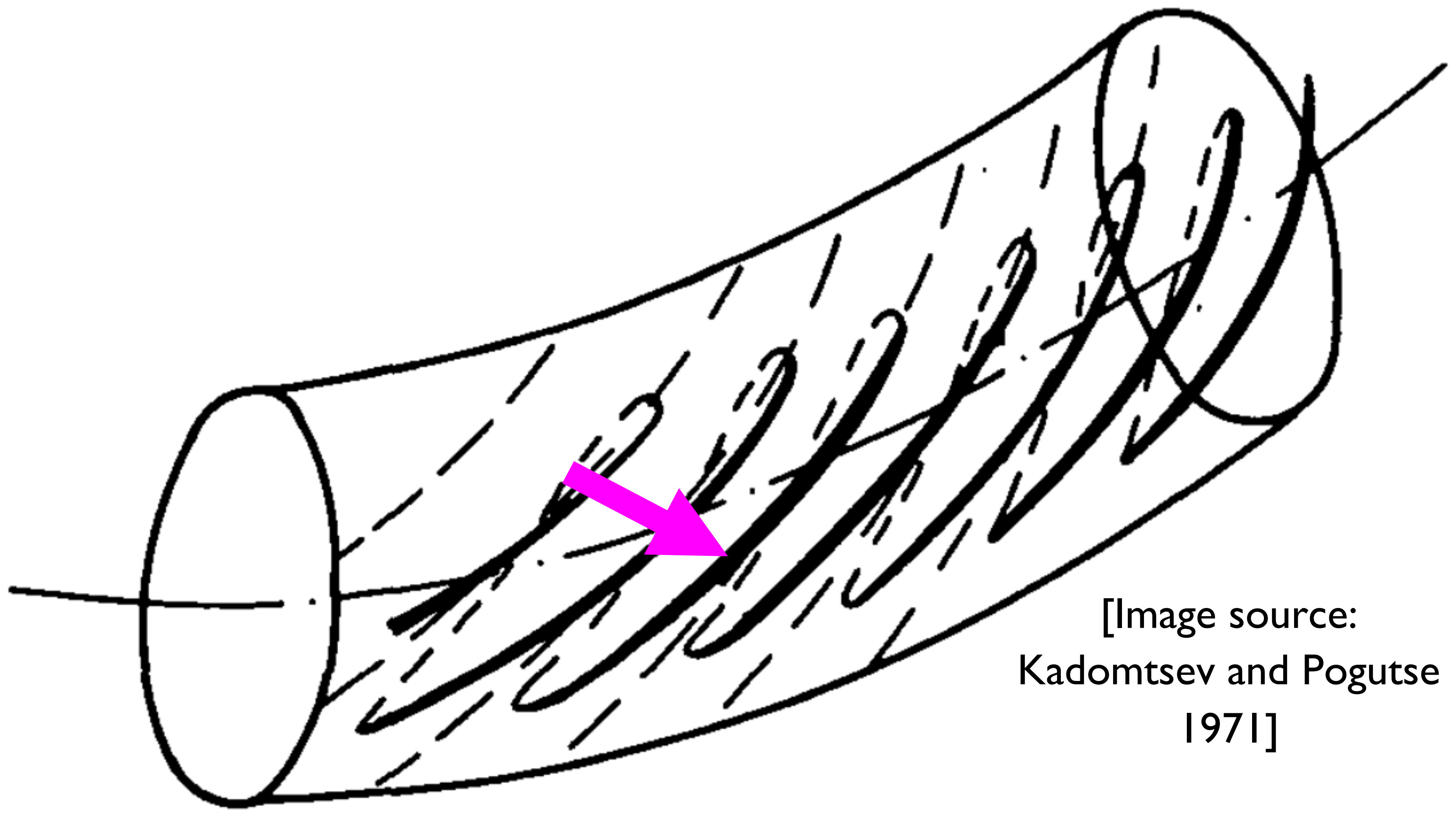
Drift kinetic equation models transport, gives h

The perturbed drift kinetic equation used to find h is:

$$\boxed{v_{\parallel}\hat{b} \cdot \nabla\vartheta \frac{\partial h}{\partial \vartheta} - i\,[\omega - n\omega_{\alpha}]h + i v_{r}\frac{\partial f_0}{\partial r}G(\vartheta) = \nu_{pas}\frac{\partial^2 h}{\partial \lambda^2}}$$

Drift of unperturbed alpha orbit in flux surface

$$\omega_{\alpha} = \overrightarrow{v_d} \cdot \nabla(\zeta - q\,\vartheta)$$



[Image source: Kadomtsev and Pogutse 1971]

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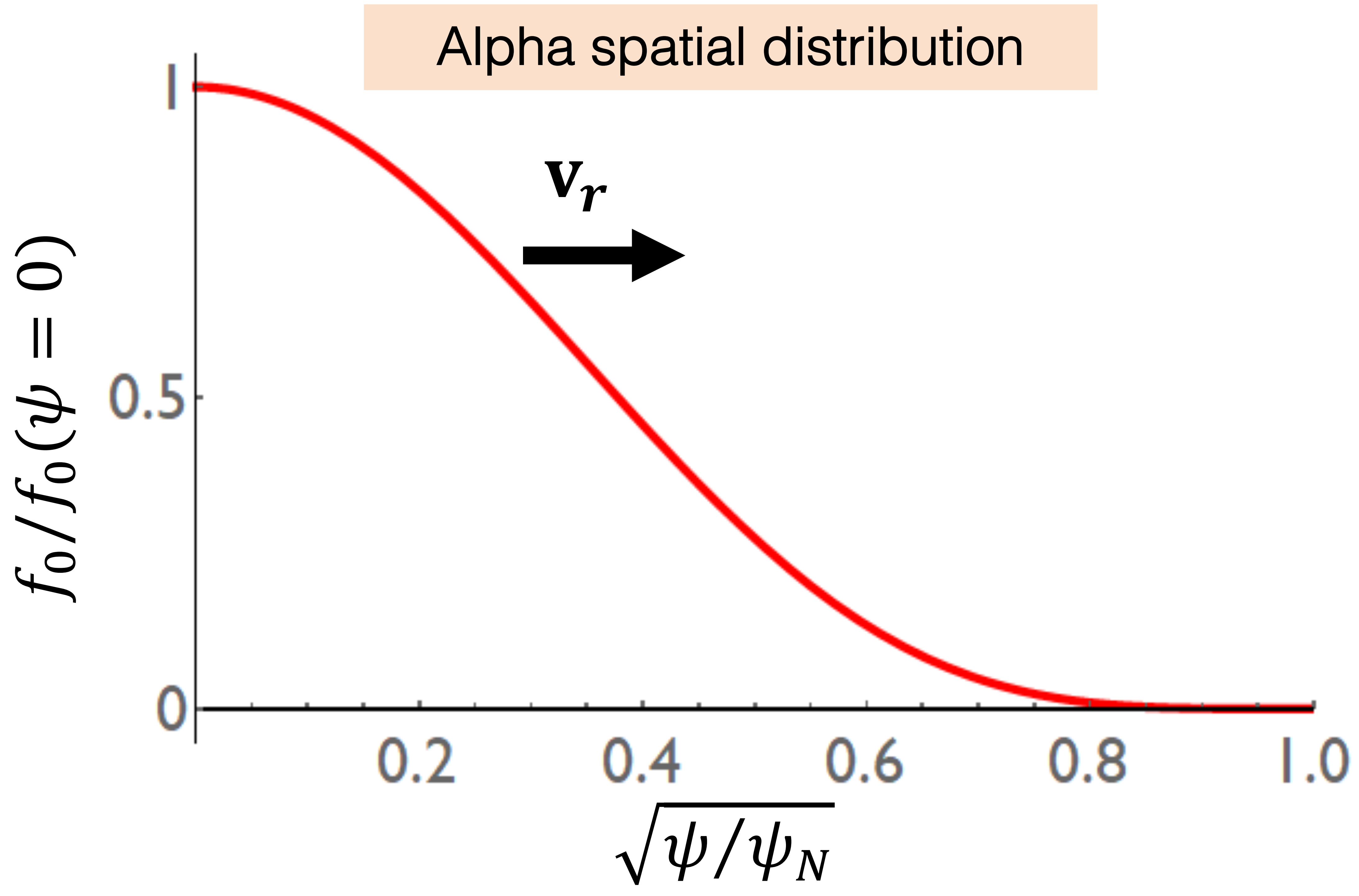
Drive from perturbation and alpha spatial gradient

- v_{r} is radial velocity caused by perturbation ($\vec{E}\times\vec{B}$ drift + grad $\vec{B}$ drift + changed B field direction)
- $\frac{\partial f_0}{\partial r}$ is the alpha spatial gradient
- $G(\vartheta)$ gives poloidal variation in strength of transport



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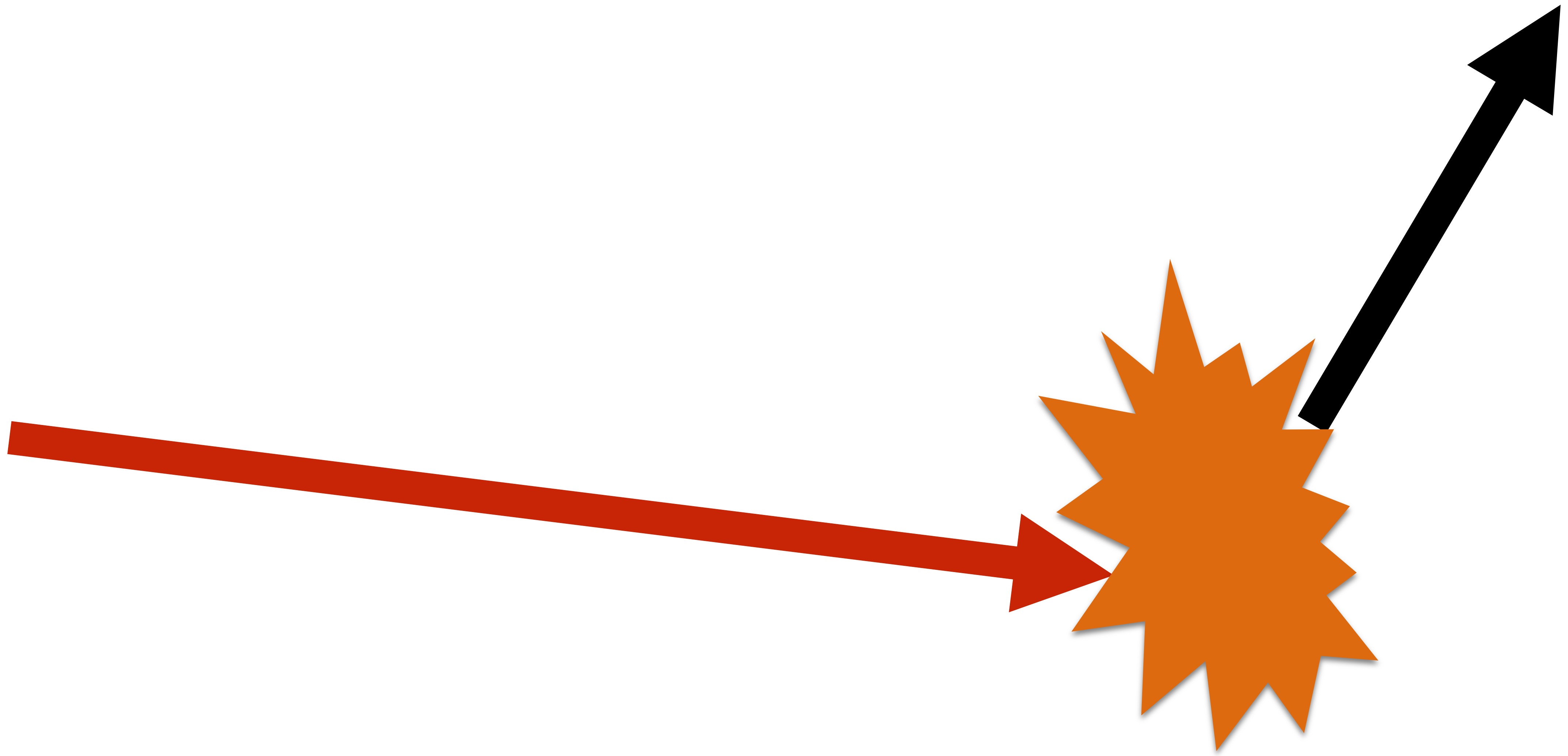
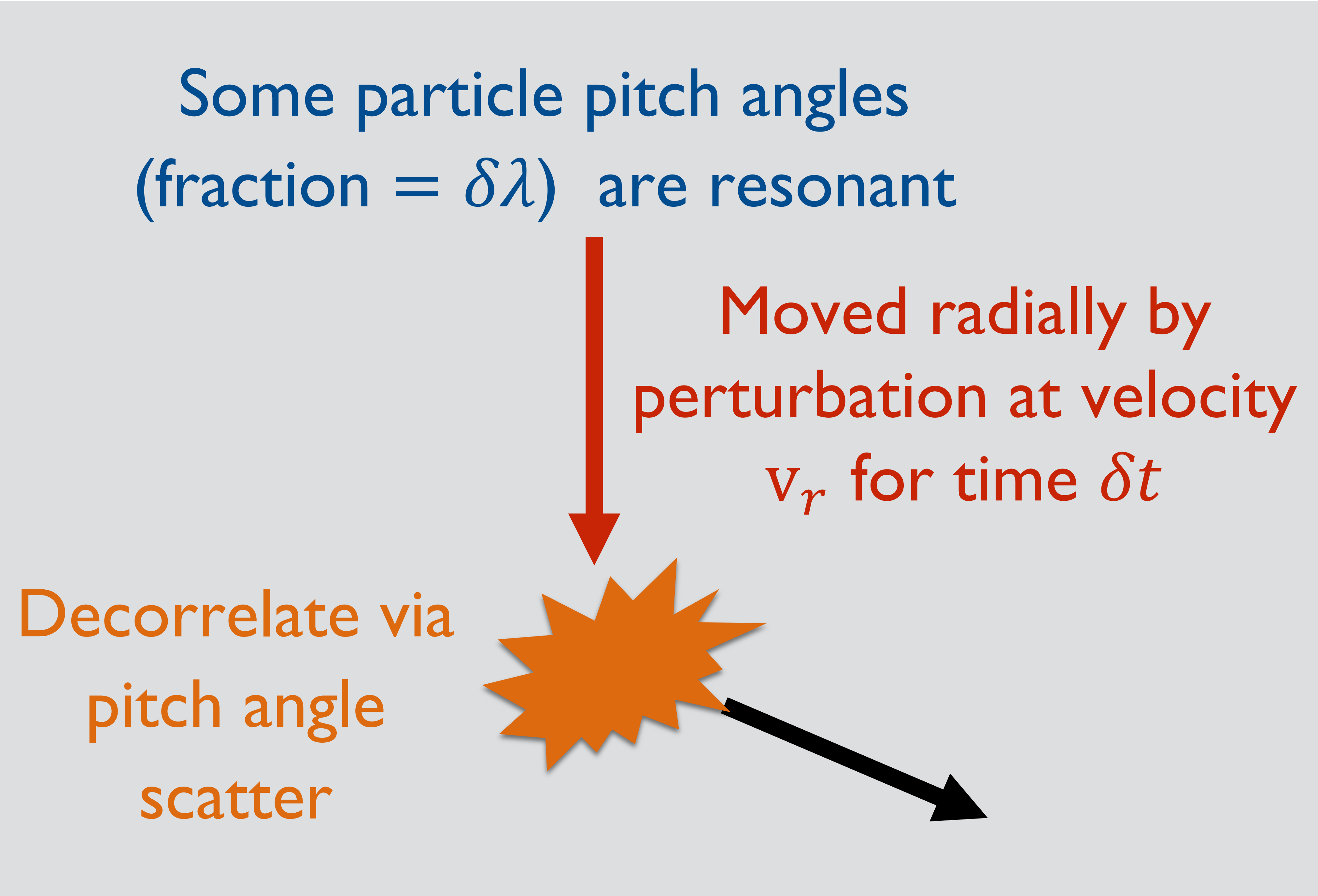
Pitch angle scattering of alpha particles

- Pitch angle is the angle between a particle's velocity and the background magnetic field
- Represented by $\lambda \equiv \frac{B_0 \mathrm{v}_{\perp}^2}{B\mathrm{v}^2}$
- Frequency of pitch angle scatter is ν_{pas}



Phenomenological estimate of transport is possible

$$\boxed{\mathrm{v}_{\parallel}\hat{b}\cdot\nabla\vartheta\,\frac{\partial h}{\partial\vartheta}-i\left[\omega-n\omega_{\alpha}\right]h+i\mathrm{v}_{\mathrm{r}}\frac{\partial f_0}{\partial r}G(\vartheta)=\nu_{pas}\frac{\partial^2 h}{\partial\lambda^2}}$$



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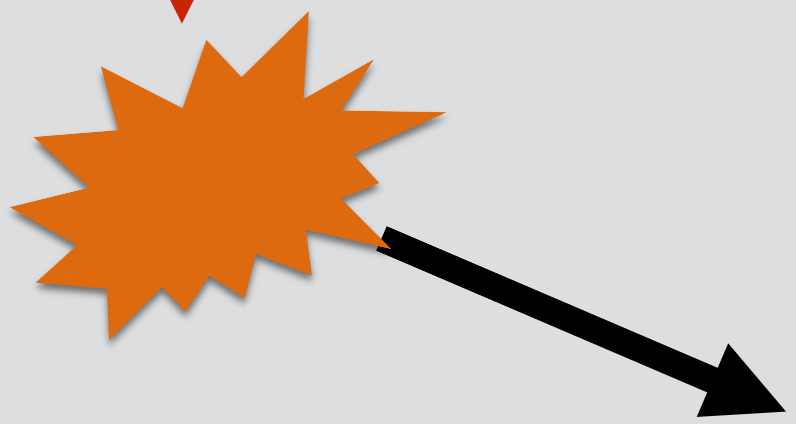
$$\boxed{v_{\parallel}\hat{b}\cdot\nabla\vartheta\,\frac{\partial h}{\partial\vartheta} - i\,[\omega - n\omega_{\alpha}]h + i v_{r}\frac{\partial f_{0}}{\partial r}G(\vartheta) = \nu_{pas}\frac{\partial^{2}h}{\partial\lambda^{2}}}$$

Some particle pitch angles (fraction $= \delta\lambda$) are resonant

Moved radially by perturbation at velocity v_r for time δt

Decorrelate via pitch angle scatter



Estimate of fraction of particles in resonance ($\delta\lambda$)

$$n\omega_{\alpha}\delta\lambda \sim \frac{\nu_{pas}}{\delta\lambda^{2}} \rightarrow \delta\lambda \sim \left(\frac{\nu_{pas}}{n\omega_{\alpha}}\right)^{1/3}$$

Higher ν_{pas} allows more particles to be resonant

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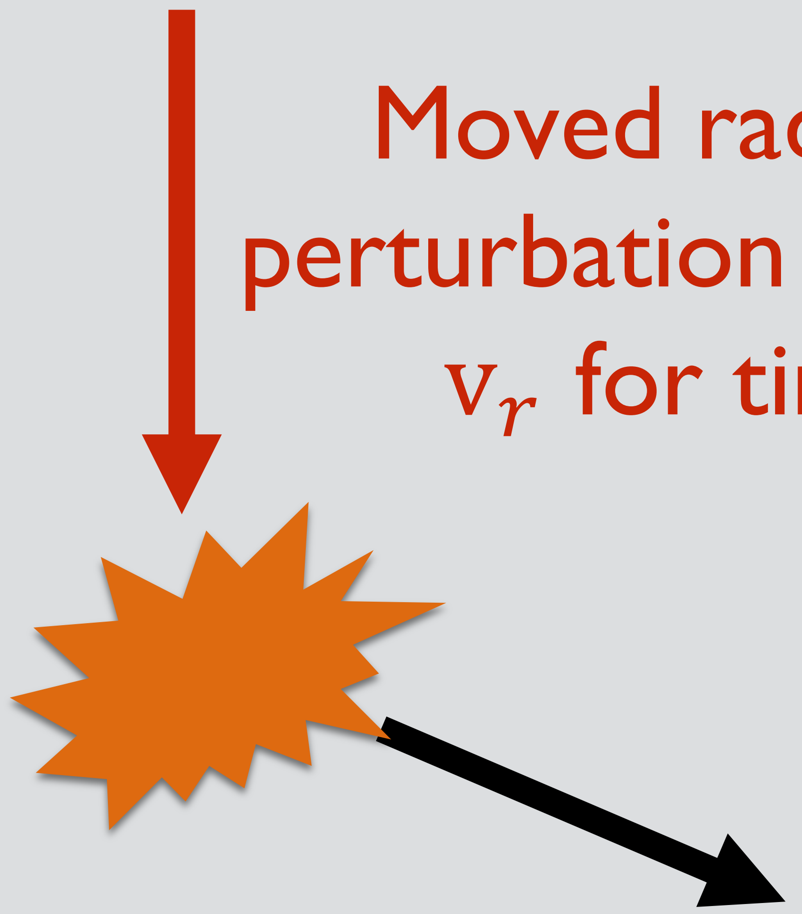

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Estimate of radial step ($\mathrm{v}_r\delta t$)

$$\delta t \sim \frac{1}{\nu_{eff}} \sim \frac{\delta\lambda^2}{\nu_{pas}}$$

Phenomenological estimate of transport is possible

$$v_{\parallel}\hat{b}\cdot\nabla\vartheta\,\frac{\partial h}{\partial\vartheta} - i\,[\omega - n\omega_{\alpha}]h + i\mathrm{v}_{\mathrm{r}}\frac{\partial f_0}{\partial r}G(\vartheta) = \nu_{pas}\frac{\partial^2 h}{\partial\lambda^2}$$

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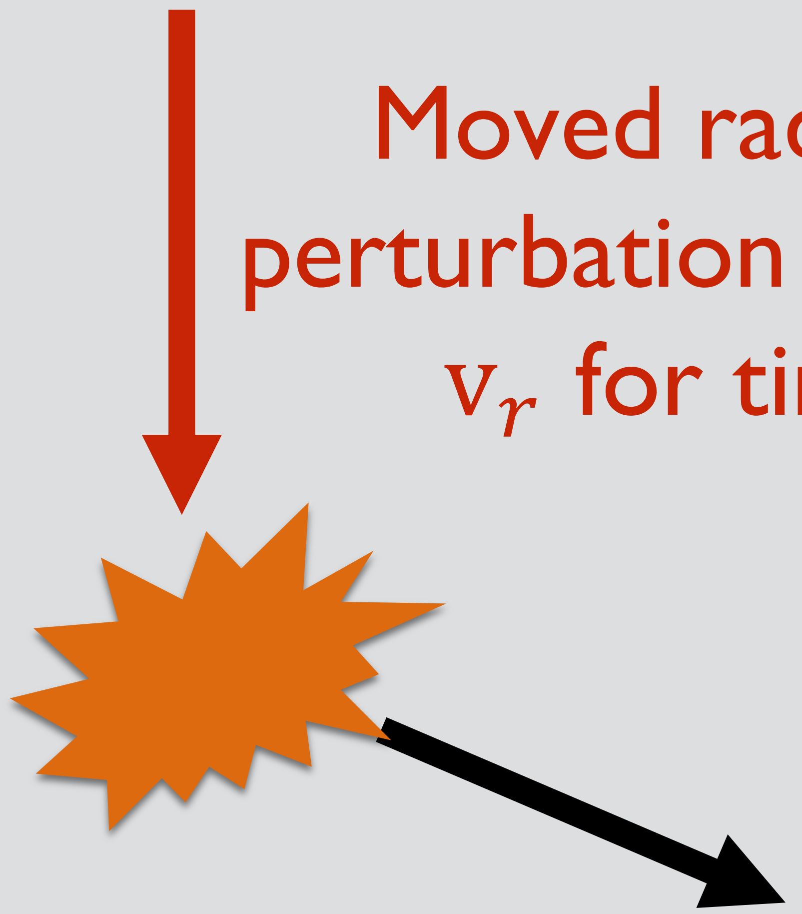

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$$\delta t \sim \frac{1}{\nu_{eff}} \sim \frac{\delta\lambda^2}{\nu_{pas}} \rightarrow \mathrm{v}_r\delta t \sim \frac{\mathrm{v}_r}{(n\omega_{\alpha})^{2/3}\nu_{pas}^{1/3}}$$

Higher ν_{pas} shortens step size

Phenomenological estimate of transport is possible

$$\boxed{\mathrm{v}_{\parallel}\hat{b}\cdot\nabla\vartheta\,\frac{\partial h}{\partial\vartheta} - i\,[\omega - n\omega_\alpha]h + i\mathrm{v}_{\mathrm{r}}\frac{\partial f_0}{\partial r}G(\vartheta) = \nu_{pas}\frac{\partial^2 h}{\partial\lambda^2}}$$

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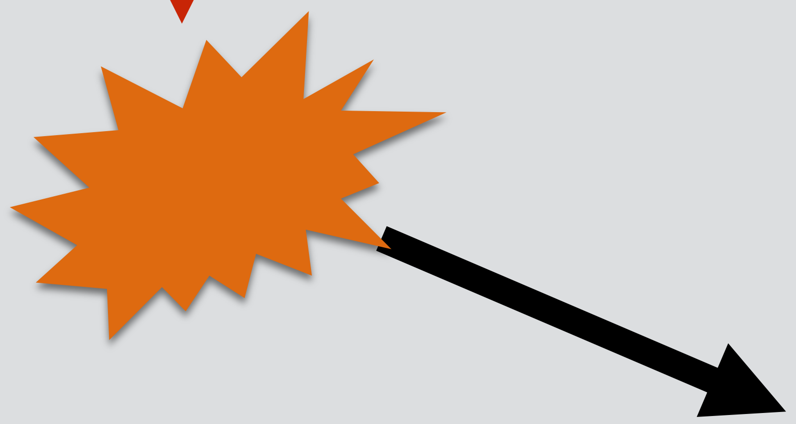

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$$\delta t \sim \frac{1}{\nu_{eff}} \sim \frac{\delta\lambda^2}{\nu_{pas}} \rightarrow \mathrm{v}_r\delta t \sim \frac{\mathrm{v}_r}{(n\omega_\alpha)^{2/3}\nu_{pas}^{1/3}}$$

Higher ν_{pas} shortens step size

Estimate of overall diffusivity (D)

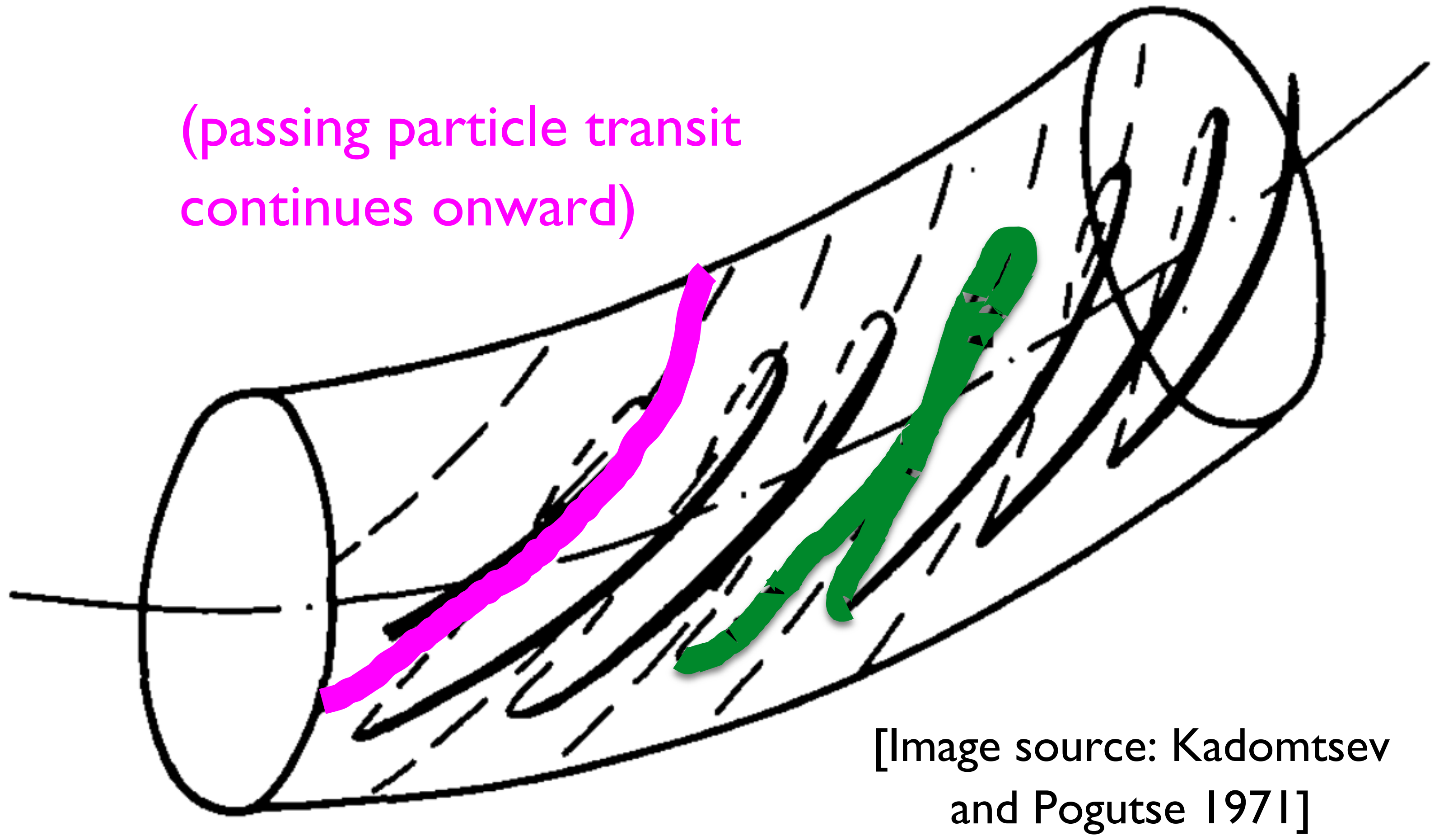
$$D \sim \delta\lambda\frac{(\mathrm{v}_r\delta t)^2}{\delta t} \sim \frac{\mathrm{v}_r^2}{n\omega_\alpha}$$

- ***D has no explicit ν_{pas} dependence***
- ***D increases with $\mathrm{v}_r^2 \propto B_{mn\omega}^2$***

Evaluation of transport

Rigorous evaluation integrates over particle trajectory

- Particle orbit is a series of **bounces (trapped particles)** or **transits (passing particles)**
- Integrate drift kinetic equation over bounce or transit to get h
- D is proportional to h times velocity outwards, v_r



[Image source: Kadomtsev and Pogutse 1971]

Rigorous evaluation integrates over particle trajectory

Estimate of overall diffusivity (D)

$$D \sim \delta\lambda \frac{(\mathrm{v}_r \delta t)^2}{\delta t} \sim \frac{\mathrm{v}_r^2}{n\,\omega_\alpha}$$

Integration of drift kinetic equation

$$D \propto \iint d\mathrm{v}\, d\lambda\, g(\mathrm{v},\lambda)\mathrm{v}_r^2 \left[\frac{\nu_{eff}\tau_b}{(\omega\tau_b - n\overline{\omega_\alpha}\,\tau_b - 2\pi l)^2 + \nu_{eff}^2\tau_b^2}\right] \times P_l^2$$

(trapped particles; passing particle expression is similar)

- $\nu_{eff} \sim \frac{\nu_{pas}}{\delta\lambda^2} \sim (n\omega_\alpha)^{2/3}\nu_{pas}^{1/3}$
- $\overline{\omega_\alpha}$ is the average value of ω_α; τ_b is the bounce or transit time
- P_l^2 is a phase factor that results from integration over trajectory

Rigorous evaluation integrates over particle trajectory

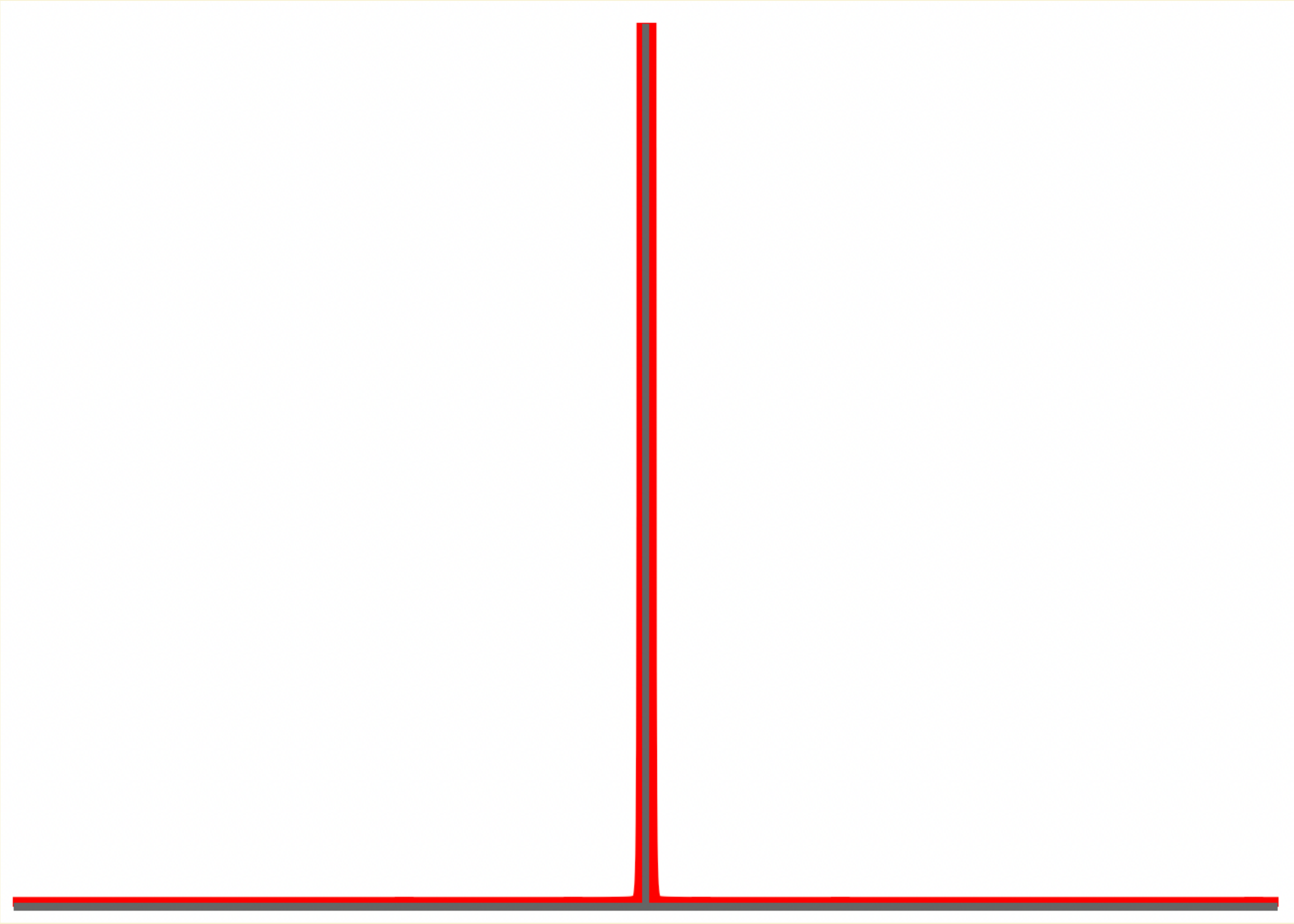
$$\boxed{D \propto \iint d\mathrm{v}\, d\lambda\, g(\mathrm{v},\lambda)\mathrm{v}_r^2 \left[\frac{\nu_{eff}\tau_b}{(\omega\tau_b - n\overline{\omega_\alpha}\,\tau_b - 2\pi l)^2 + \nu_{eff}^2\tau_b^2}\right] \times P_l^2}$$

- For some values of $\lambda(v)$, $\omega\tau_b - n\overline{\omega_\alpha}\tau_b - 2\pi l$ vanishes
- These are resonant velocities
- Recall drift kinetic equation: $\mathrm{v}_\parallel \hat{b} \cdot \nabla\vartheta \frac{\partial h}{\partial \vartheta} - i\,[\omega - n\omega_\alpha]h + i\mathrm{v}_\mathrm{r}\frac{\partial f_0}{\partial r}P(\vartheta) = \nu_{pas}\frac{\partial^2 h}{\partial \lambda^2}$
  - At resonant velocities, the averaged value of the blue terms vanishes up to $2\,\pi l$

Without collisionality, resonance is unresolved

$$D \propto \iint d\mathrm{v}\, d\lambda\, g(\mathrm{v},\lambda)\mathrm{v}_r^2 \left[\frac{\nu_{eff}\tau_b}{(\omega\tau_b - n\overline{\omega_\alpha}\,\tau_b - 2\pi l)^2 + \nu_{eff}^2\tau_b^2}\right] \times P_l^2$$

Particle response to perturbation (integrand above),
$\nu_{eff} = 0$



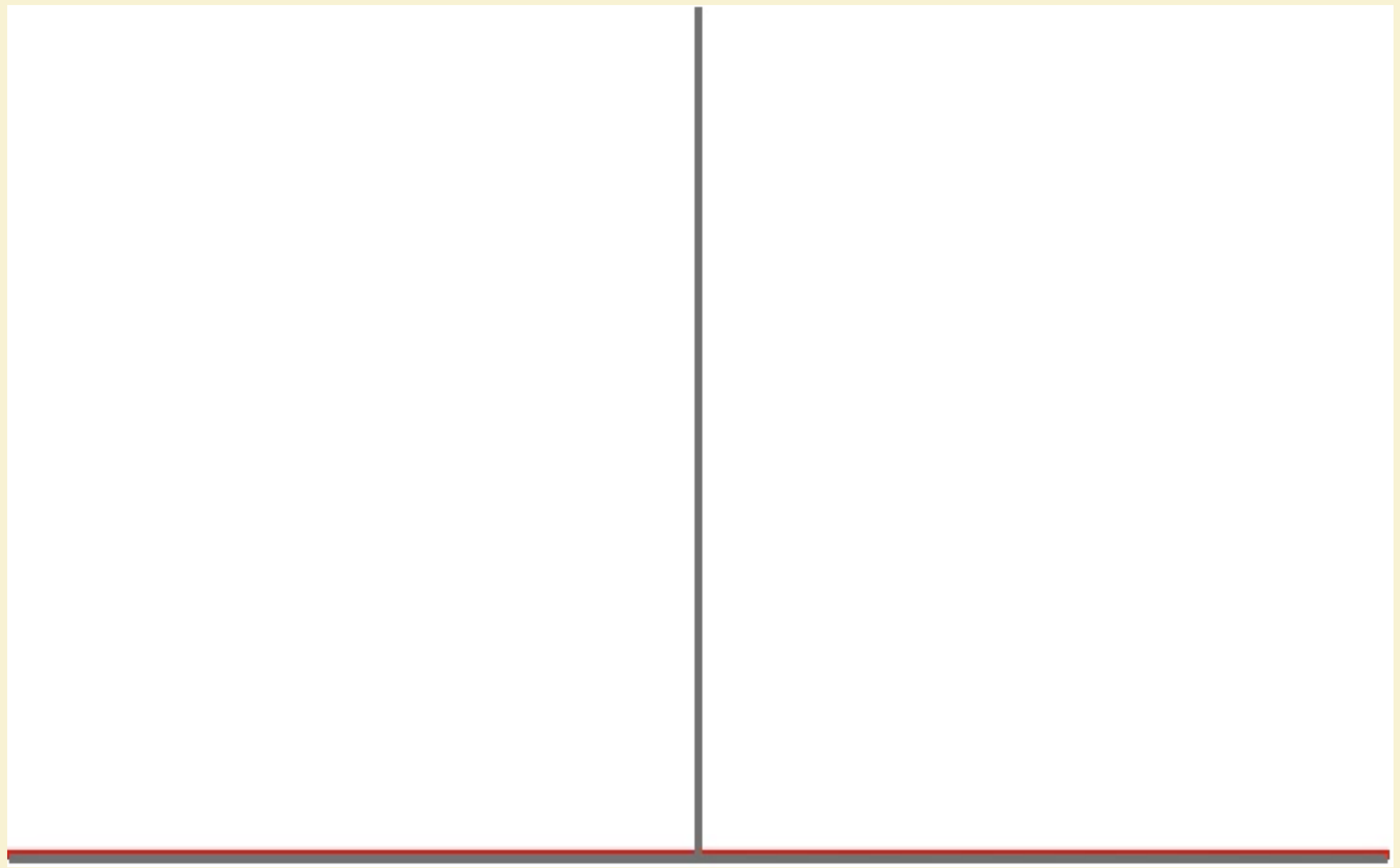
$(\omega\tau_b - n\overline{\omega_\alpha}\tau_b - 2\pi l\,)[\lambda]$

- $\mathrm{v}_\parallel \hat{b} \cdot \nabla\vartheta \cancelto{0}{\frac{\partial h}{\partial\vartheta}} - i\,[\omega - n\omega_\alpha]h + i\mathrm{v}_\mathrm{r}\frac{\partial f_0}{\partial r}P(\vartheta) = \cancelto{0}{\nu_{pas}}\frac{\partial^2 h}{\partial\lambda^2}$
- Collisionality resolves resonance

Collisionality resolves resonance

$$\boxed{D \propto \iint d\mathrm{v}\, d\lambda\, g(\mathrm{v},\lambda)\mathrm{v}_r^2 \left[\frac{\nu_{eff}\tau_b}{(\omega\tau_b - n\overline{\omega_\alpha}\,\tau_b - 2\pi l)^2 + \nu_{eff}^2\tau_b^2}\right] \times P_l^2}$$

Particle response to perturbation (integrand above), ν_{eff} increasing from zero



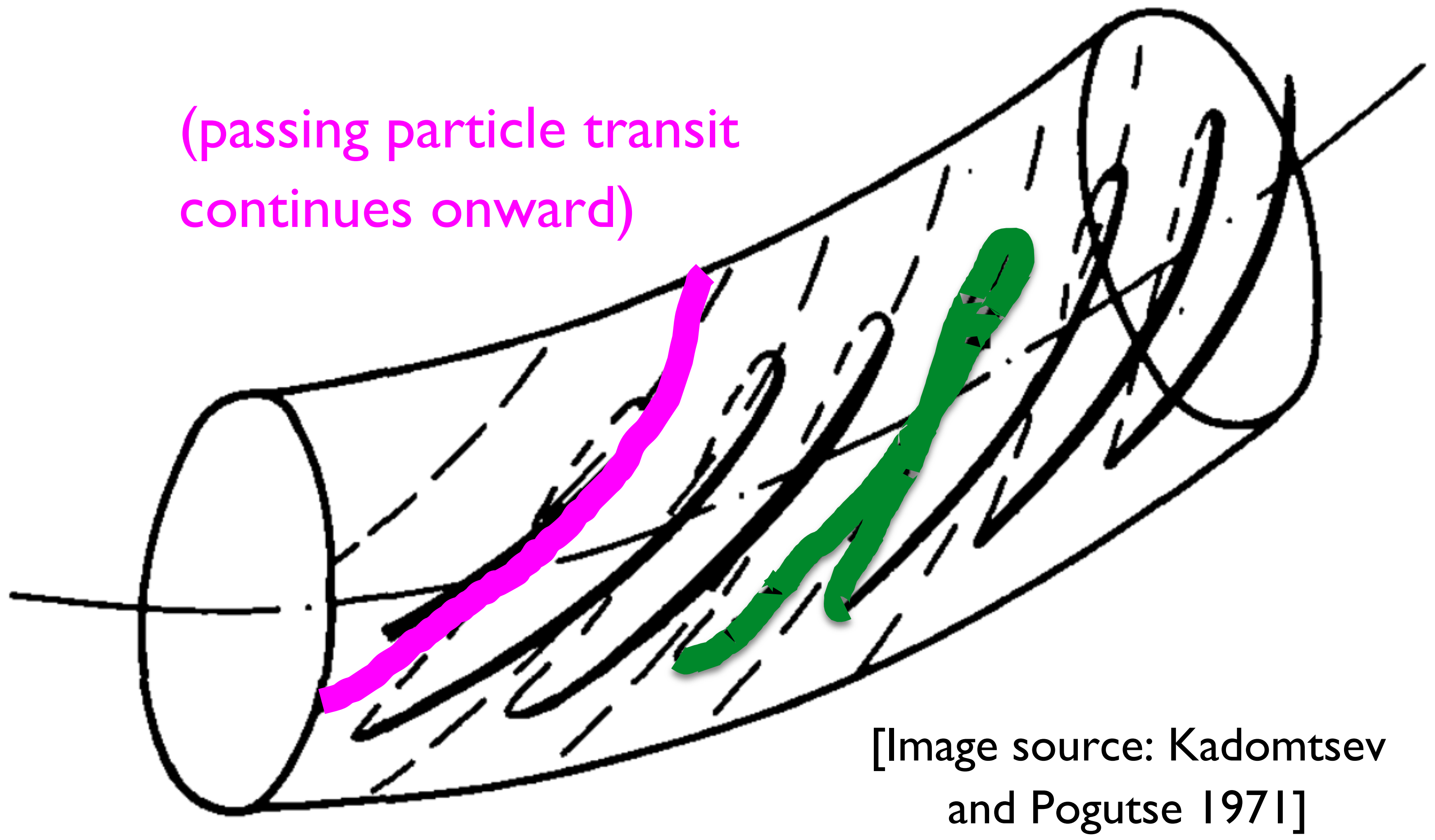
$(\omega\tau_b - n\overline{\omega_\alpha}\tau_b - 2\pi l\ )[\lambda]$

- $\cancelto{0}{\mathrm{v}_\| \hat{b}\cdot\nabla\vartheta\,\frac{\partial h}{\partial\vartheta}} - i\,[\omega - n\omega_\alpha]h + i\mathrm{v}_\mathrm{r}\frac{\partial f_0}{\partial r}P(\vartheta) = \nu_{pas}\frac{\partial^2 h}{\partial\lambda^2}$

- Delta function becomes Lorentzian
  - Width $\delta\lambda \sim \left(\frac{\nu_{pas}}{n\omega_\alpha}\right)^{\frac{1}{3}} \sim \frac{\nu_{eff}}{n\omega_\alpha}$
- Sharp variation of h with respect to λ explains importance of pitch angle scattering $\nu_{pas}\frac{\partial^2 h}{\partial\lambda^2}$
- Consistent with causality

Particles moving in phase with perturbation resonate

- Integration reveals resonance condition
  - $\omega\tau_b - n\overline{\omega_\alpha}\tau_b - 2\pi l = 0$
  - Particles that drift in resonance with ω, n participate in transport



[Image source: Kadomtsev and Pogutse 1971]

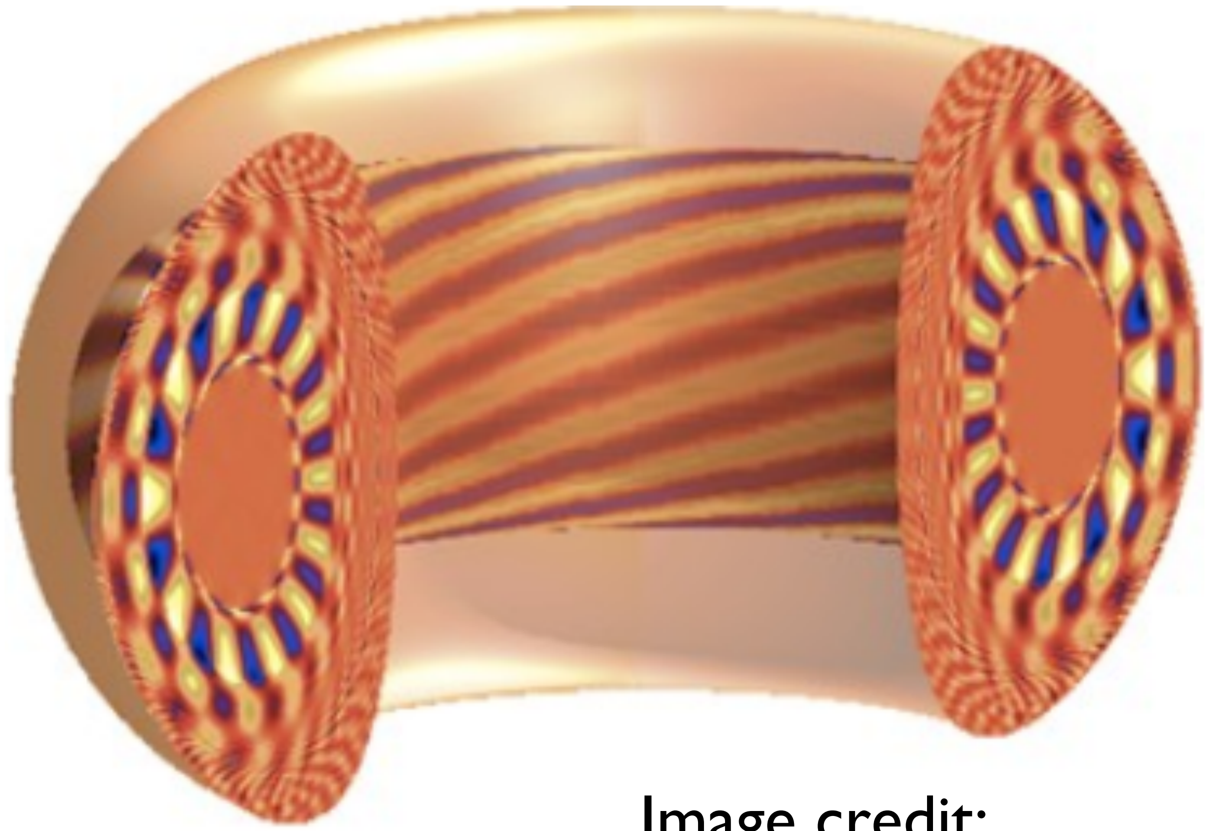


Image credit:
Heidbrink APS 2007

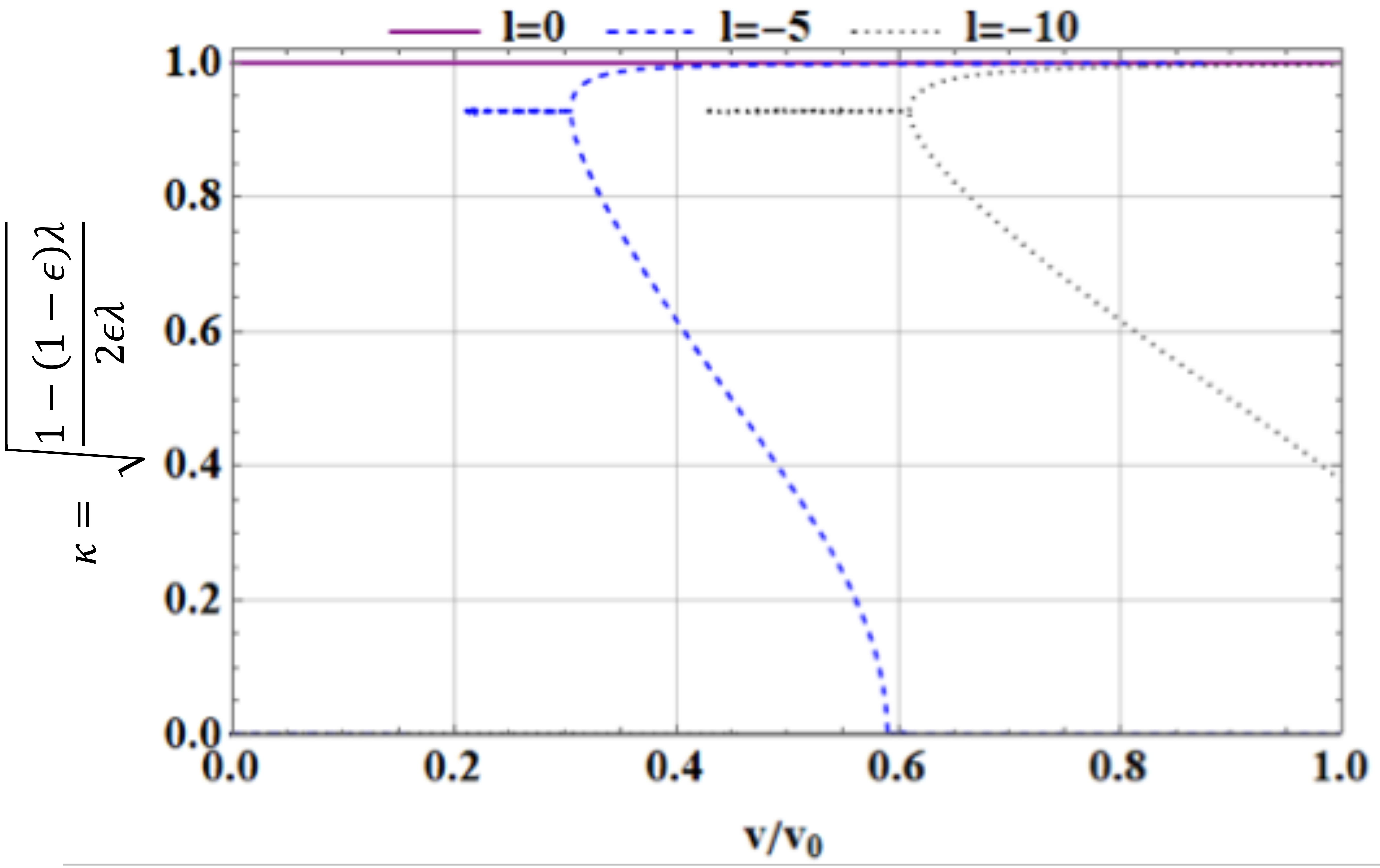
Phase factor reduces effect of high $|l + (nq - m)|$

$$D \propto \iint d\text{v}\, d\lambda\, g(\text{v},\lambda)\text{v}_r^2 \left[\frac{\nu_{eff}\tau_b}{(\omega\tau_b - n\overline{\omega_\alpha}\,\tau_b - 2\pi l)^2 + \nu_{eff}^2\tau_b^2}\right] \times P_l^2$$

- Rigorous integration gives a "phase factor" $P_l^2 \sim \frac{1}{|l+(nq-m)|^2}$
  - More discussion of phase factors found in paper
  - They are a lot more complicated than $\sim \frac{1}{|l+(nq-m)|^2}$
- Higher values of $|l + (nq - m)|$ get "washed out"
- Contribute very little transport

Ripple resonance structure shows phase factor is small

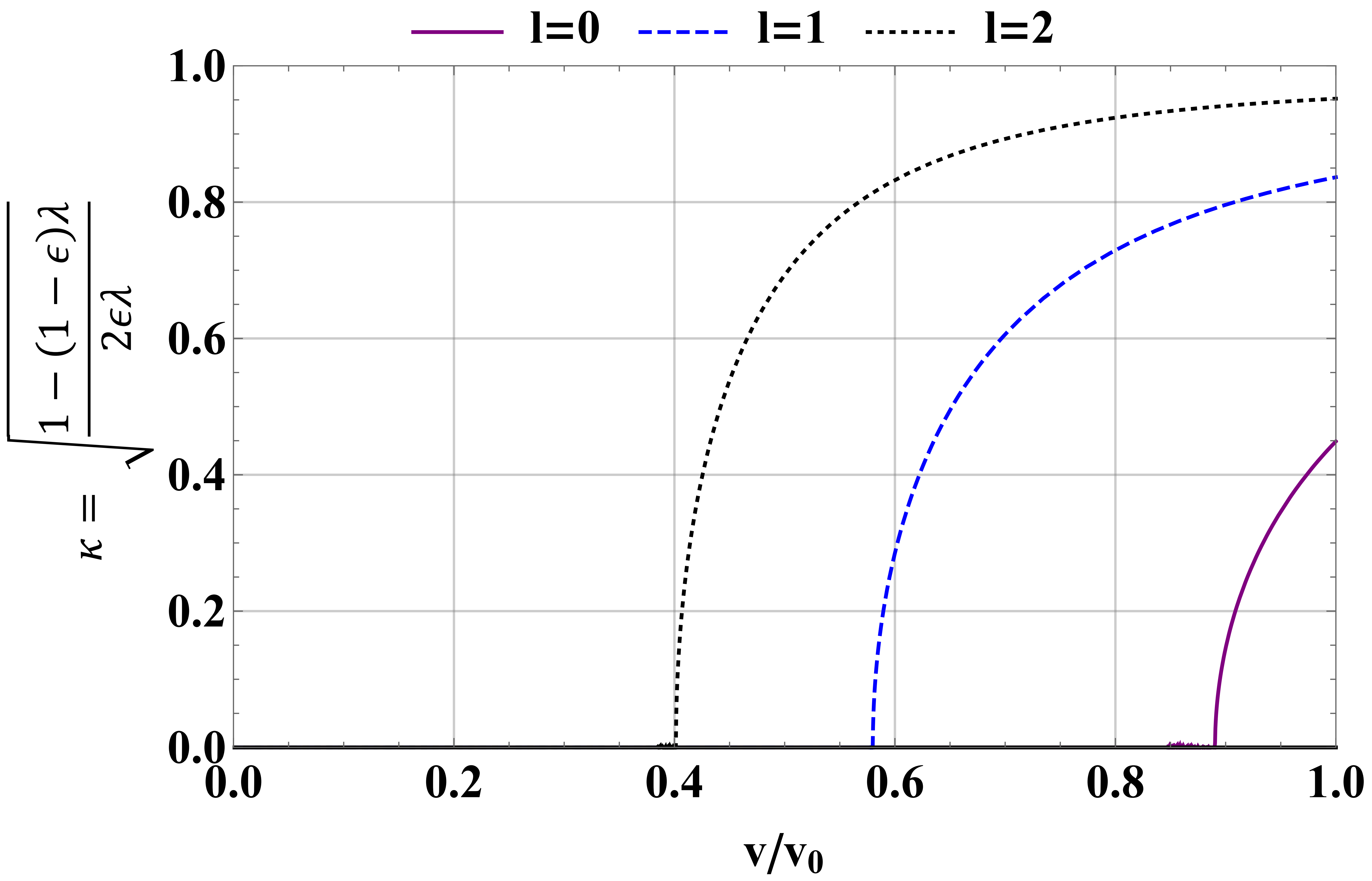
Trapped particle ripple resonance structure, SPARC-like parameters



- Ripple resonances have low to moderate $|l|$
- For ripple, nq is very high (≈ 60)
- Phase factor $P_l^2 \sim \frac{1}{|l+(nq-m)|^2}$ is very small
- Negligible transport
  - Significant transport still possible via mechanisms outside this theory

TAE resonance structure shows phase factor is high

Trapped particle TAE resonance structure, SPARC-like parameters



- TAE resonances have low to moderate $|l|$
- For TAE, $nq - m = 1/2$
- Phase factor $P_l^2 \sim \frac{1}{|l+(nq-m)|^2}$ is large
- Significant transport

Strength of TAE transport

Evaluation agrees with phenomenological estimate

- Full evaluation of D integral gives:

$$D_{trapped} \sim \sqrt{\epsilon}\,\frac{\mathrm{v_r^2}}{n\omega_\alpha}$$

$$D_{passing} \sim \frac{\mathrm{v_r^2}}{n\omega_\alpha}$$

- Compare to estimate $D \sim \frac{\mathrm{v_r^2}}{n\omega_\alpha}$
  - $\sqrt{\epsilon} = \sqrt{\frac{r}{R}}$ fraction of particles are trapped

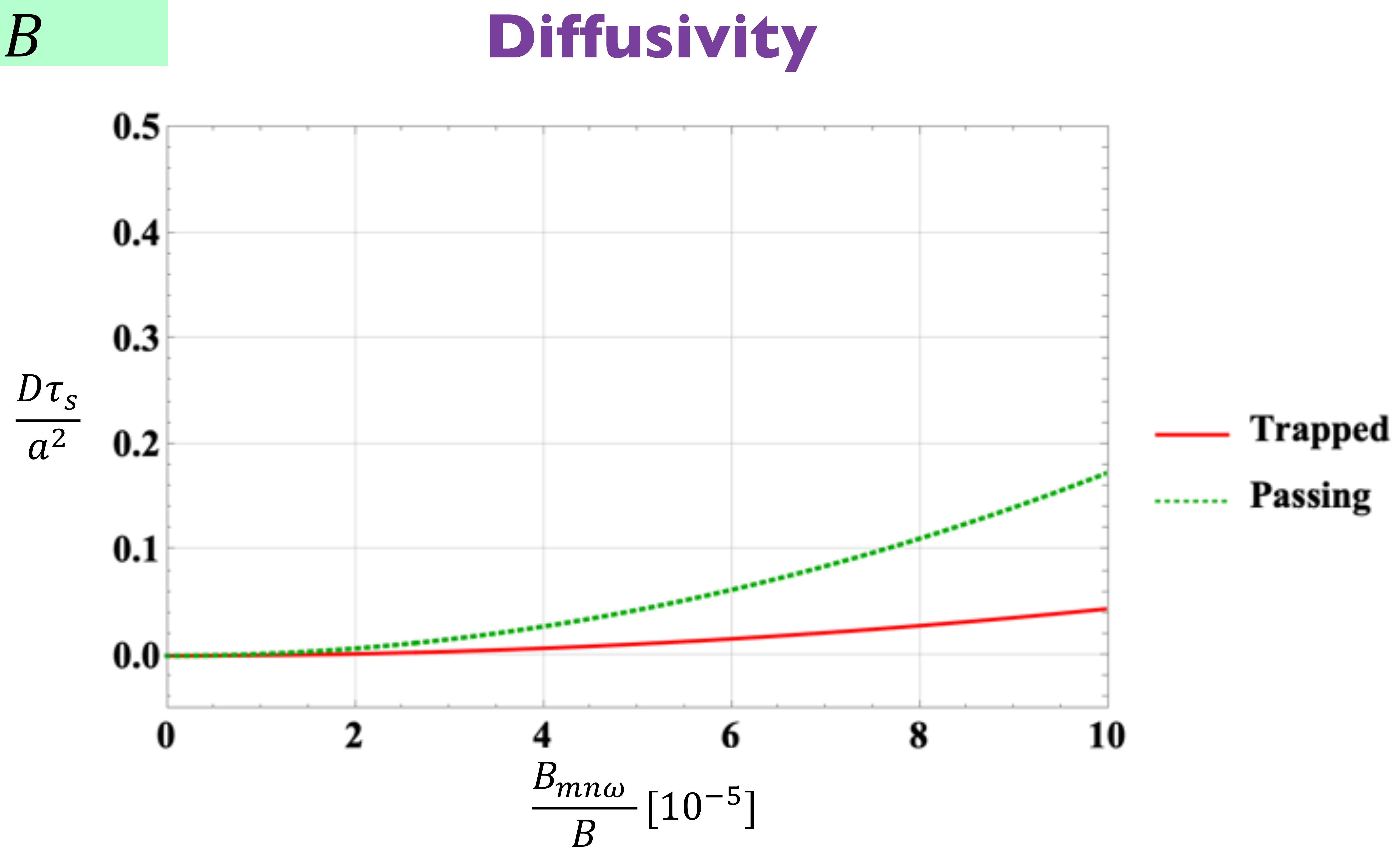
Diffusivity is significant, grows with amplitude squared

$$D_{trapped} \sim \sqrt{\epsilon}\,\frac{\mathrm{v_r^2}}{n\omega_\alpha}$$

$$D_{passing} \sim \frac{\mathrm{v_r^2}}{n\omega_\alpha}$$

$$\mathrm{v_r} \sim \mathrm{v}_A \frac{B_{mn\omega}}{B}$$

- D is normalized with slowing down time τ_s and device minor radius a
- Plot shows normalized D as function of TAE amplitude at SPARC-like parameters
  - R = 1.85 m, n = 10, $\omega \approx 2{\times}10^6\ s^{-1}, \mathrm{v}_A \approx 8\times 10^6 \frac{m}{s}$

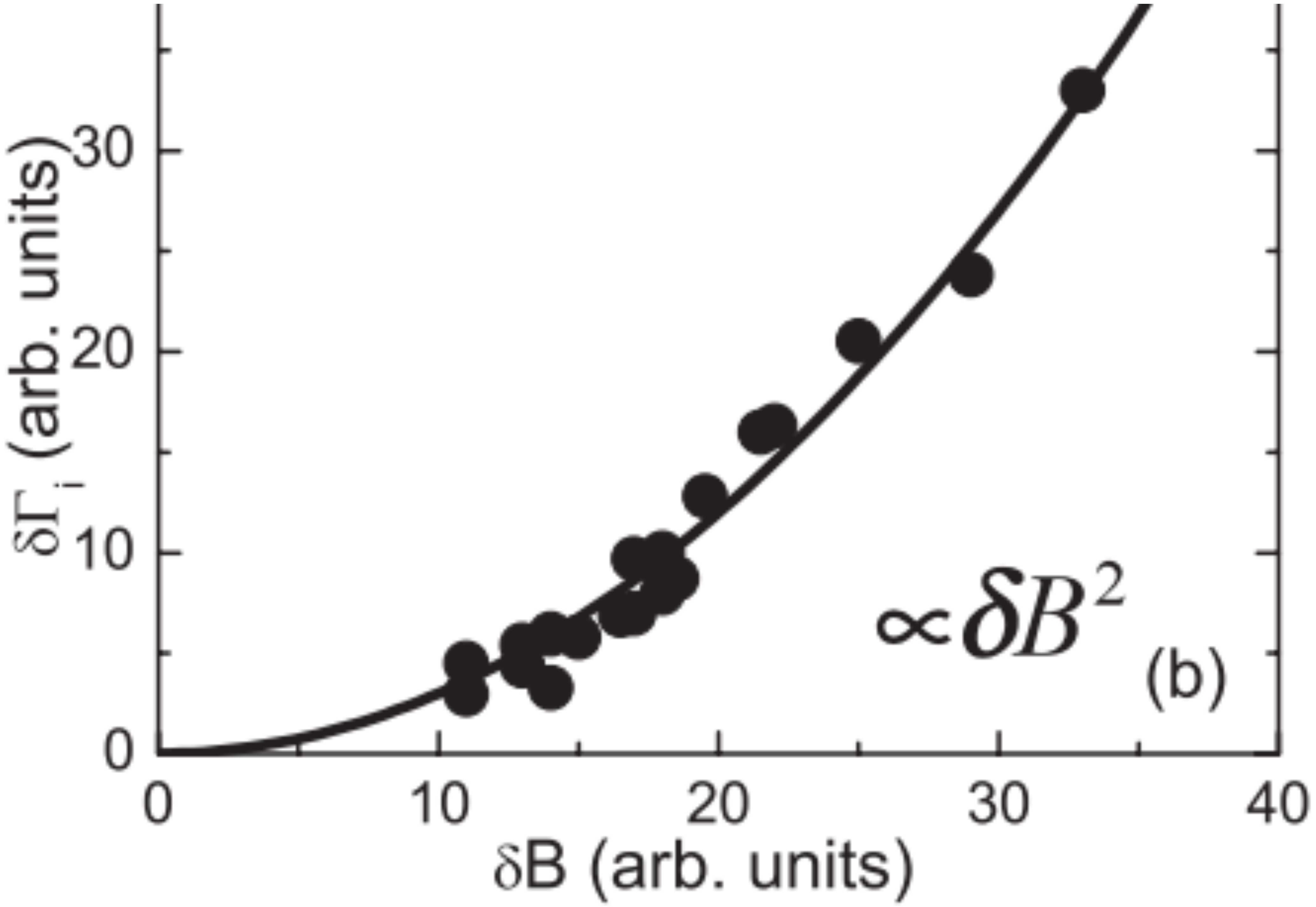


Experimental literature shows similar trends

- Because of specialized form of alpha distribution, can't definitively validate theory until DT
- Multiple types of energetic particle transport observed in modern experiments
- Diffusive transport is observed

Energetic particle flux from EPMs in the Compact Helical System

(from Nagaoka et al. PRL 2008)



Saturation condition needed to determine diffusion

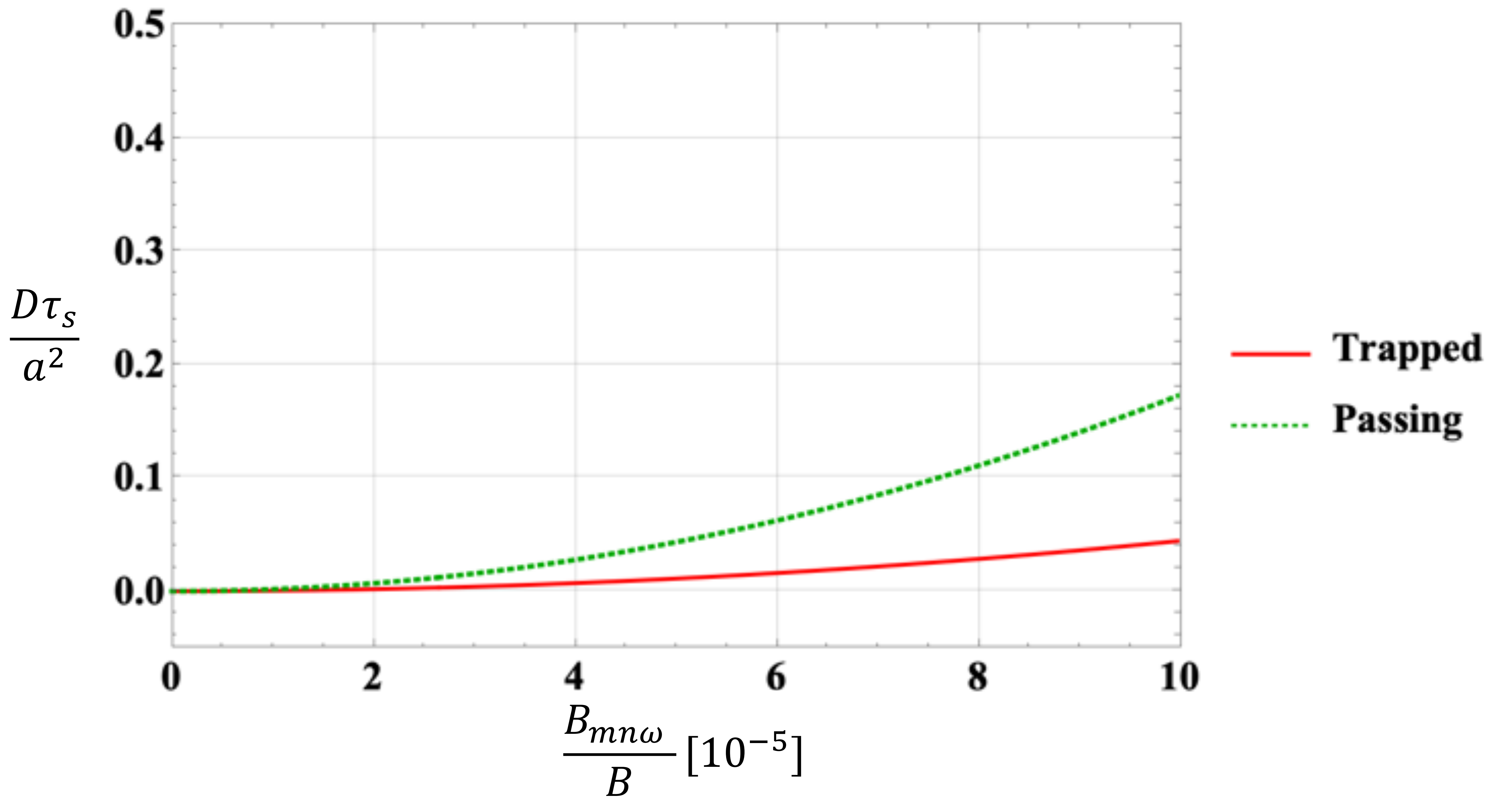
$$D_{trapped} \sim \sqrt{\epsilon}\,\frac{\mathrm{v}_\mathrm{r}^2}{n\omega_\alpha}$$

$$D_{passing} \sim \frac{\mathrm{v}_\mathrm{r}^2}{n\omega_\alpha}$$

$$\mathrm{v}_\mathrm{r} \sim \mathrm{v}_A \frac{B_{mn\omega}}{B}$$

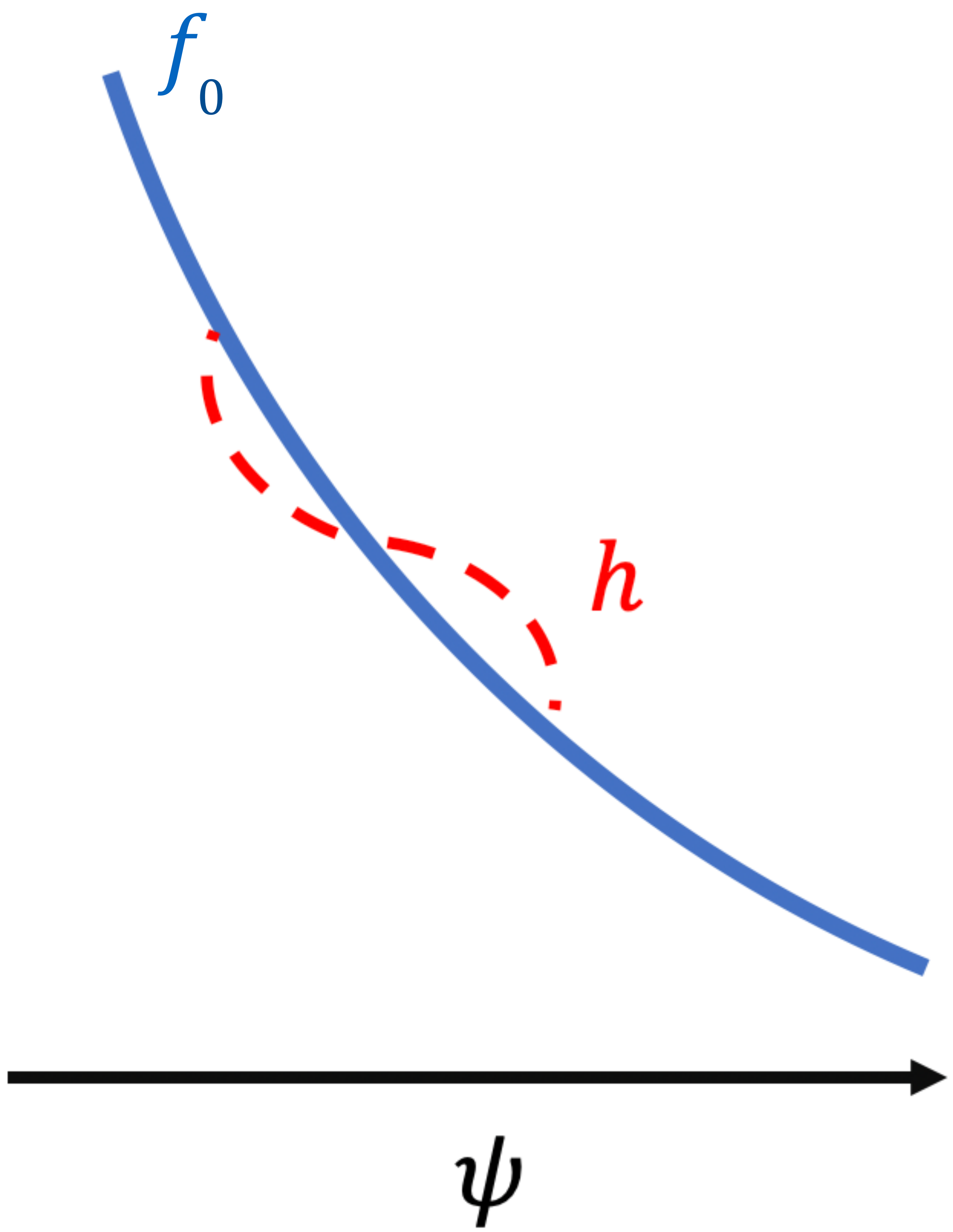
- D is normalized with slowing down time τ_s and device minor radius a
- Plot shows normalized D as function of TAE amplitude at SPARC-like parameters
  - R = 1.85 m, n = 10, $\omega \approx 2{\times}10^6\ s^{-1}, \mathrm{v}_A \approx 8\times10^6\ \frac{m}{s}$

Diffusivity



Saturation condition balances flattening with refilling

- Saturation estimated in many ways throughout literature (wave trapping, nonlinear mode couplings, etc.)
- One simple method balances nonlinear drive reduction with collisional refilling

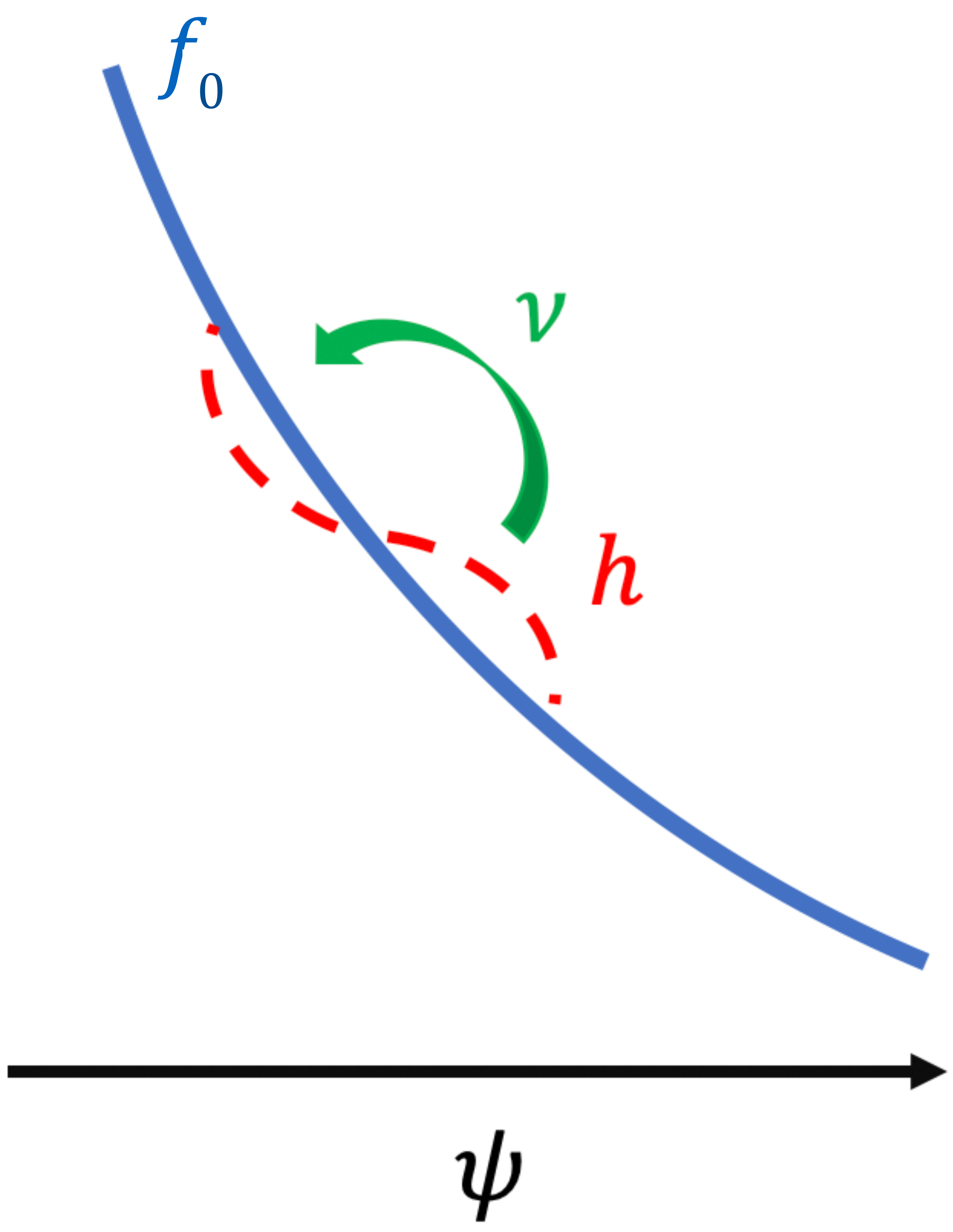


Saturation condition balances flattening with refilling

- Saturation estimated in many ways throughout literature (wave trapping, nonlinear mode couplings, etc.)

- One simple method balances nonlinear drive reduction with collisional refilling

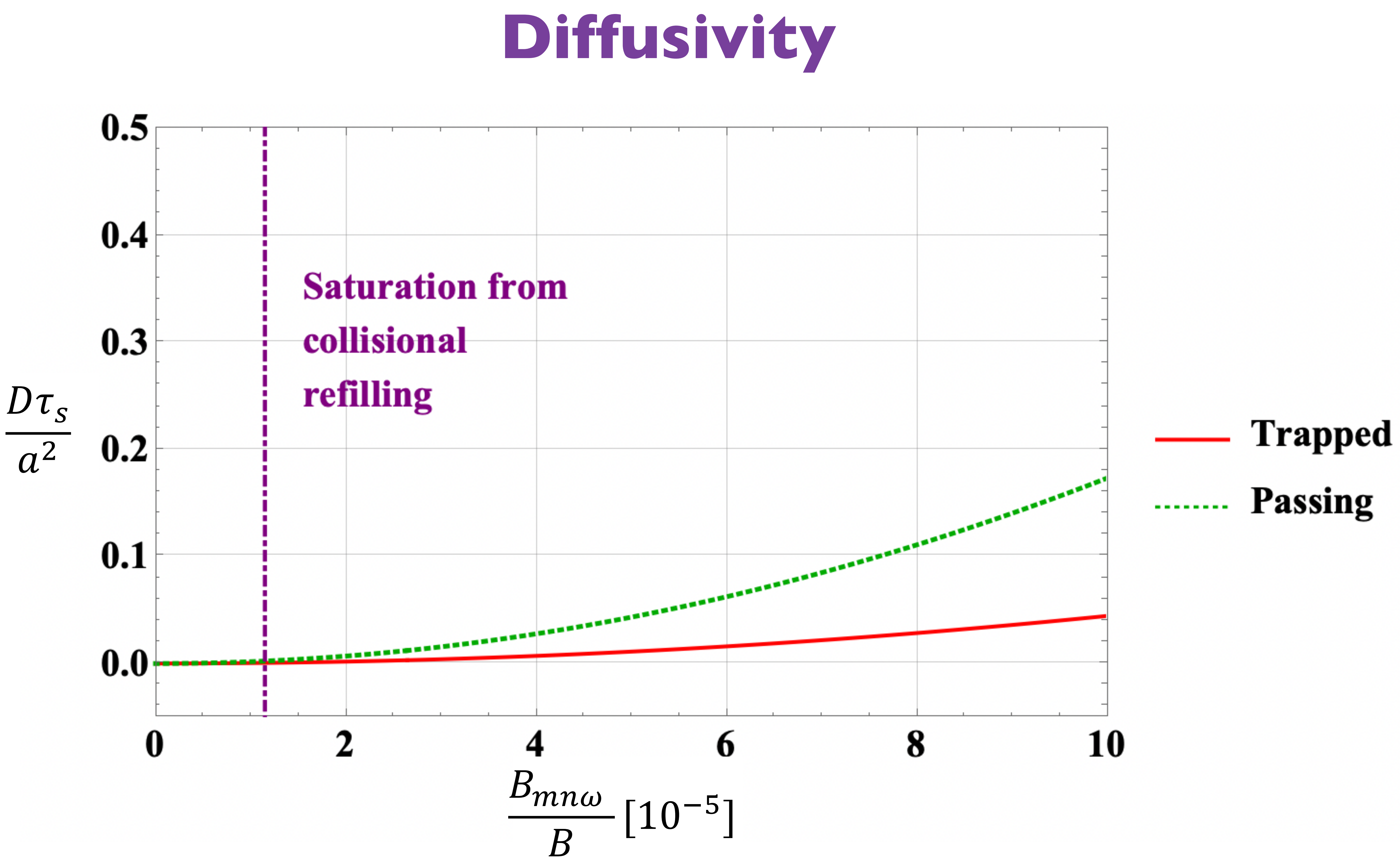
$$\mathrm{v}_A \frac{B_{mn\omega}}{\mathrm{B}} \frac{\partial h}{\partial r} \sim \nu_{pas} \frac{\partial^2 h}{\partial \lambda^2}$$



Saturation condition suggests insignificant diffusion

- At the amplitude predicted by simple condition, diffusion is insignificant

- **Caveats**:
  - Coupling with other *m* will change *D*
  - Saturation amplitudes in the literature can be as large as 100 x ours
  - Onset of full stochasticity at higher amplitude could further enhance transport



Conclusions

- Alpha transport by tokamak perturbations can be calculated drift kinetically
- Drift kinetic calculation, plus simple saturation estimate, suggests TAE transport in SPARC-like tokamak could be small
- Caveat: saturation at a higher level, possibly accompanied by onset of stochasticity, could lead to significant transport
  - Strong motivation for experimental exploration, numerical simulations!

- *Based on paper accepted to JPP: "Drift kinetic theory of alpha transport by tokamak perturbations," Tolman and Catto. Available as arXiv:2011.04920, should appear shortly in JPP*
- *Slides available after the presentation at* elizabethtolman.com

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