Drift kinetic formulation of alpha particle transport by tokamak MHD perturbations

APS DPP
November 13th, 2020

Based on paper in review: "Drift kinetic theory of alpha transport by tokamak perturbations," E.A. Tolman and P.J. Catto. Available on arXiv:2011.04920

Elizabeth A. Tolman

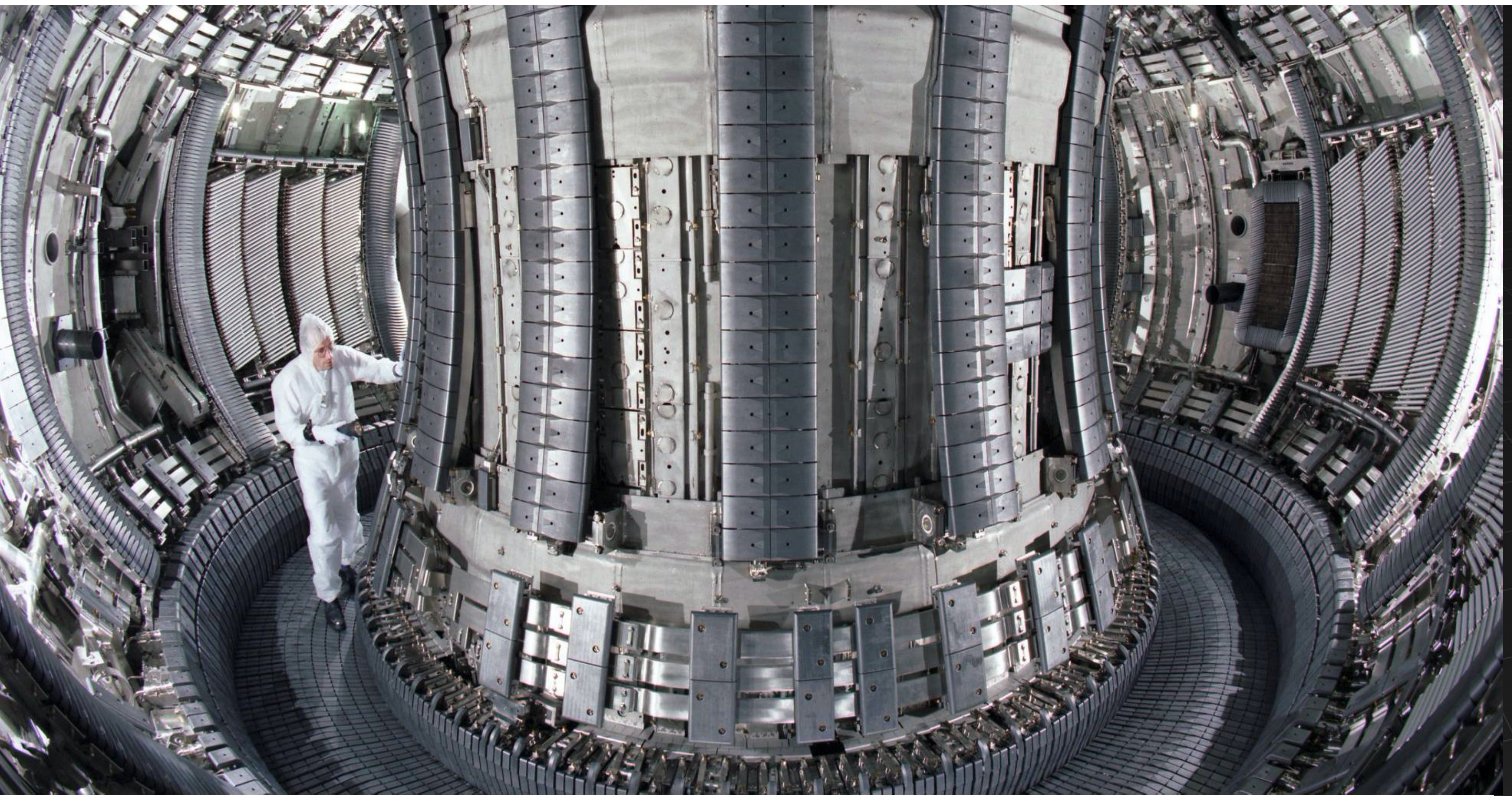
Bezos Member, School of Natural Sciences

Institute for Advanced Study, Princeton, NJ USA

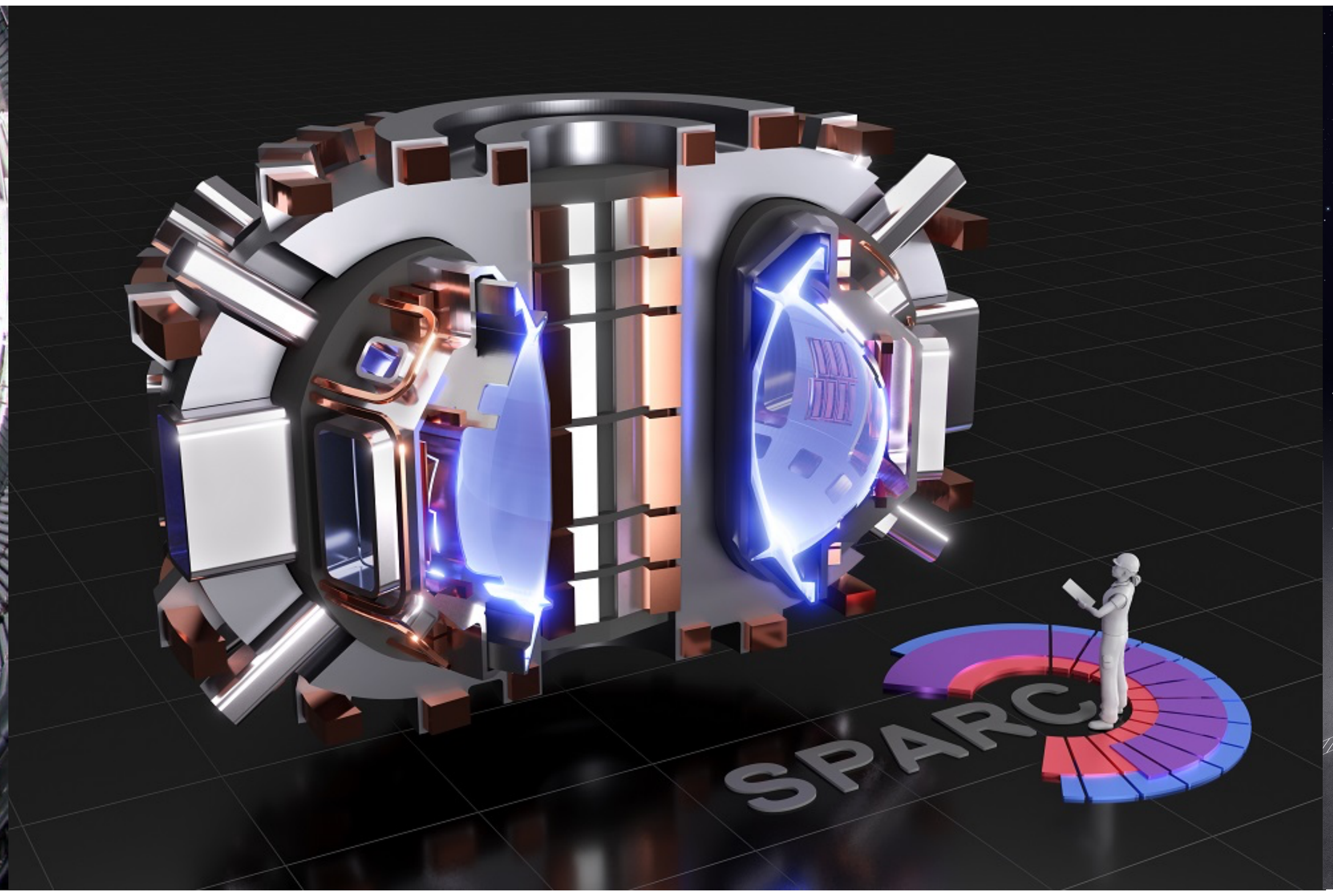
(Work primarily completed at MIT, Cambridge, MA USA)

MIT PSFC U.S. DEPARTMENT OF ENERGY NSF INSTITUTE FOR ADVANCED STUDY

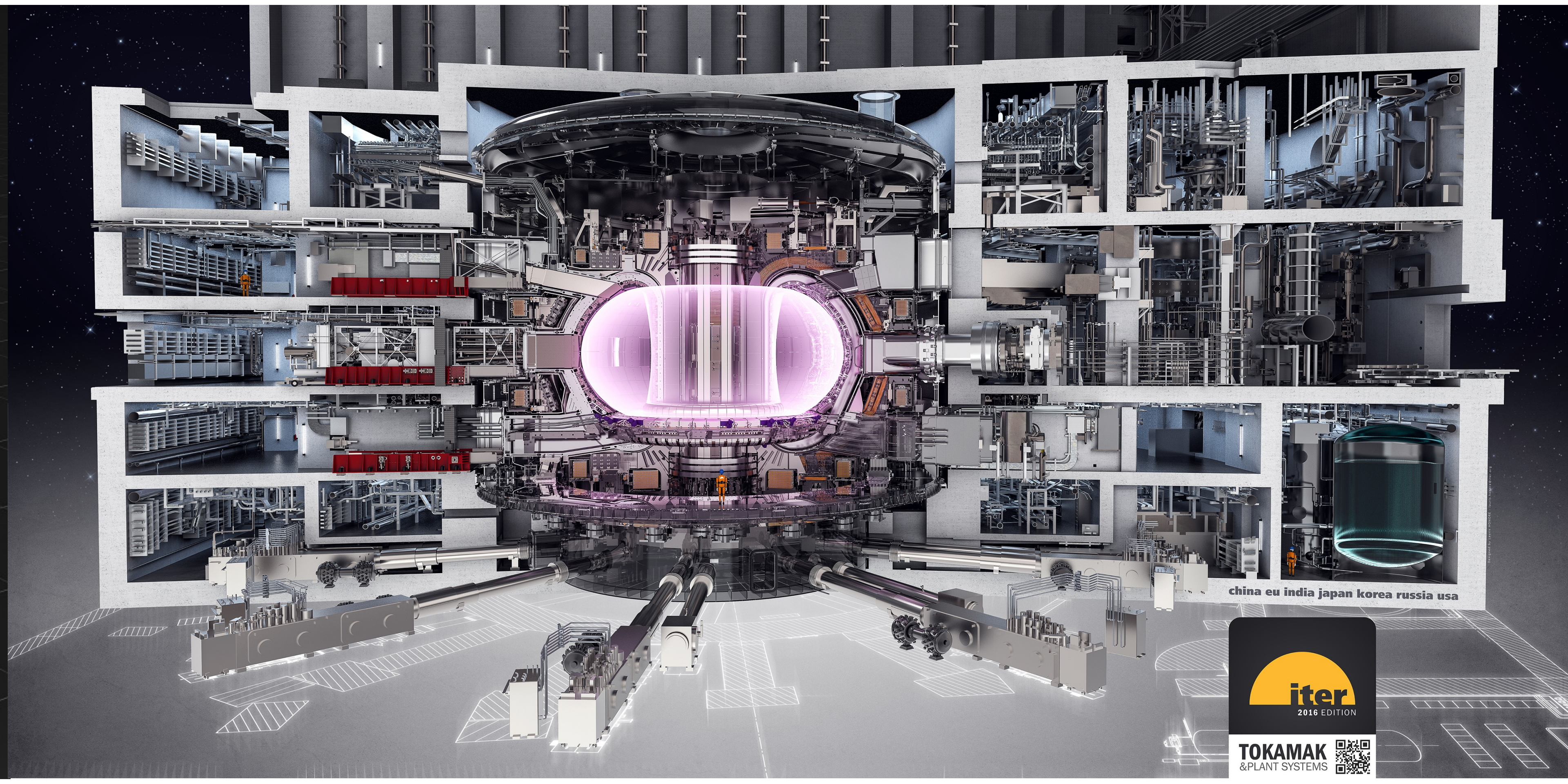
Multiple upcoming tokamak experiments will run DT plasmas



The Joint European Torus (JET) chamber
Source: CCFE



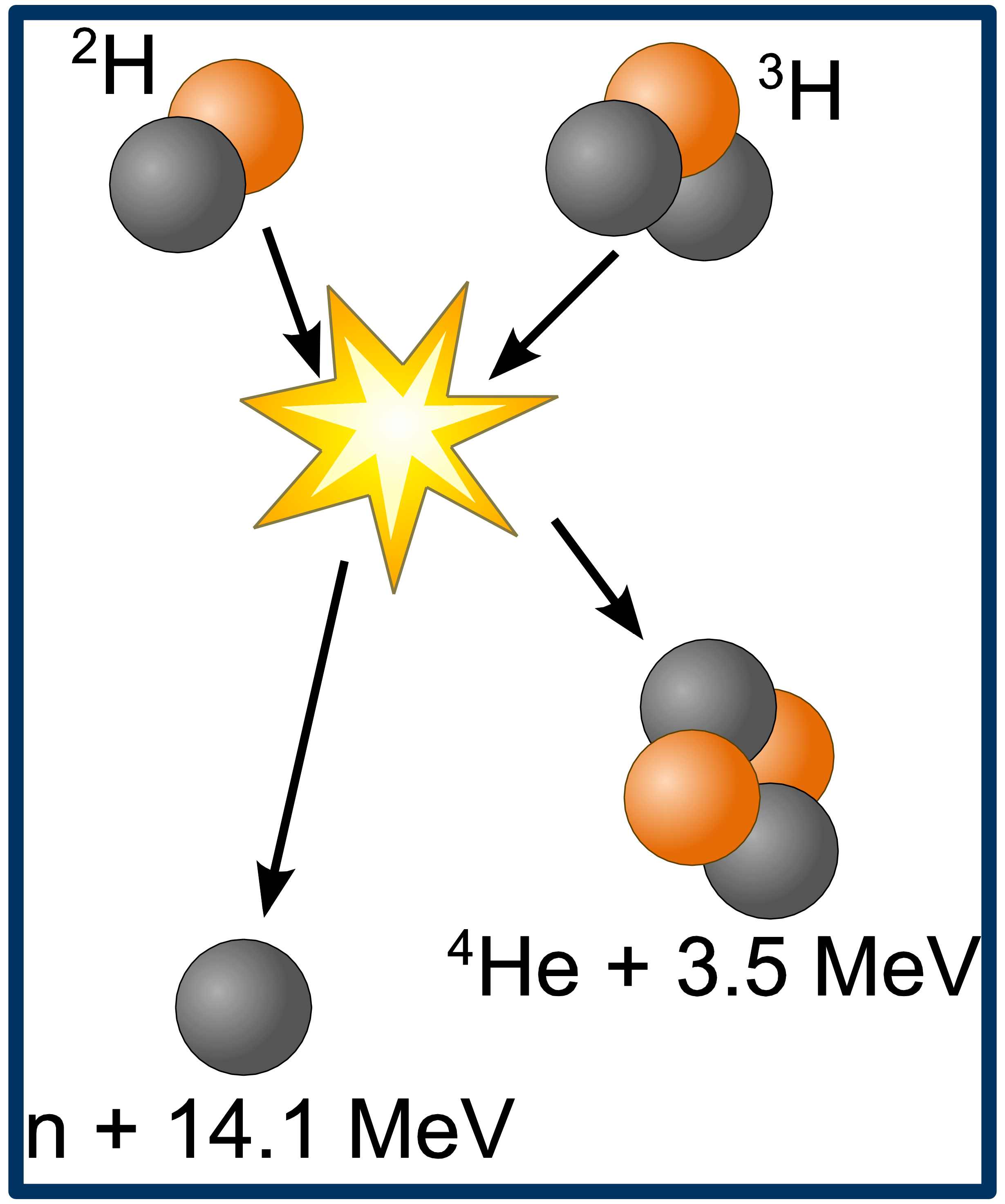
Rendering of the SPARC tokamak
Source: CFS/MIT-PSFC - CAD Rendering by T. Henderson



Rendering of the ITER tokamak
Source: ITER Organization, http://www.iter.org/

- A series of upcoming tokamak experiments plan to run with DT fuel
  - JET DT campaign
  - SPARC
  - ITER
- First DT experiments since the 1990's
  - (with exception of trace tritium experiments)
- Exciting plasma physics motivates new attention to relevant theory

Alpha physics is novel part of next-generation DT tokamaks



- One novel, important part of DT tokamak physics is alpha particle behavior

- Alphas heat bulk plasma, help maintain its temperature
  - For SPARC primary reference plasma: $P_\alpha \approx 28$ MW , $P_{rf,coupled} + P_{ohmic} \approx 12.8$ MW [Creely et al. DPP 2020]

- Alphas can interact with perturbations to tokamak electric and magnetic fields, causing transport
  - Transport can modify heat deposition
  - Loss can degrade performance, damage device

Unperturbed alpha distribution is peaked in core

- Unperturbed alpha population given by slowing down distribution:

$$f_\alpha(v,\psi) = \frac{S_{fus}(\psi)\tau_s(\psi)H(v-v_0)}{4\,\pi[v^3 + v_c^3(\psi)]}$$

$$S_{fus}\tau_s = n_D n_T \langle\sigma v\rangle\,\tau_s \propto nT^{7/2}$$

Parameter definitions

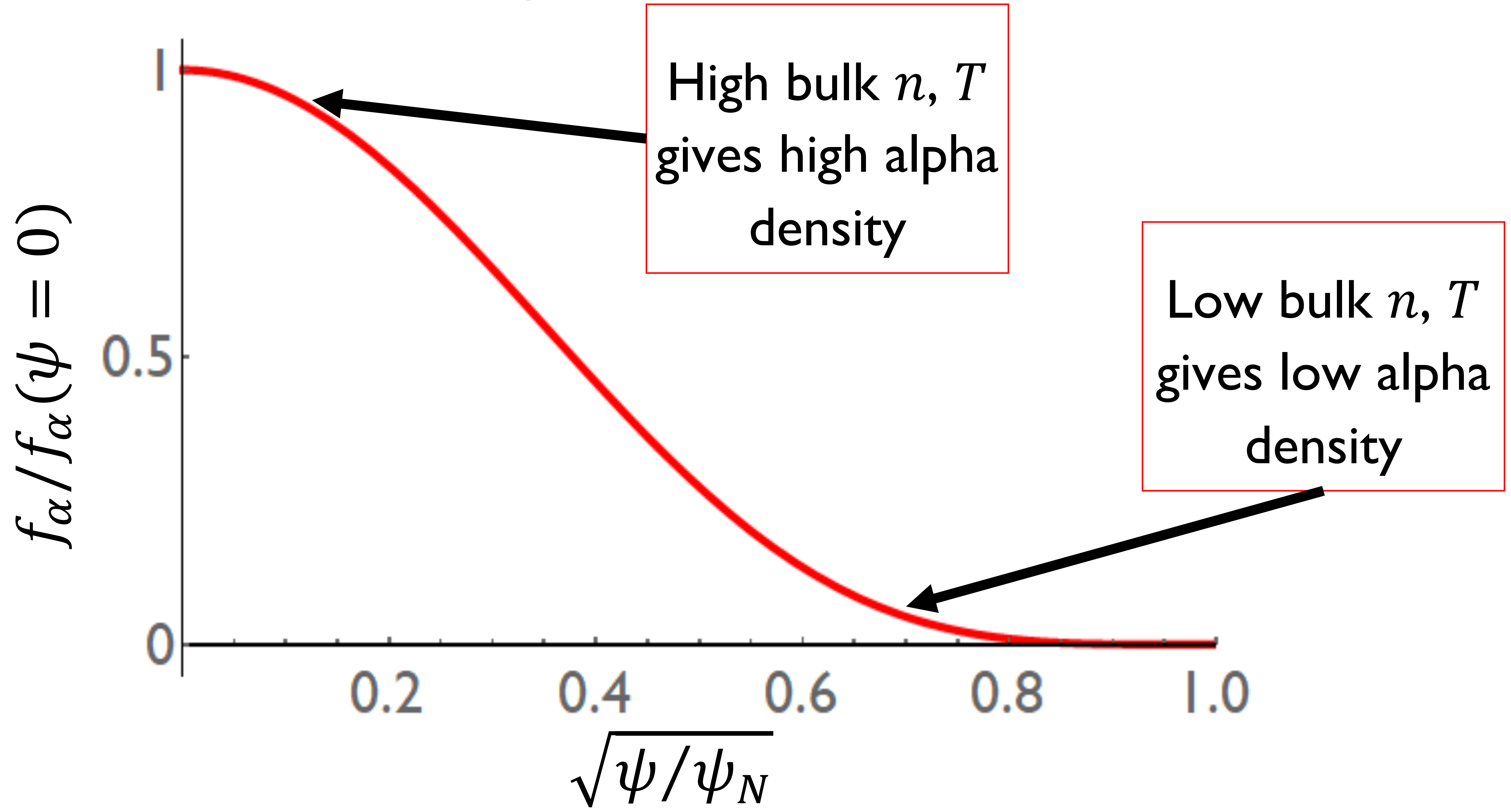
S_{fus}: fusion rate

$H(v-v_0)$: Heaviside step function

v_c : critical speed

$\tau_s \sim T_e^{\frac{3}{2}}/n_e$: slowing down time

- Many alphas toward core; few towards edge

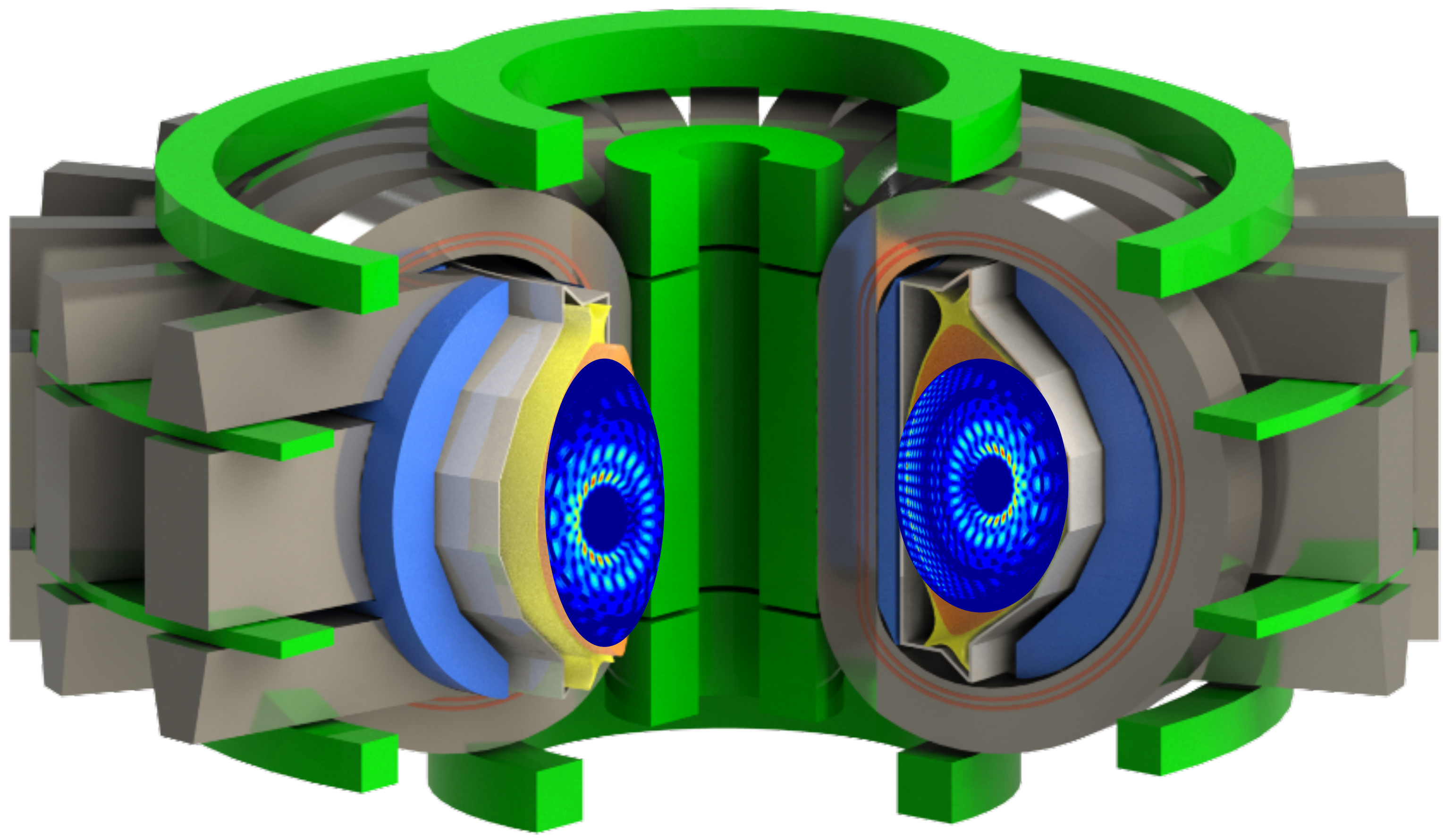


Interaction of alphas and perturbations leads to transport

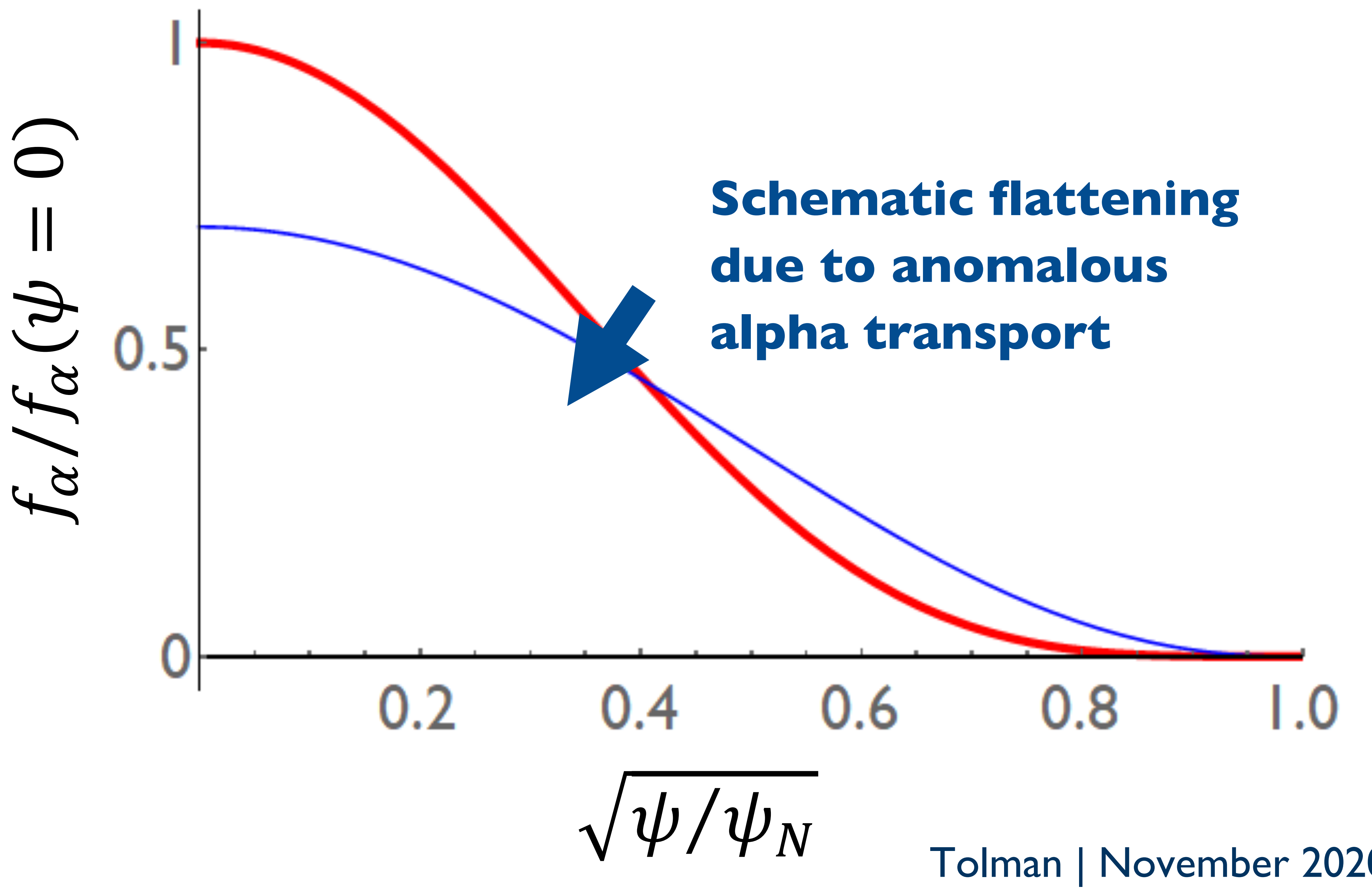
- Tokamak fields experience a variety of perturbations
  - Ripple
  - MHD modes (Alfvén eigenmodes, NTMs, etc.)
  - RMP coils

- These perturbations create perturbed:
  - Alpha distribution: f_1
  - Alpha radial velocity: v_{AE}

- Leads to transport of alphas from core to edge
  - Excessive transport could lead to loss of necessary alpha heat or to damage to device



[Image sources: Mumgaard APS 2018 + Snicker et al. NF 2013]



This presentation focuses on Alfvén eigenmode transport

- In a tokamak, Alfvén waves can exist as eigenmodes (AEs)

$$B_1 = \sum_m B_{mn\omega}(\psi)\cos(n\zeta - m\vartheta - \omega t)$$

- AEs exist at discrete frequencies
  - $\omega_{TAE} = \frac{v_A}{2qR}$

- AEs driven by spatial gradient of alpha population through resonance with alpha orbits
  - Resonant speed depends on harmonic and particle pitch angle, increases with with Alfvén speed

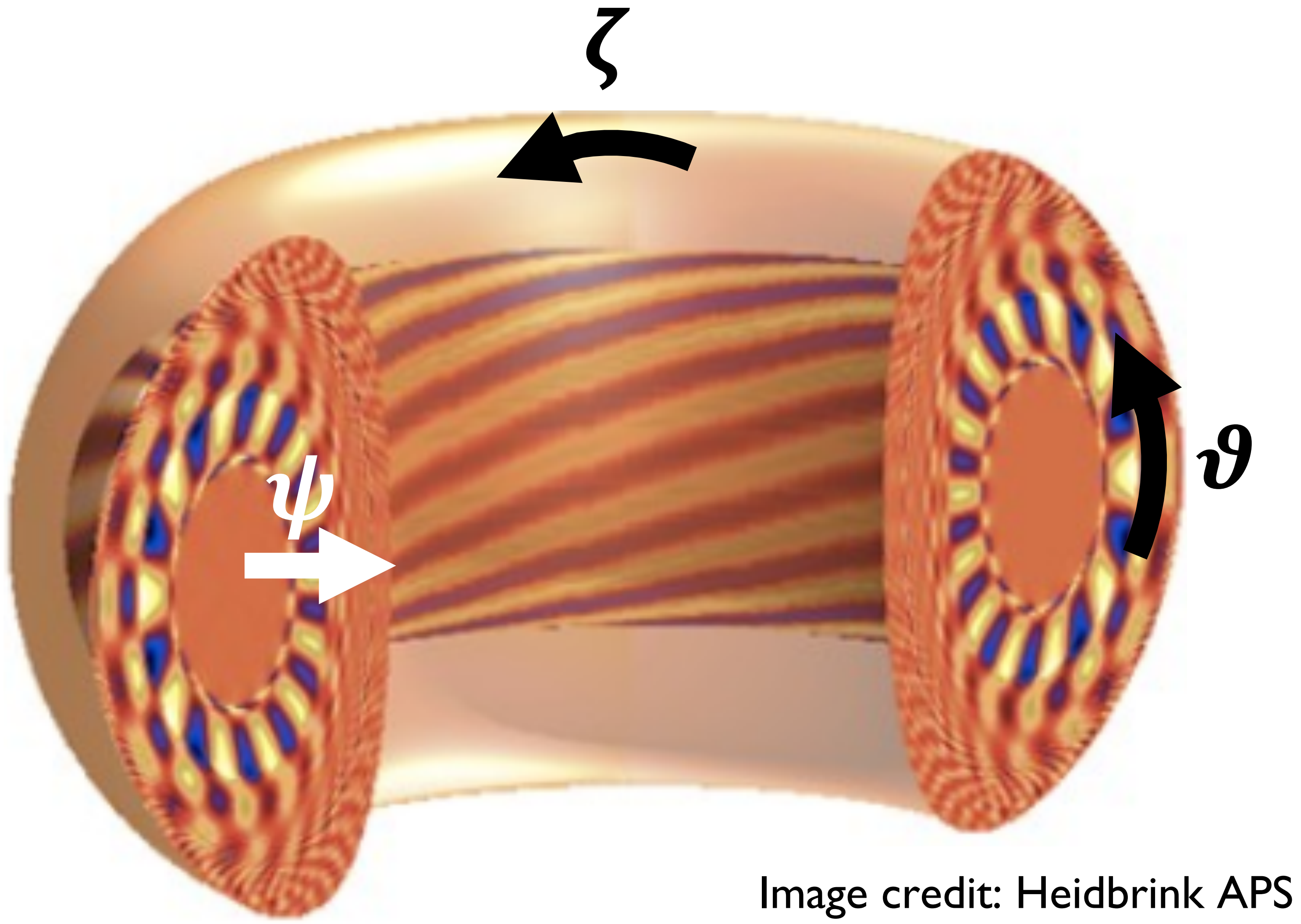
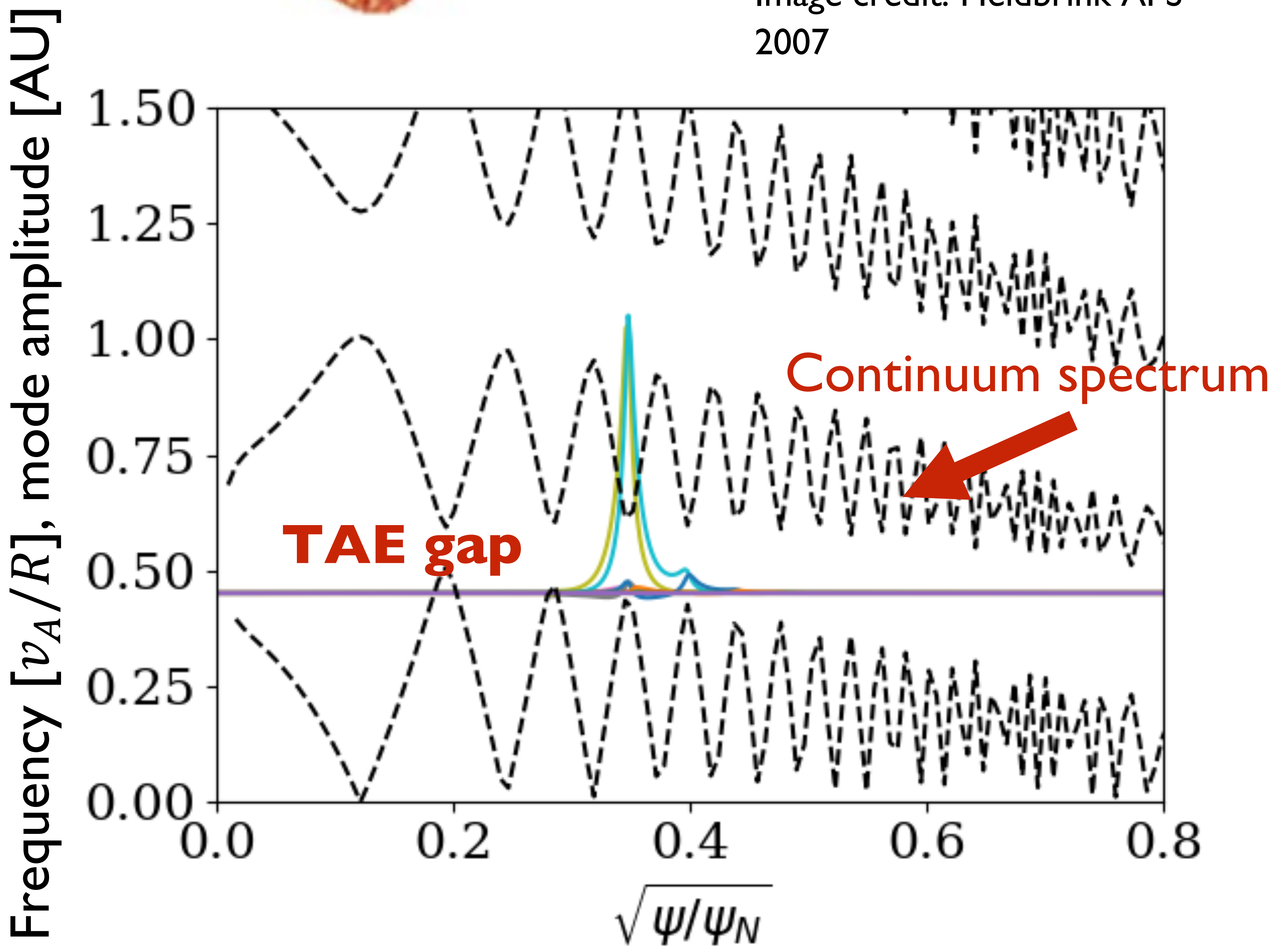


Image credit: Heidbrink APS 2007



We develop drift kinetic theory of transport

- Theory of alpha transport by perturbations focuses on single alpha trajectories
  - Codes used to study alpha distribution

We develop a drift kinetic theory for ***D***, the diffusivity caused by TAEs, as a function of local TAE characteristics

•*Can also be used to model transport from other perturbations (other AEs, ripple, etc.)*

•*See Tolman and Catto* In Review *(available on arXiv:2011.04920)*

TAEs include electric and magnetic field perturbations

- TAE includes magnetic field and electric field perturbations

Electric potential is given by:

$$\Phi_1 = \Phi_{mn\omega} e^{i(n\zeta - m\vartheta - \omega t)} e^{i\int d\psi \frac{k_\psi}{RB_p}}$$

Amplitude — Wave phase — Radial variation

Magnetic vector potential is given by:

$$A_{\parallel 1} = \frac{c\Phi_{mn\omega}}{v_A} e^{i(n\zeta - m\vartheta - \omega t)} e^{i\int d\psi \frac{k_\psi}{RB_p}} \hat{b}$$

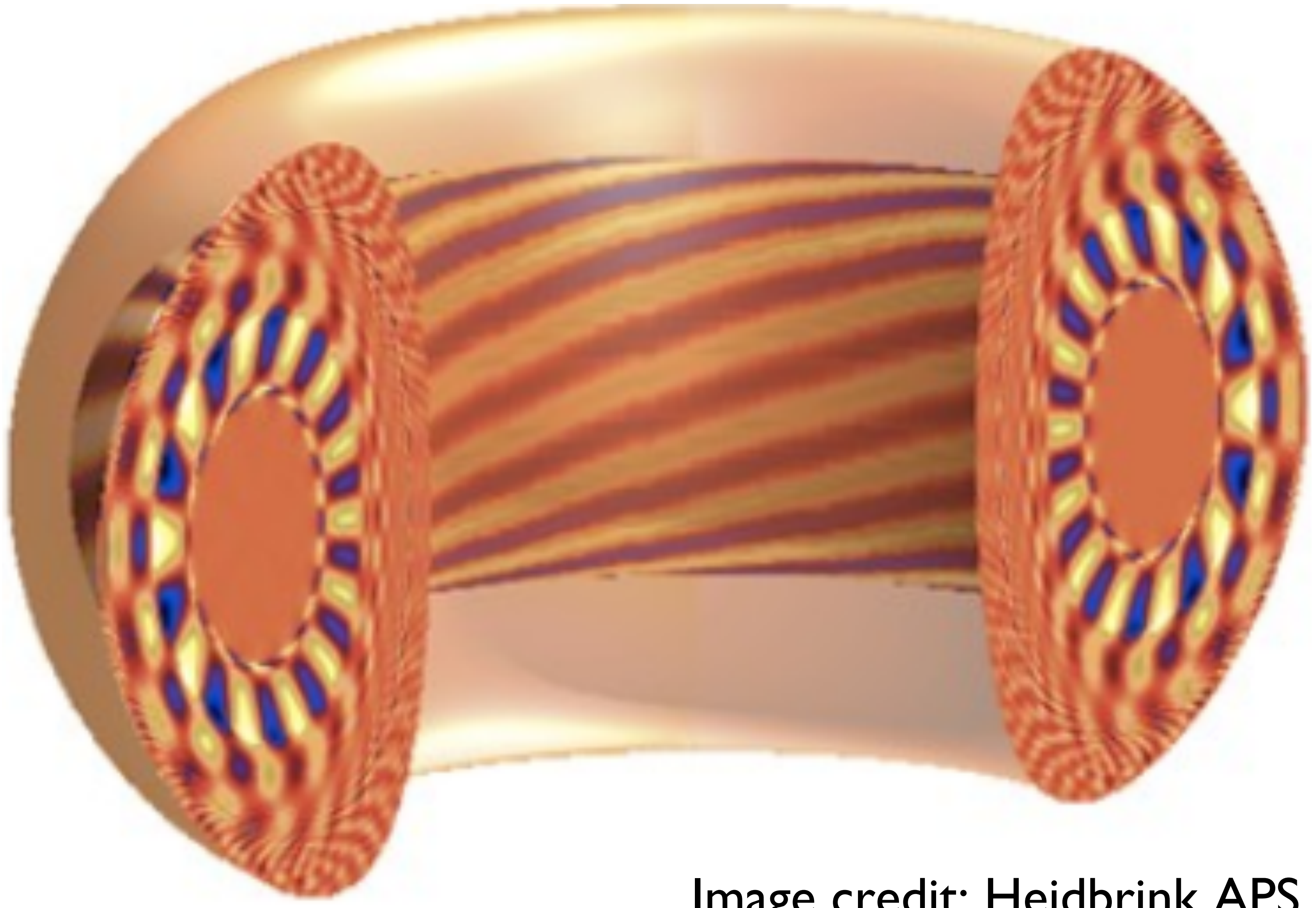


Image credit: Heidbrink APS 2007

Multiple TAEs work together to cause transport

- In realistic tokamak, multiple poloidal harmonics per TAE and multiple TAEs

- TAEs at different radial locations work together to cause transport across cross section

- For perturbations with significant radial overlap:

$n \neq n'$	$n = n', m \neq m'$
• Transport does not couple • Diffusion is superimposed	• Transport couples for similar m • Difficult to treat analytically • See discussion in paper

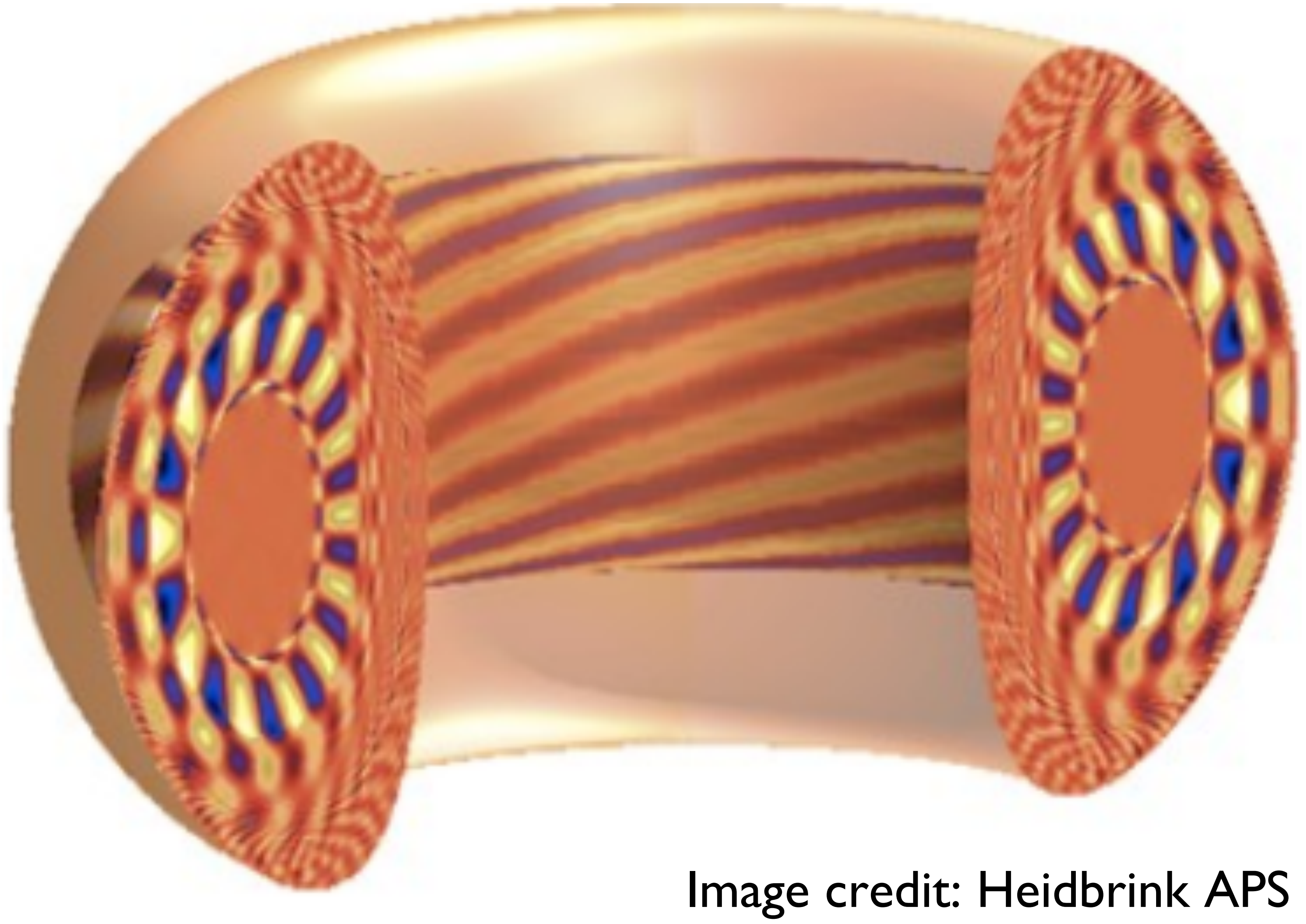


Image credit: Heidbrink APS 2007

Alpha distribution develops similar perturbation

- TAE includes magnetic field and electric field perturbations

Electric potential is given by:

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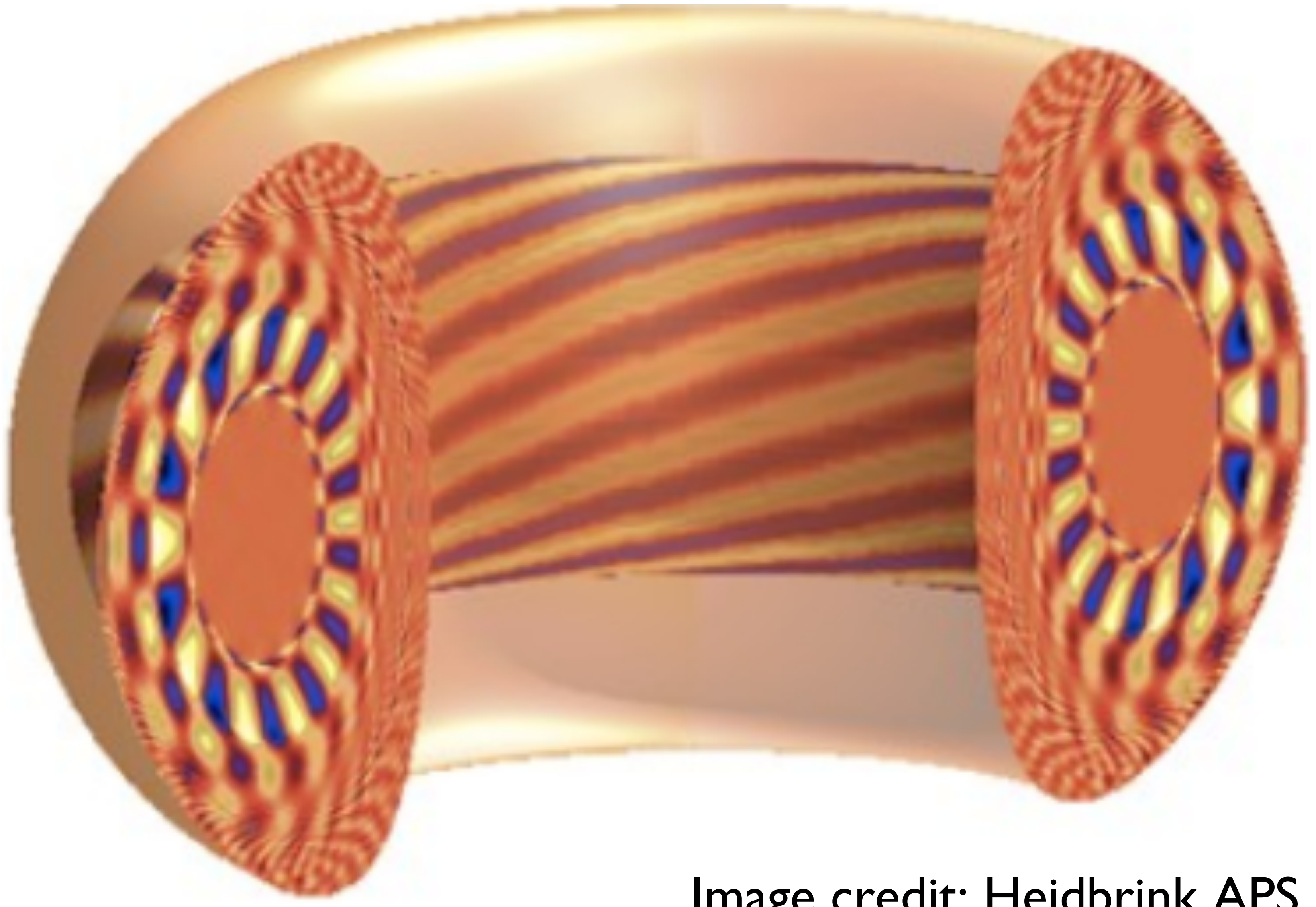

Image credit: Heidbrink APS 2007

- TAE creates a corresponding perturbation to f_α

$$f_1 = \frac{Ze\,\Phi_1}{M}\frac{\partial f_\alpha}{\partial \mathcal{E}} + h(\vartheta) e^{i[n(\zeta - q\,\vartheta) - \omega t]} e^{i\int d\psi \frac{k_\psi}{RB_p}}$$

Adiabatic response, $\mathcal{E}$ is energy

Response that causes transport

Drift kinetic equation models transport, gives *h*

The perturbed drift kinetic equation used to find *h* is:

$$v_\parallel \hat{b} \cdot \nabla\vartheta \frac{\partial h}{\partial\vartheta} - i\left[\omega - n\overrightarrow{v_d} \cdot \nabla(\zeta - q\,\vartheta)\right]h + i\nu_{\mathrm{AE}} \frac{\partial f_\alpha}{\partial r}\left(1 - \frac{v_\parallel}{v_A}\right) e^{i\left[(nq-m)\vartheta + \frac{k_\psi v_\parallel}{\Omega_p}\right]} = \nu_{pas} \frac{\partial^2 h}{\partial \lambda^2}$$

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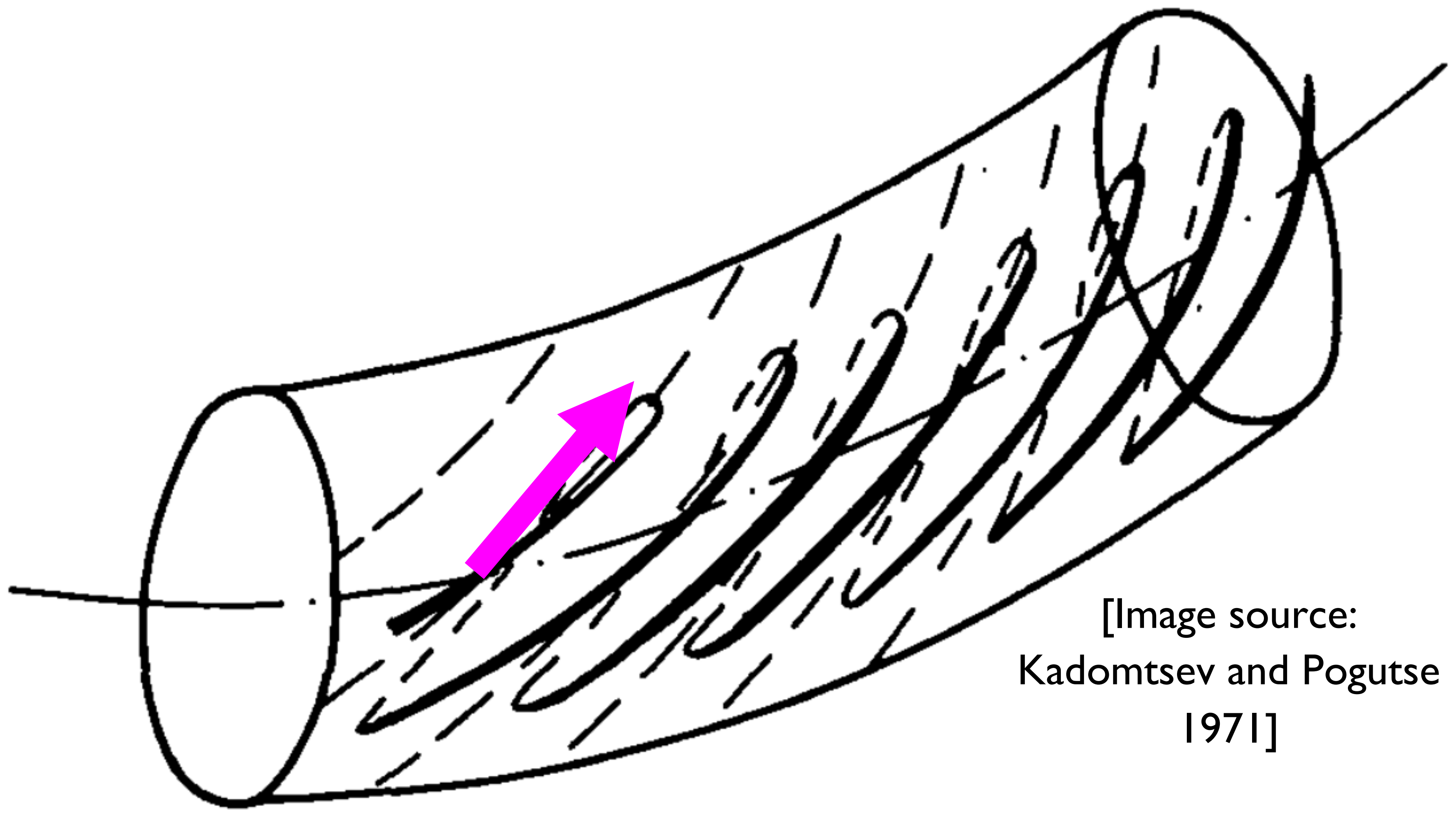
Streaming of unperturbed alpha orbit along magnetic field



[Image source: Kadomtsev and Pogutse 1971]

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TAE frequency

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Toroidal mode number

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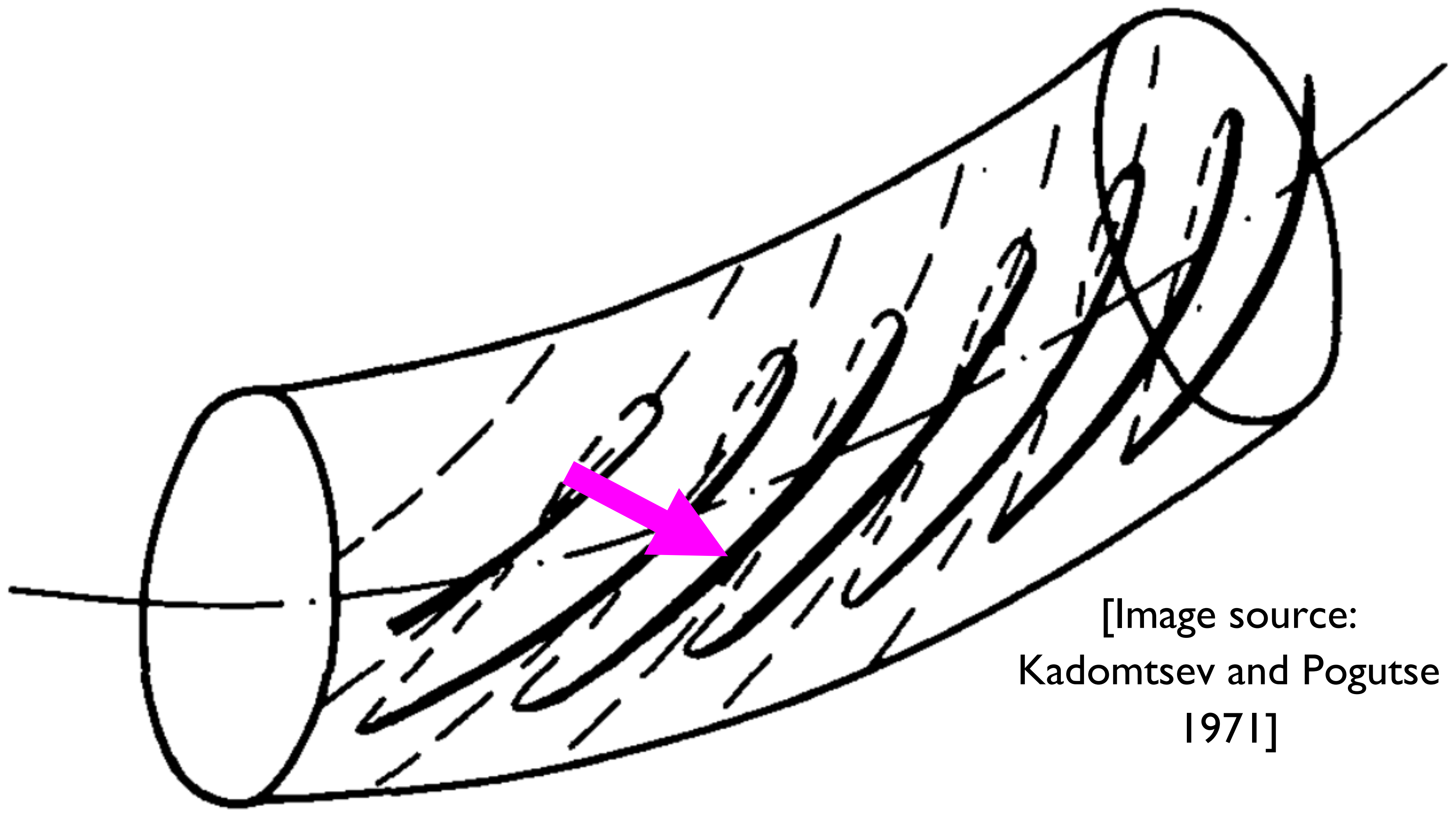
Drift of unperturbed alpha orbit in flux surface



[Image source: Kadomtsev and Pogutse 1971]

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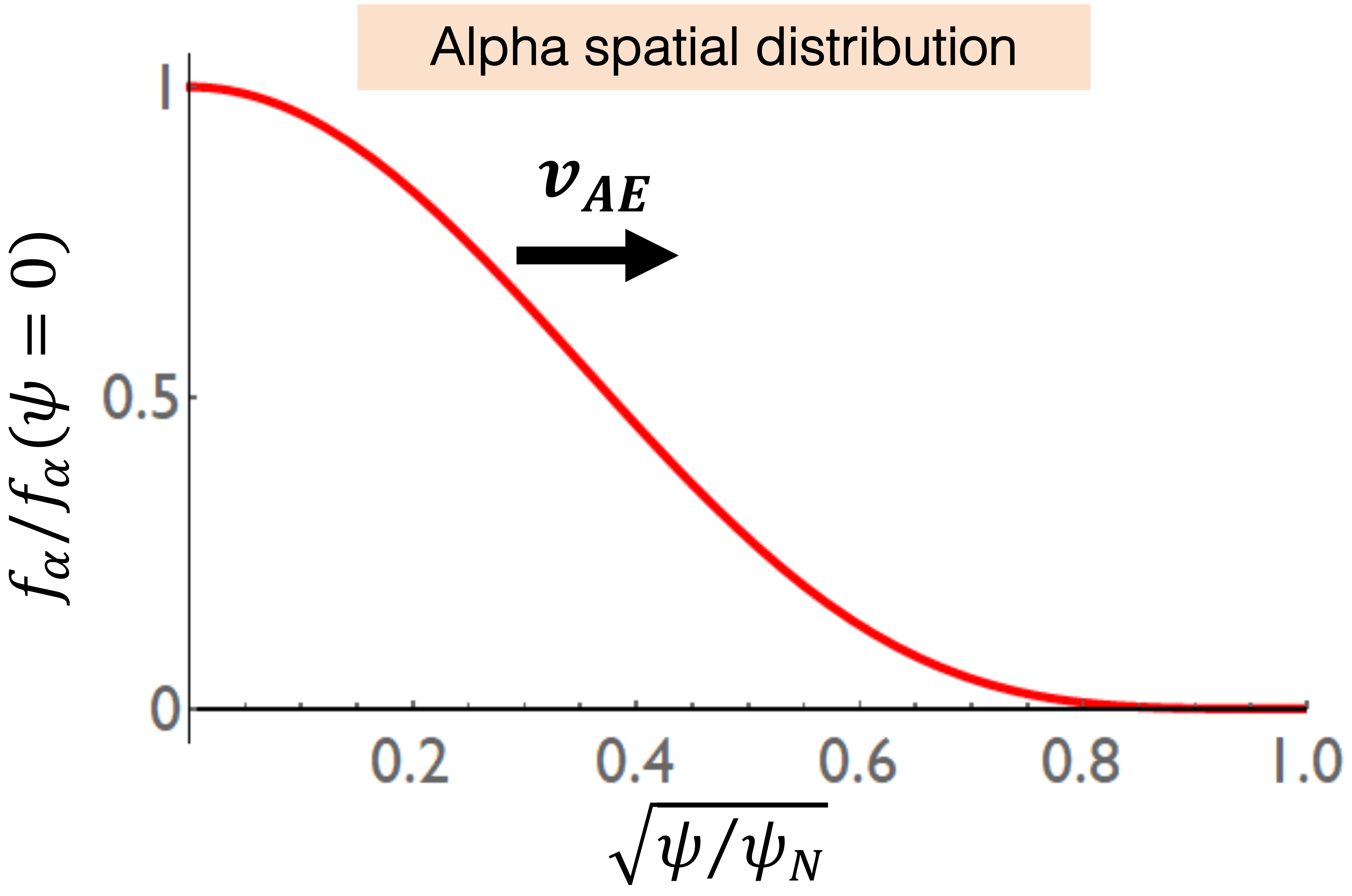
Drive from TAE and alpha spatial gradient

- v_{AE} is radial velocity caused by TAE ($\vec{E}\times\vec{B}$ drift + changed B field direction)
- $\frac{\partial f_\alpha}{\partial r}$ is the alpha spatial gradient
- Other terms give poloidal variation in strength of transport



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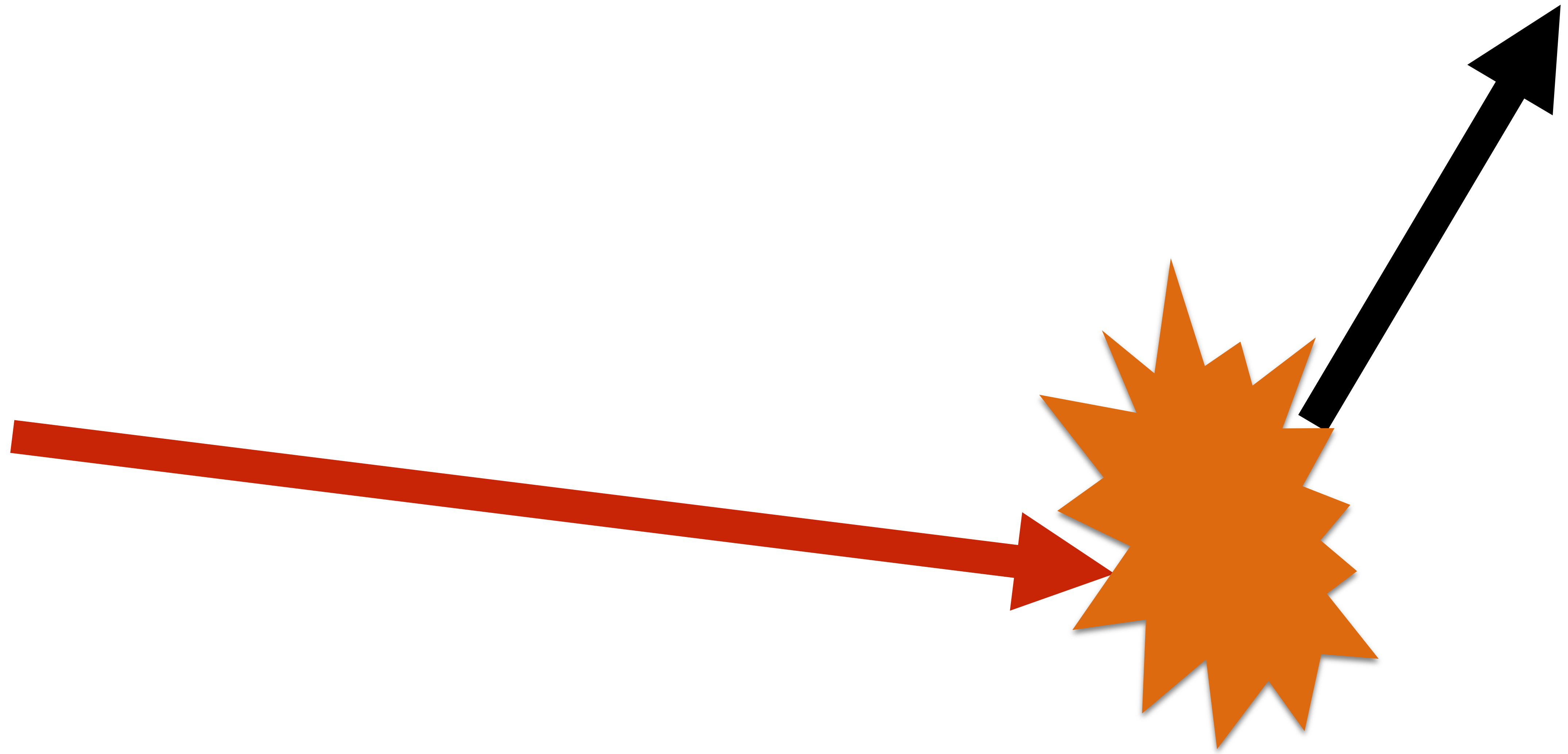
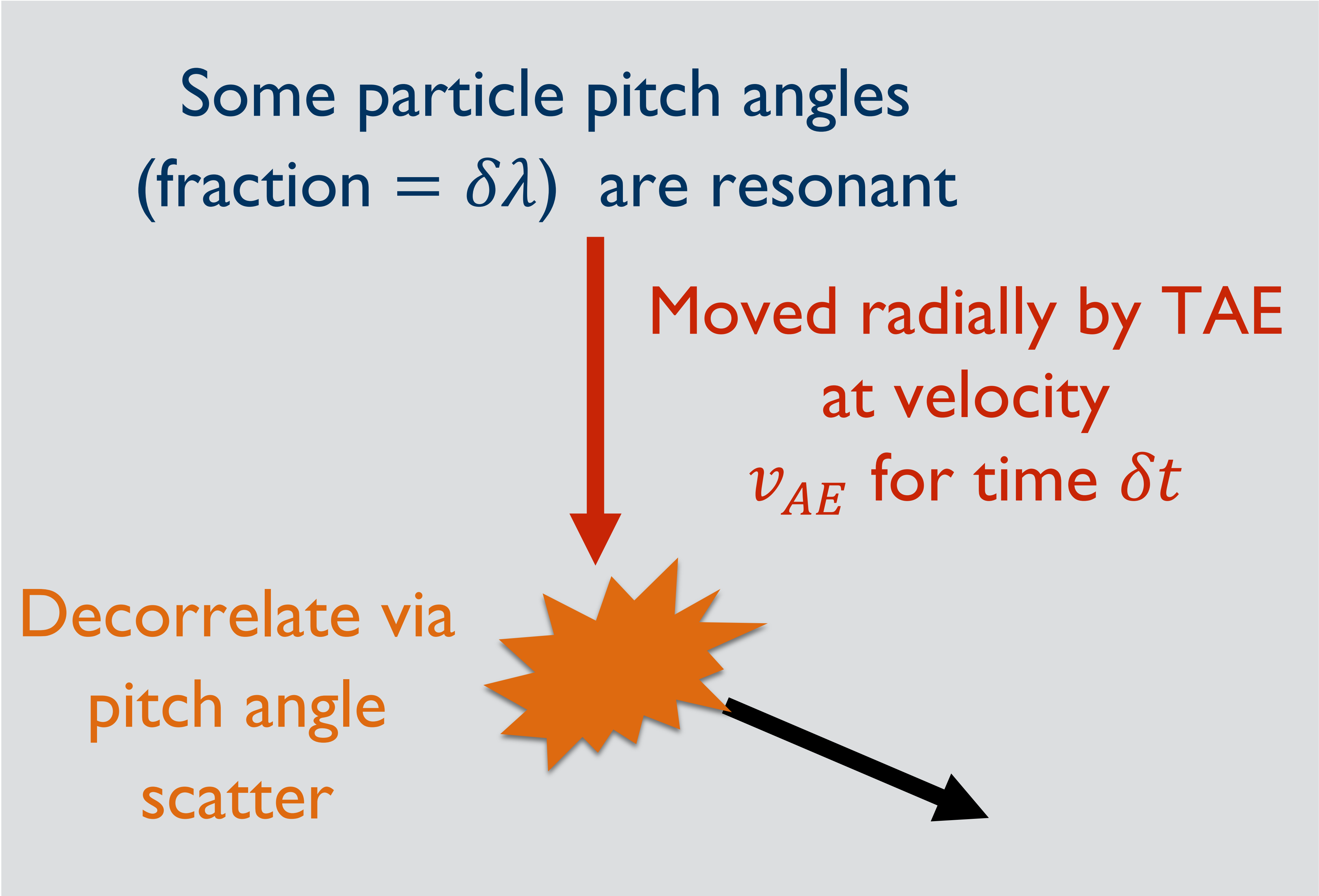
Pitch angle scattering of alpha particles

- Pitch angle is the angle between a particle's velocity and the background magnetic field
- Represented by $\lambda\equiv\frac{B_0 v_\perp^2}{Bv^2}$
- Frequency of pitch angle scatter is ν_{pas}



Phenomenological estimate of transport is possible

$$v_{\parallel}\hat{b}\cdot\nabla\vartheta\,\frac{\partial h}{\partial\vartheta}-i\,[\omega-n\overrightarrow{v_d}\cdot\nabla(\zeta-q\,\vartheta)]h+iv_{\mathrm{AE}}\frac{\partial f_{\alpha}}{\partial r}\left(1-\frac{v_{\parallel}}{v_A}\right)e^{i\left[(nq-m)\vartheta+\frac{k_{\psi}v_{\parallel}}{\Omega_p}\right]}=\nu_{pas}\frac{\partial^2 h}{\partial\lambda^2}$$



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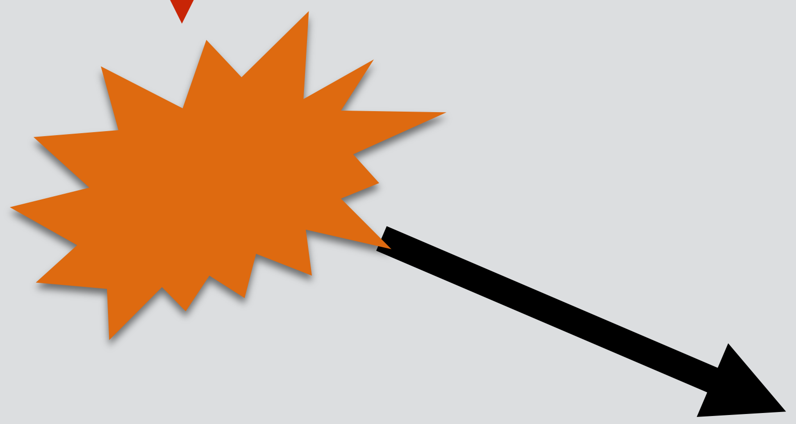
Some particle pitch angles (fraction $= \delta\lambda$) are resonant

Moved radially by TAE at velocity v_{AE} for time δt

Decorrelate via pitch angle scatter



Estimate of fraction of particles in resonance ($\delta\lambda$)

$$\omega\delta\lambda \sim \frac{\nu_{pas}}{\delta\lambda^2} \rightarrow \delta\lambda \sim \left(\frac{\nu_{pas}}{\omega}\right)^{1/3}$$

Higher ν_{pas} allows more particles to be resonant

Phenomenological estimate of transport is possible

$$v_\parallel \hat{b} \cdot \nabla\vartheta \frac{\partial h}{\partial \vartheta} - i\left[\omega - n\overrightarrow{v_d} \cdot \nabla(\zeta - q\,\vartheta)\right]h + i v_{\mathrm{AE}} \frac{\partial f_\alpha}{\partial r}\left(1 - \frac{v_\parallel}{v_A}\right) e^{i\left[(nq-m)\vartheta + \frac{k_\psi v_\parallel}{\Omega_p}\right]} = \nu_{pas} \frac{\partial^2 h}{\partial \lambda^2}$$

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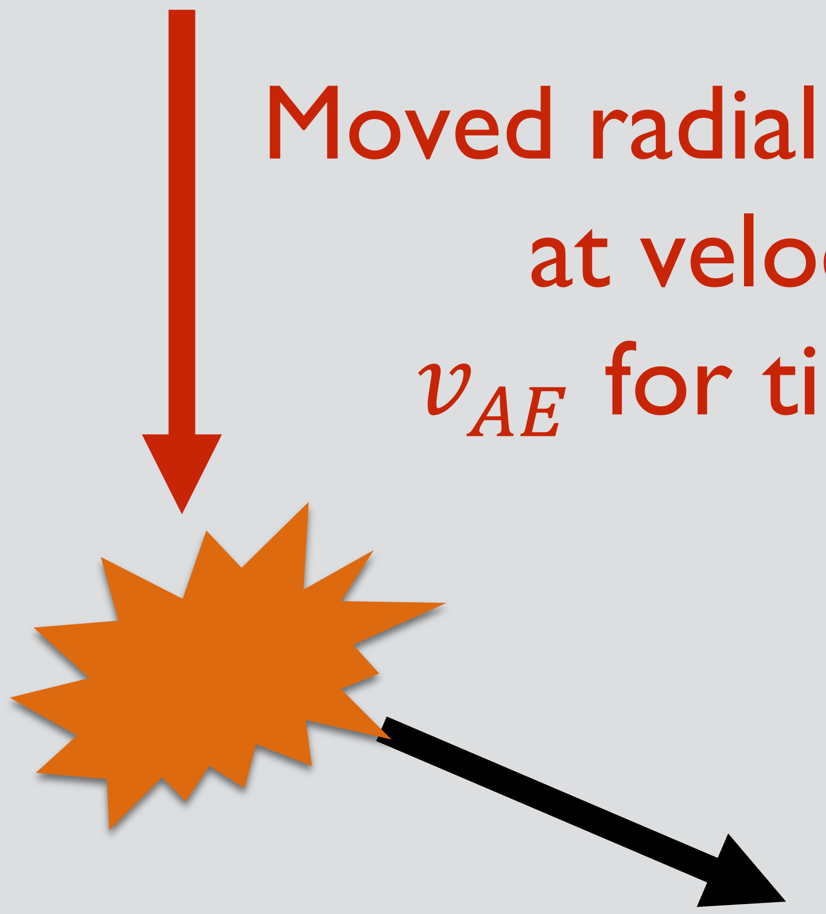

Decorrelate via pitch angle scatter

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Estimate of radial step ($v_{AE}\delta t$)

$$\delta t \sim \frac{\delta\lambda^2}{\nu_{pas}} \to v_{AE}\delta t \sim \frac{v_{AE}}{\omega^{2/3}\nu_{pas}^{1/3}}$$

Higher ν_{pas} shortens step size

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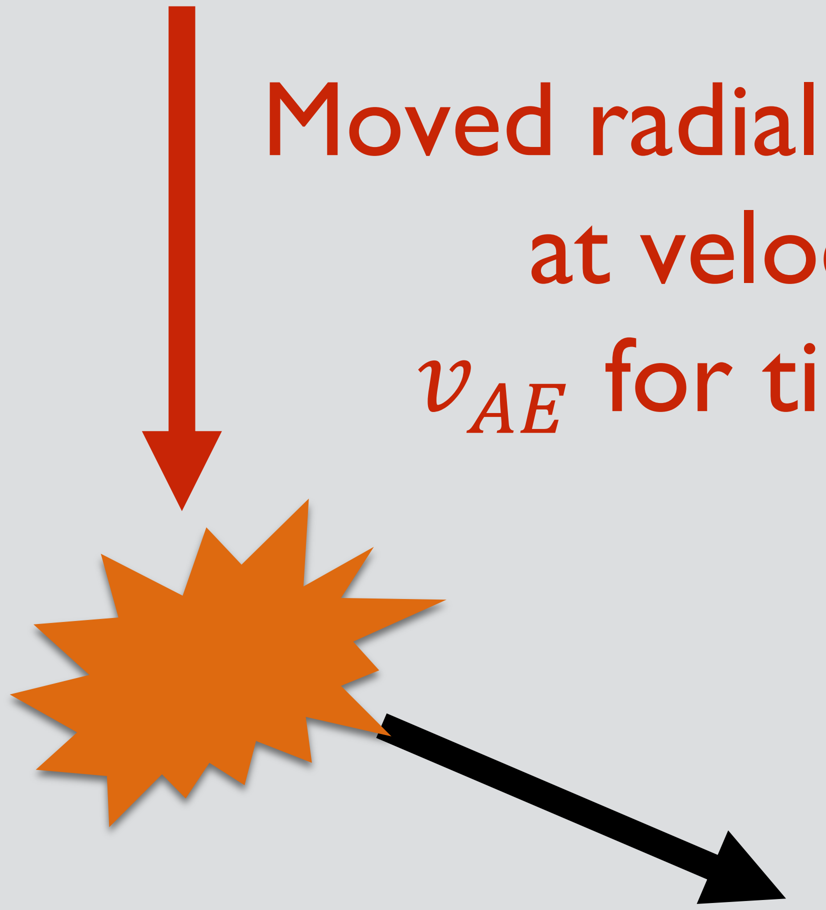

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Higher ν_{pas} shortens step size

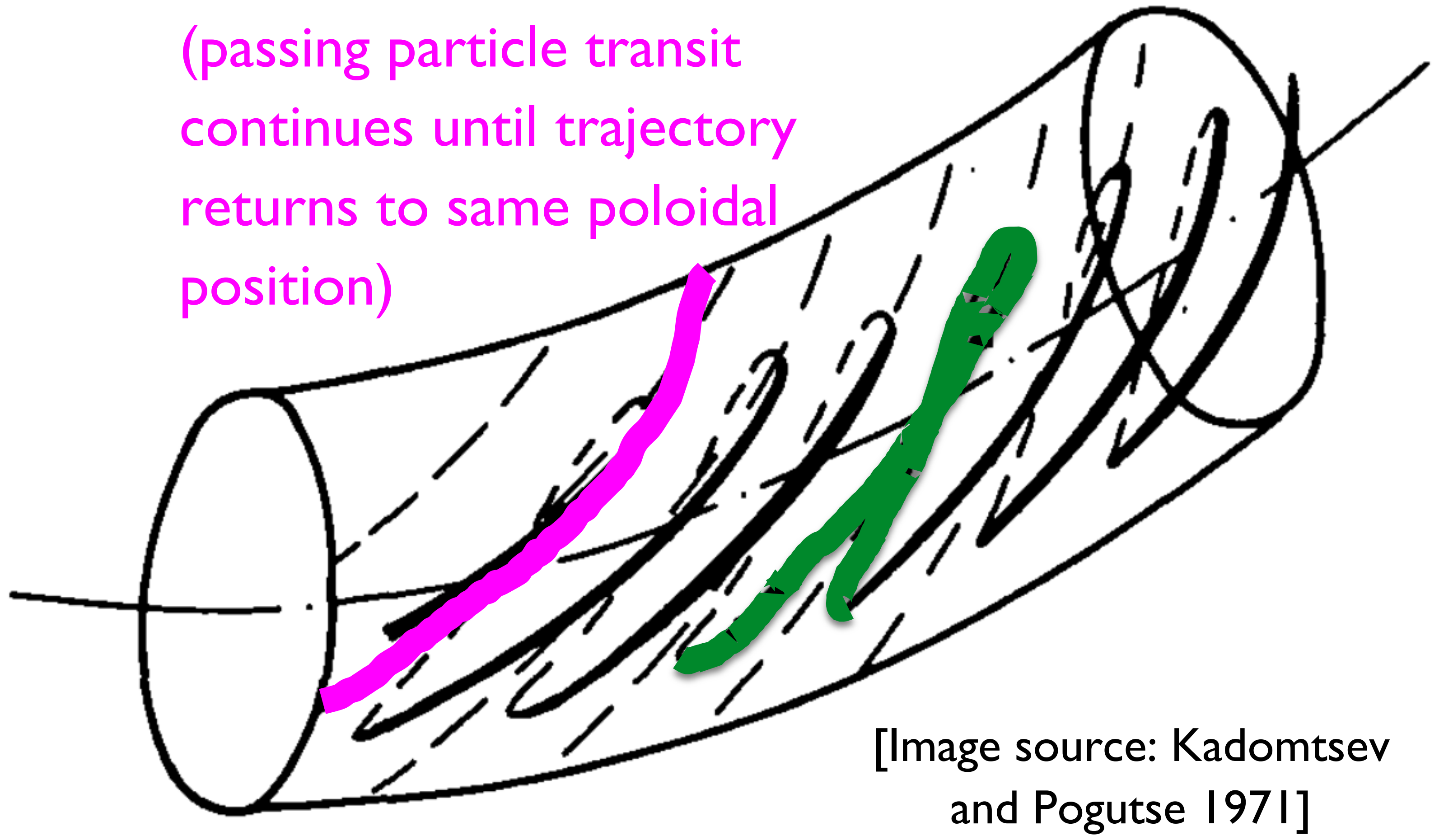
Estimate of overall diffusivity (D)

$$D \sim \delta\lambda \frac{(v_{AE}\delta t)^2}{\delta t} \sim \frac{v_{AE}^2}{\omega}$$

- ***D has no explicit ν_{pas} dependence***
- ***D increases with $v_{AE}^2 \propto B_{mn\omega}^2$***

Rigorous evaluation integrates over particle trajectory

- Particle orbit is a series of **bounces (trapped particles)** or **transits (passing particles)**
- Integrate drift kinetic equation over bounce or transit to get *h*



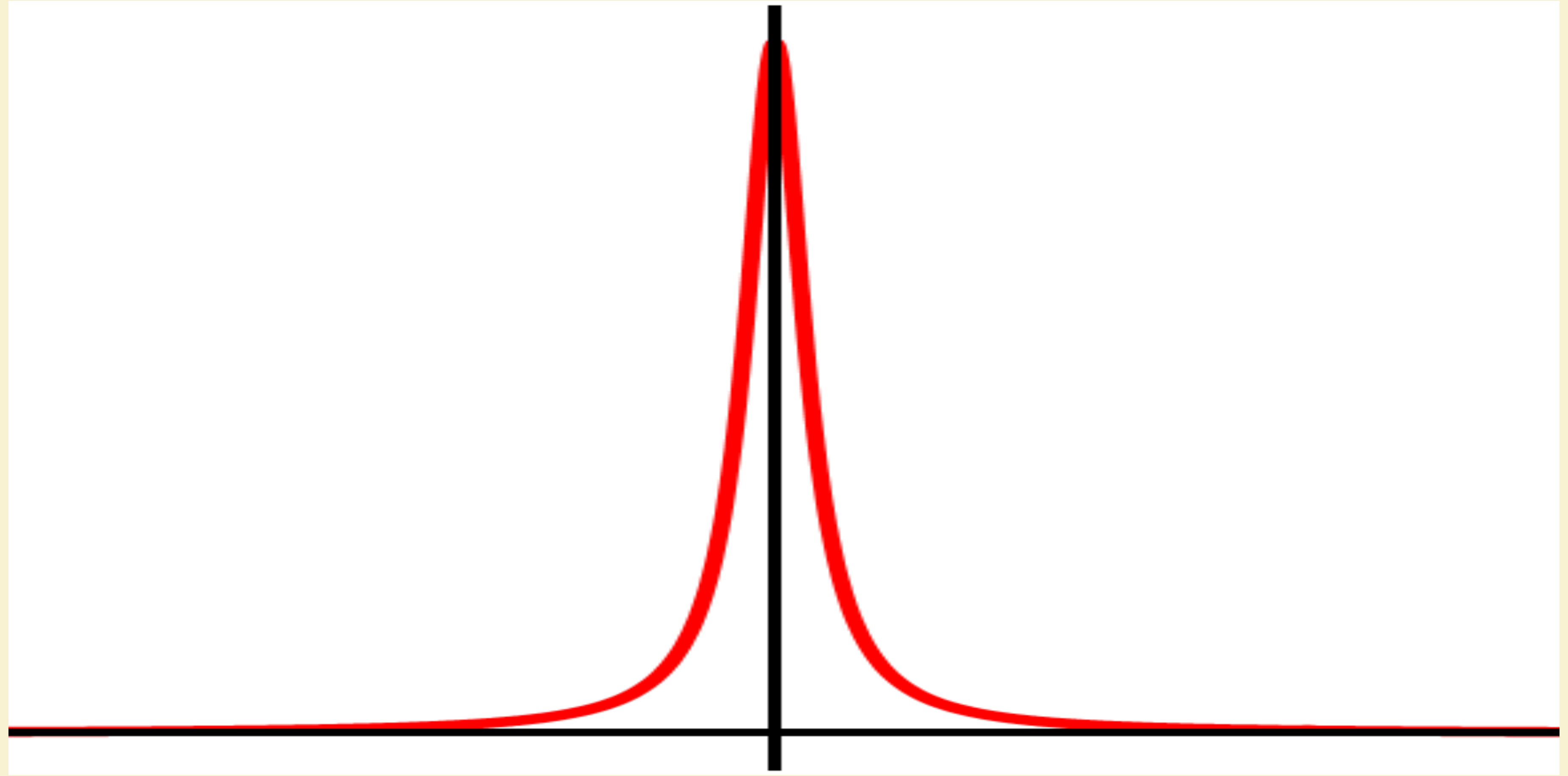
[Image source: Kadomtsev and Pogutse 1971]

Rigorous evaluation reveals resonance condition

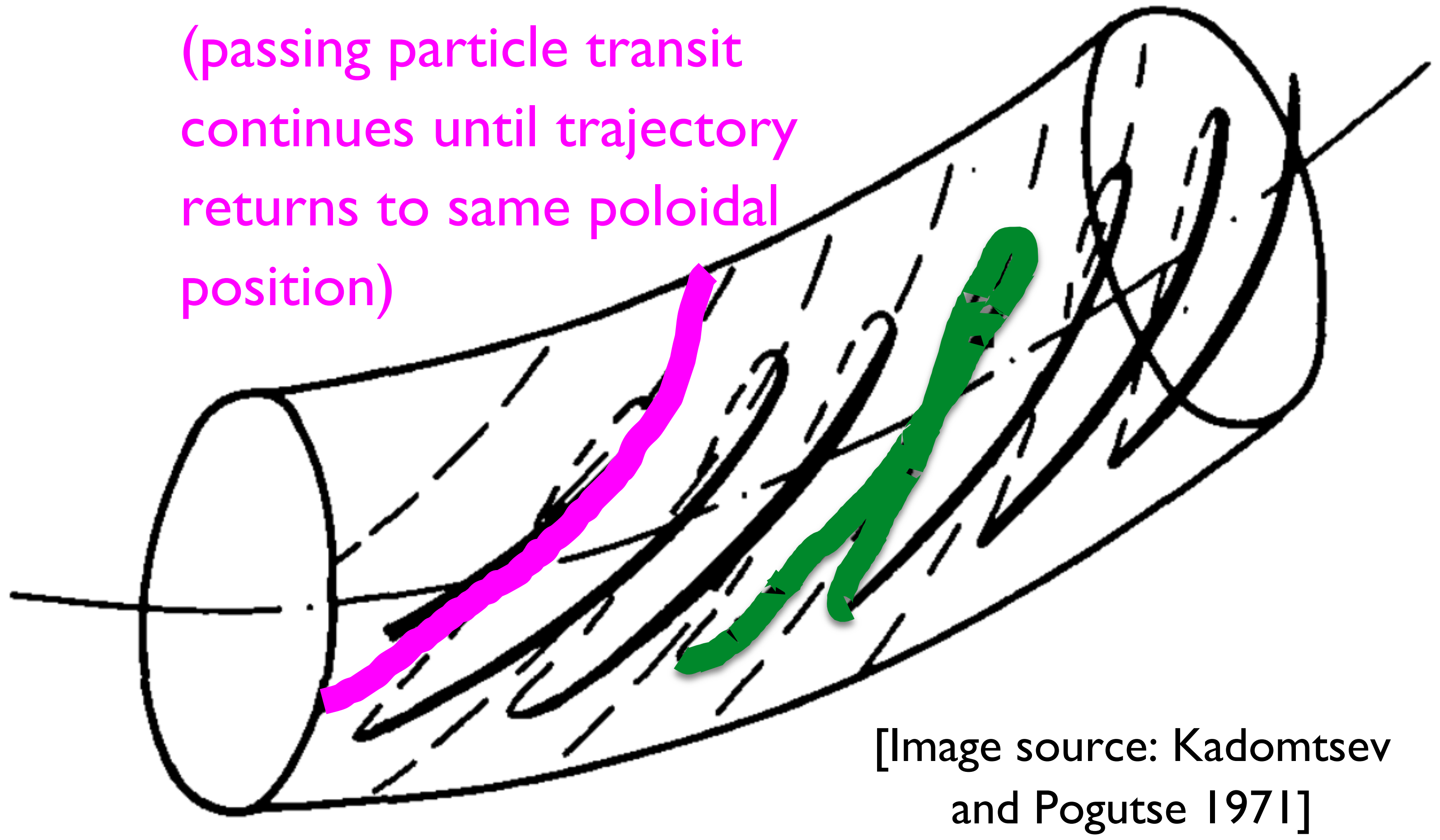
- Integration reveals resonance condition: particles that drift in resonance with ω, n participate in transport

Distribution response $\boldsymbol{h}$ for trapped particles

(harmonic l = 0)



$$\omega - n\overline{\vec{v_d} \cdot \nabla(\zeta - q\,\vartheta)}(\lambda)$$



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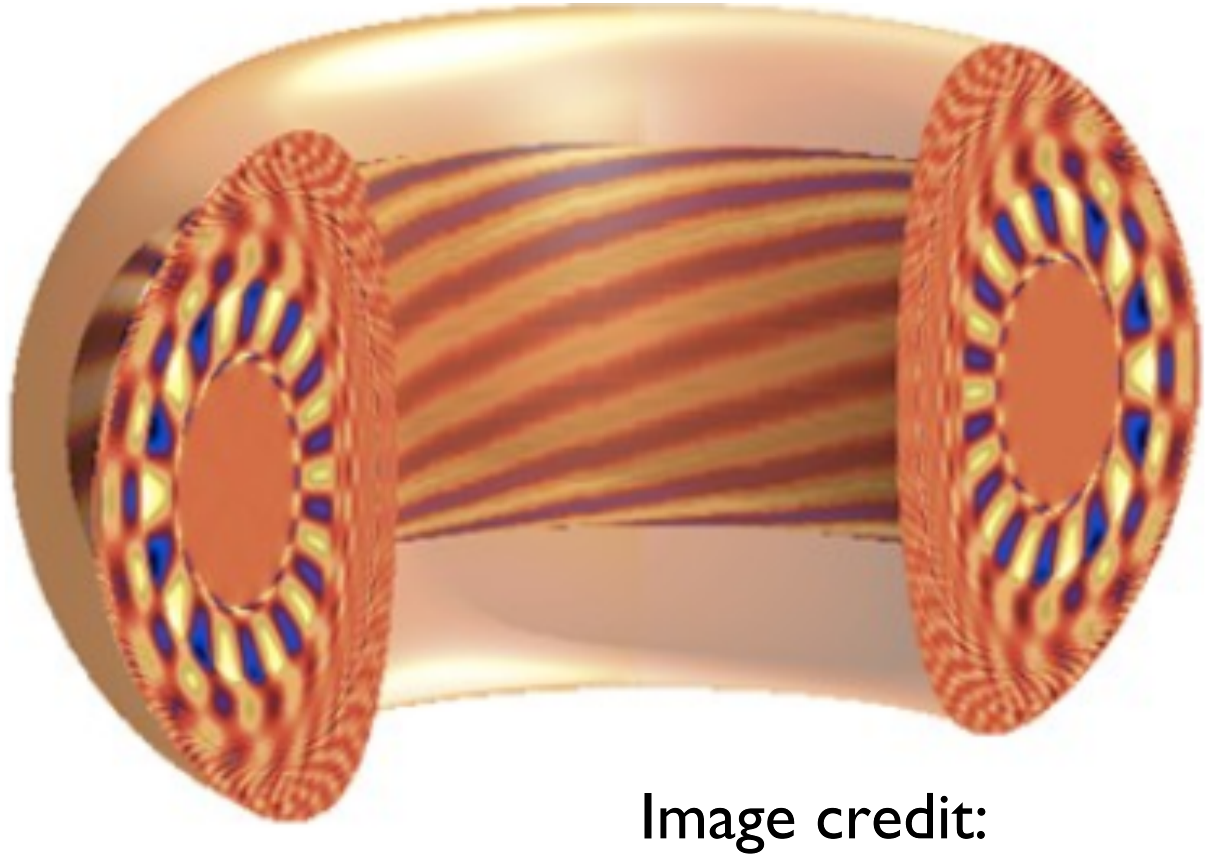


Image credit: Heidbrink APS 2007

- Sharp variation of h with respect to λ explains importance of pitch angle scattering $\nu_{pas} \frac{\partial^2 h}{\partial \lambda^2}$

Rigorous evaluation integrates over particle trajectory

- h averaged over flux surface to get D

- Rigorous evaluation gives:

$$D_{trapped} \sim \sqrt{\epsilon}\,\frac{v_{\mathrm{AE}}^2}{\omega}$$

$$D_{passing} \sim \frac{v_{\mathrm{AE}}^2}{\omega}$$

- Compare to estimate $D \sim \frac{v_{\mathrm{AE}}^2}{\omega}$
  - $\sqrt{\epsilon} = \sqrt{\frac{r}{R}}$ fraction of particles are trapped

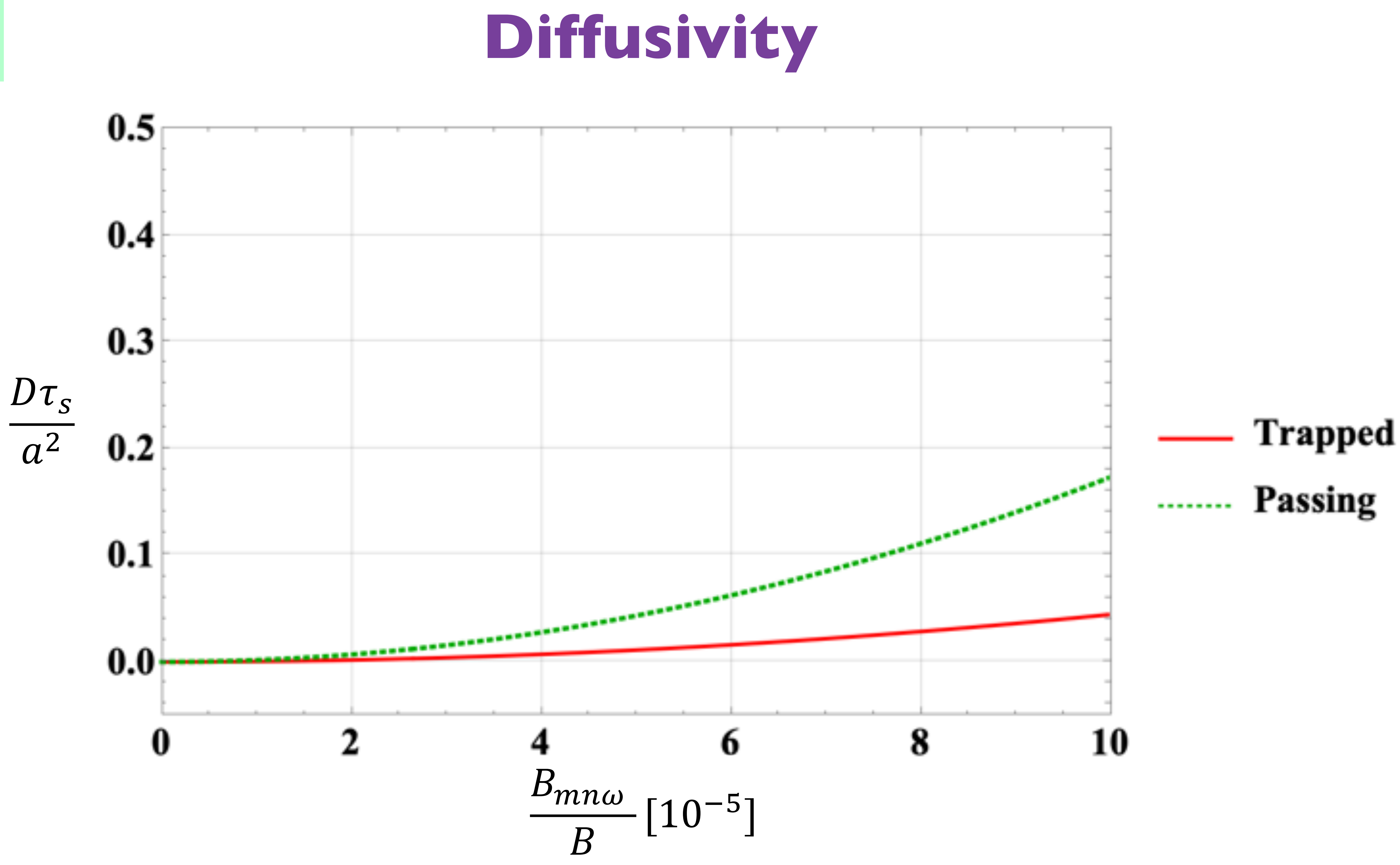
Diffusivity is significant, grows with amplitude squared

$$D_{trapped} \sim \frac{\sqrt{\epsilon} v_{\mathrm{AE}}^2}{\omega},$$

$$D_{passing} \sim \frac{v_{\mathrm{AE}}^2}{\omega}$$

$$v_{\mathrm{AE}} \sim v_A \frac{B_{mn\omega}}{B}$$

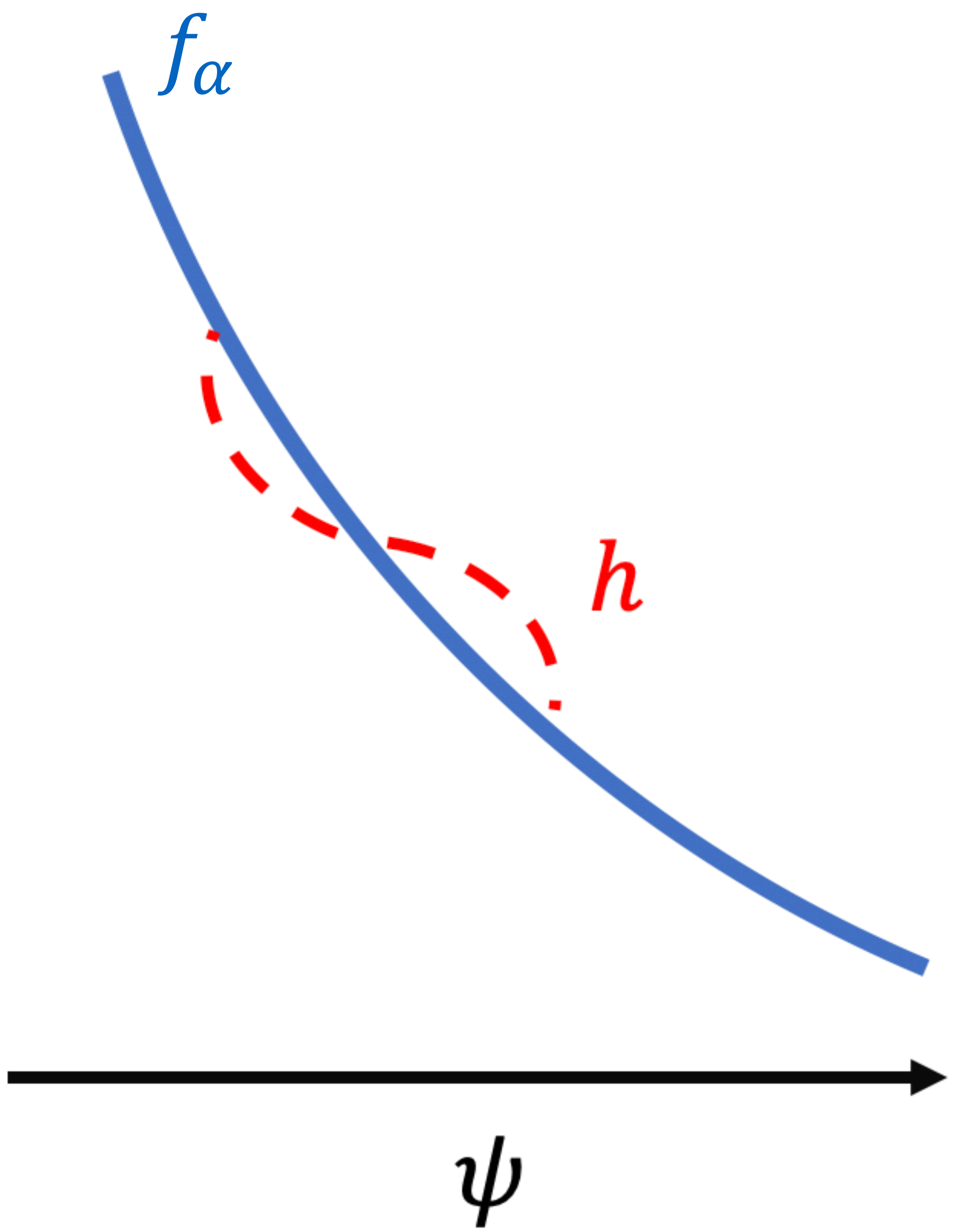
- D is normalized with slowing down time τ_s and device minor radius a
- Plot shows normalized D as function of TAE amplitude at SPARC-like parameters
  - R = 1.85 m, n = 10, $\omega \approx 2 \times 10^6\ s^{-1}, v_A \approx 8 \times 10^6 \frac{m}{s}$



Saturation condition balances flattening with refilling

- Saturation estimated in many ways throughout literature (wave trapping, nonlinear mode couplings, etc.)

- One simple method balances nonlinear drive reduction with collisional refilling



Saturation condition balances flattening with refilling

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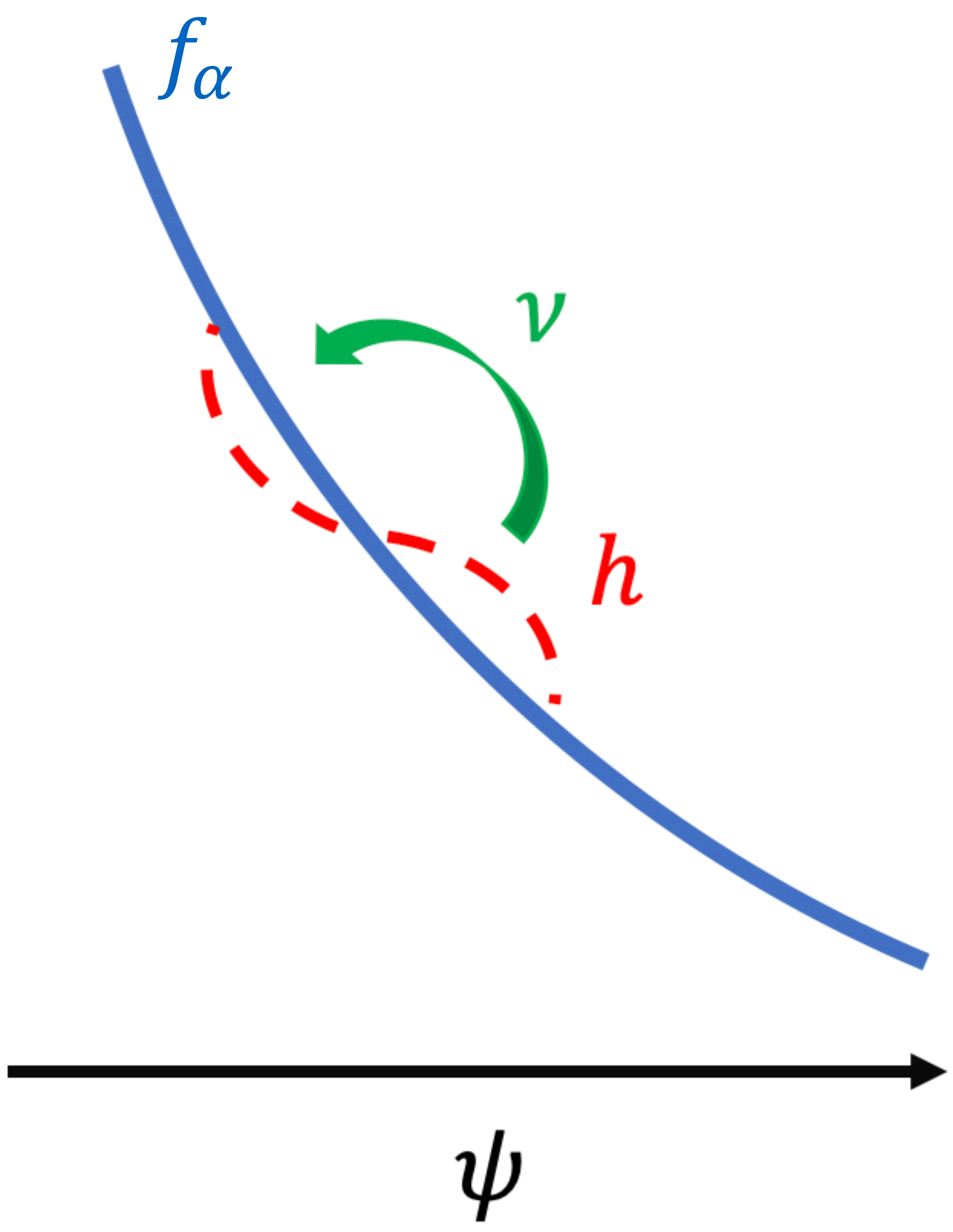
- One simple method balances nonlinear drive reduction with collisional refilling

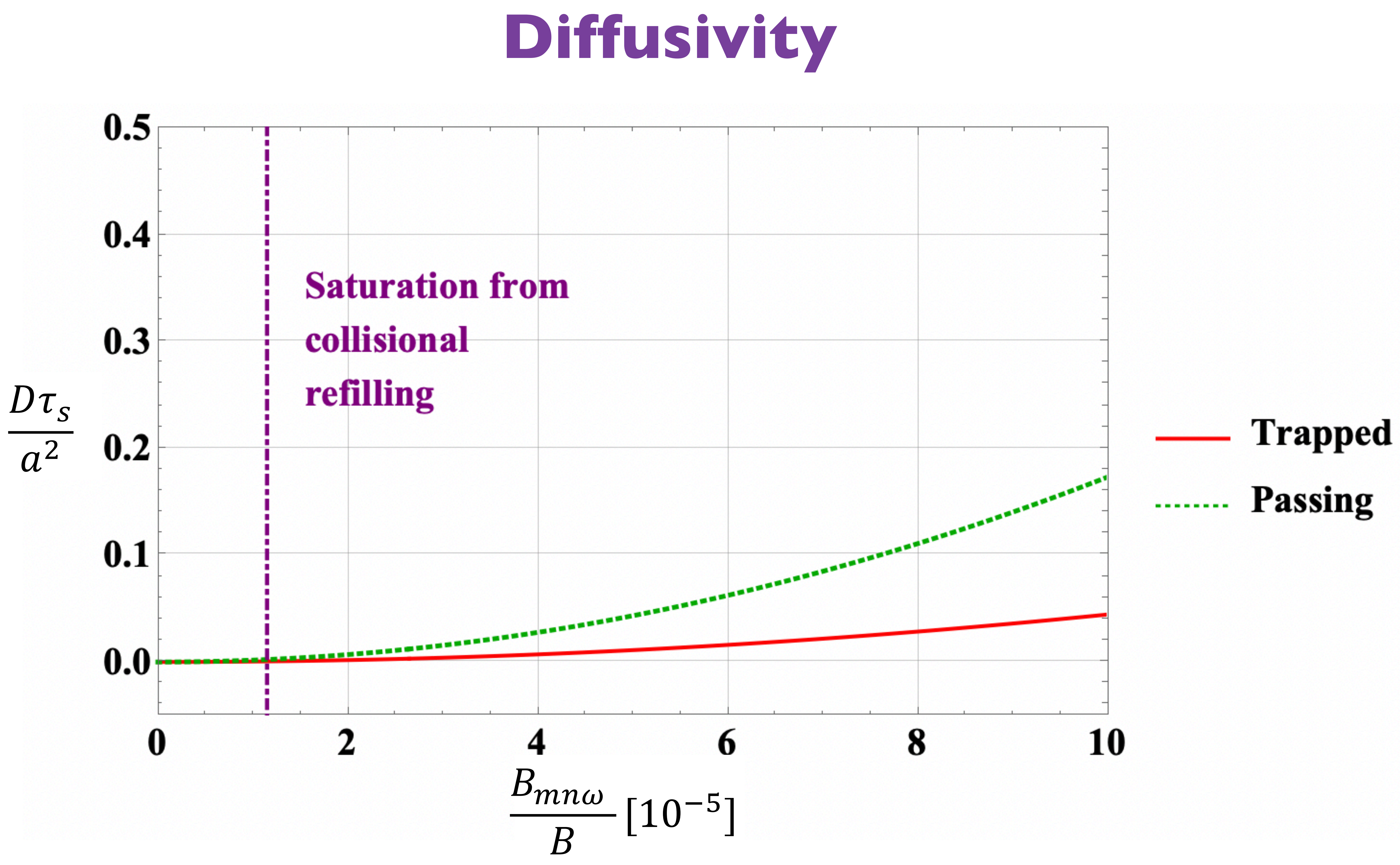
$$v_A \frac{B_{mn\omega}}{\mathrm{B}} \frac{\partial h}{\partial r} \sim \nu_{pas} \frac{\partial^2 h}{\partial \lambda^2}$$



Saturation condition suggests insignificant diffusion

- At the amplitude predicted by simple condition, diffusion is insignificant

- **Caveats**:
  - Coupling with other *m* will change *D*
  - Saturation amplitudes in the literature can be as large as 100 x ours
  - Onset of full stochasticity at higher amplitude could further enhance transport



Conclusions

- Alpha transport by tokamak perturbations can be calculated drift kinetically
- Drift kinetic calculation, plus simple saturation estimate, suggests transport in SPARC-like tokamak could be small
- Caveat: saturation at a higher level, possibly accompanied by onset of stochasticity, could lead to significant transport
  - Strong motivation for experimental exploration!

Based on Tolman and Catto *In Review* 2020, available on arXiv:2011.04920

The author thanks Steven Scott, Ryan Sweeney, Abhay Ram, Ian Abel, Ian Hutchinson, Eero Hirvijoki, Antti Snicker, Konsta Särkimäki, Paulo Rodrigues, Earl Marmar, Felix Parra, Iván Calvo, Vinícius Duarte, Lucio Milanese, and Martin Greenwald for helpful conversations. The author is especially grateful to Nuno Loureiro for suggesting P.J.C. become involved in the problem and for several insightful suggestions. The authors acknowledge support from the National Science Foundation Graduate Research Fellowship under Grant No. DGE-1122374, support from US Department of Energy award DE-FG02-91ER54109, and support from the Bezos Membership at the Institute for Advanced Study.