Tearing Instability of a Current Sheet Forming by Sheared Incompressible Flow

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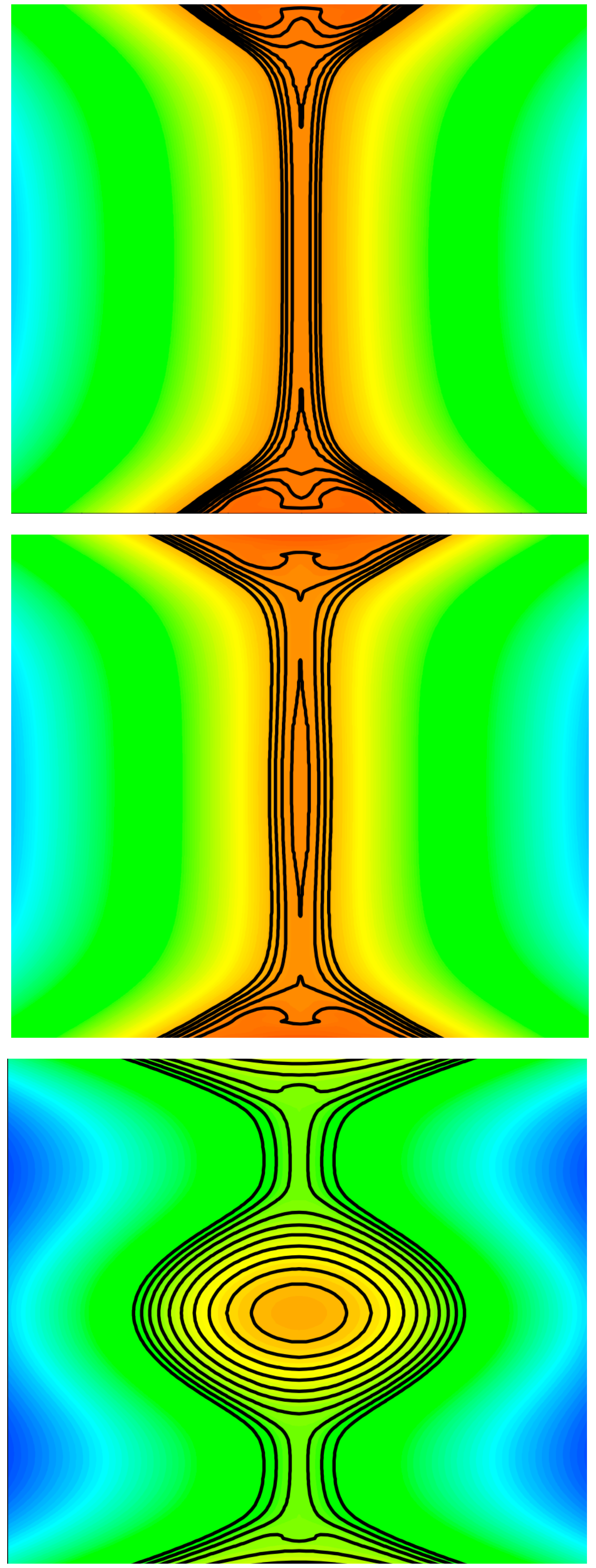
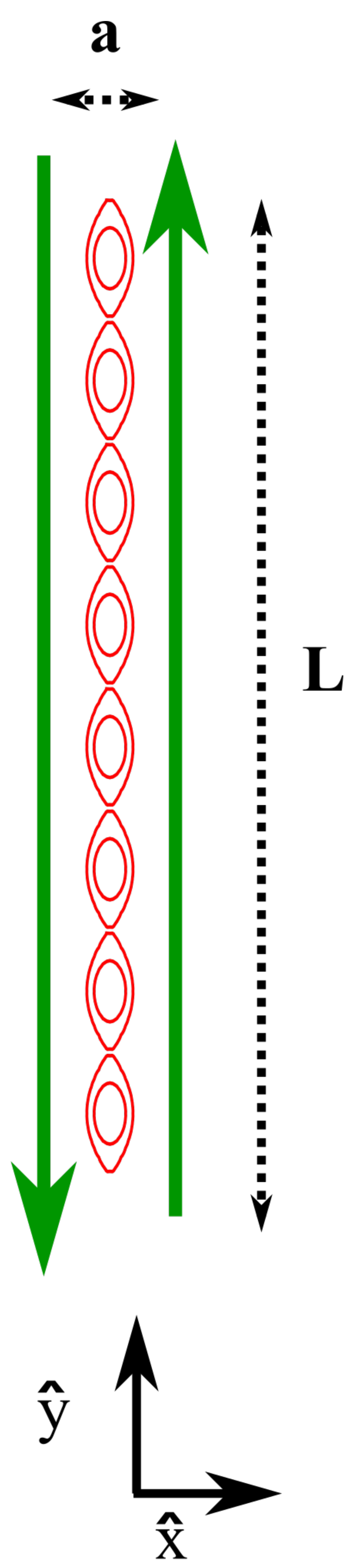
Understanding reconnection requires model for tearing in forming current sheet



- Magnetic reconnection is a plasma process behind solar flares, sawtooth crashes, and other phenomena.
- In MHD, reconnection has historically been studied using the Sweet-Parker steady-state model.
- Loureiro *et al.* 2007 and others have shown that the Sweet-Parker current sheet is strongly unstable, suggesting it will not form in nature in high-Lundquist-number plasmas.
- To better understand magnetic reconnection, need to study tearing mode in forming current sheet.

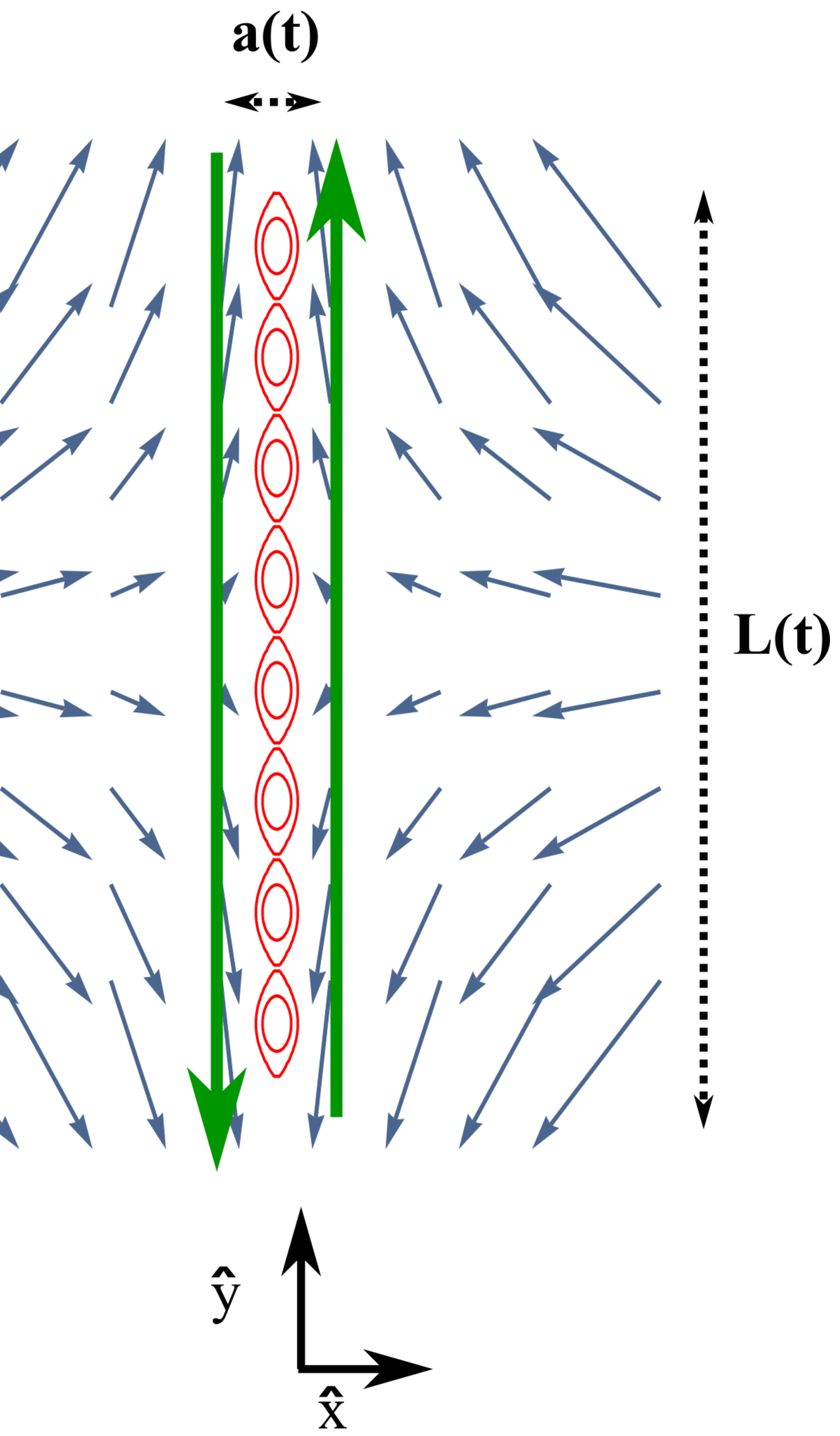
ψ contours in current sheet undergoing plasmoid instability, from Loureiro et. al., *2005*

Traditional tearing mode growth rates derived in static current sheet



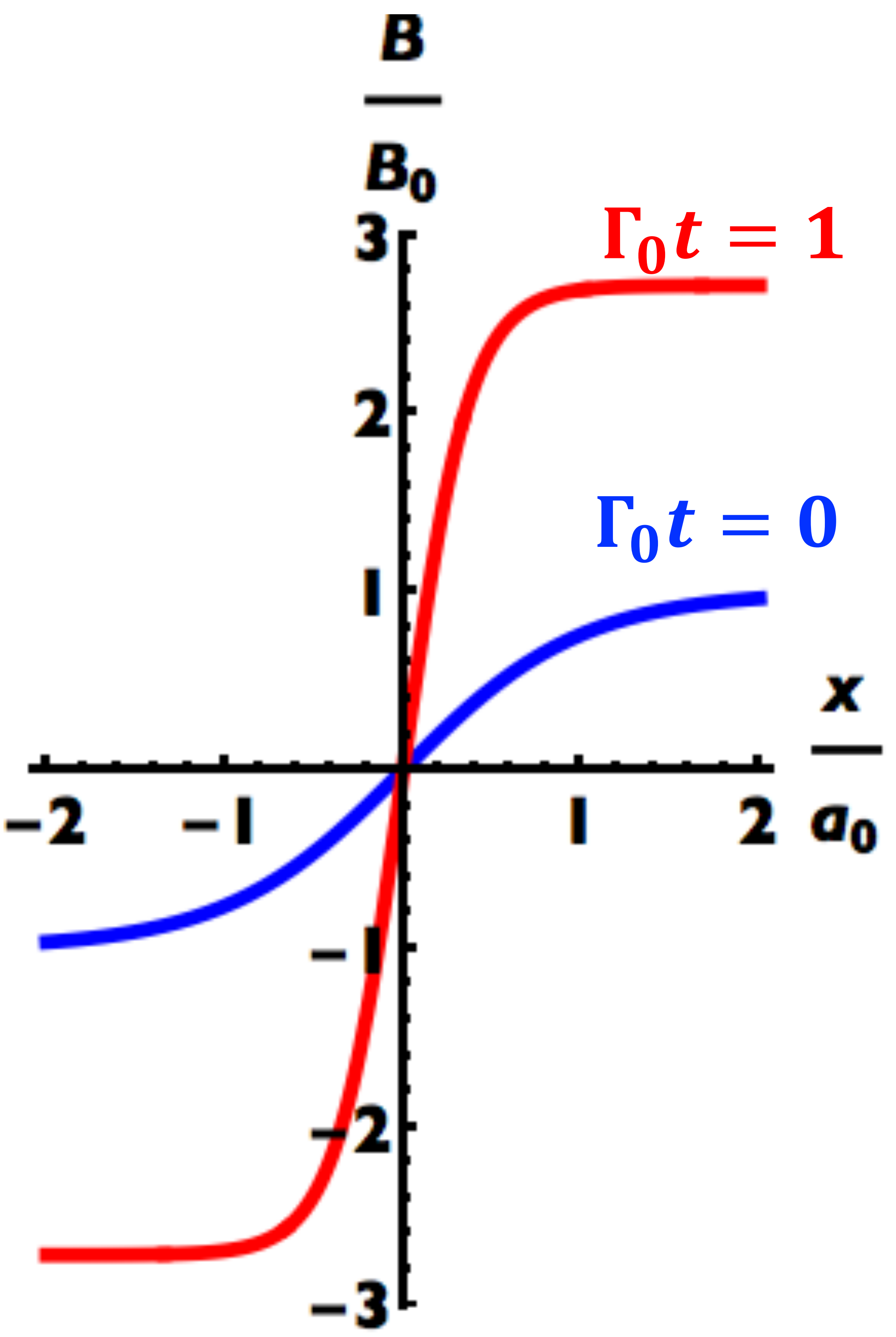
- Traditional linear tearing mode theory (Furth *et al.* 1963, Coppi *et al.* 1976) studies the stability of a stationary magnetic field $B(x)\hat{y}$ which reverses direction.

Traditional tearing mode growth rates derived in static current sheet



- Traditional linear tearing mode theory (Furth *et al.* 1963, Coppi *et al.* 1976) studies the stability of a stationary magnetic field $B(x)\hat{y}$ which reverses direction.
- In many physical systems, tearing will occur in sheets with magnetic configurations that **change** due to **flow**.
- The effect of flows that do not change the magnetic configuration has been studied (Bulanov *et al.* 1978, Chen and Morrison 1990, and others).
- We study the tearing mode in a current sheet that changes self-consistently due to flow.

Self-consistent model for current sheet formation under shear flow developed



- In an incompressible plasma with a strong and uniform z-directed guide magnetic field, the behavior of the x-y flow and magnetic field can be modeled by the resistive RMHD equations:

$$\partial_t \nabla_\perp^2 \phi + \{\phi, \nabla_\perp^2 \phi\} = \{\psi, \nabla_\perp^2 \psi\}$$
$$\partial_t \psi + \{\phi, \psi\} = \eta \nabla_\perp^2 \psi$$

$$\vec{u} = (-\partial_y \phi, \partial_x \phi)$$
$$\vec{B} = (-\partial_y \psi, \partial_x \psi)$$
$$\{P, Q\} = \partial_x P \partial_y Q - \partial_x P \partial_y Q$$

- Equilibrium is a current sheet with a magnetic field and flow that together obey the RMHD equations with $\eta = 0$.
- $\psi_0(x,t) = B_0 a_0 \log\left(\cosh \frac{x}{a(t)}\right) \Rightarrow \vec{B}(x,t) = \left(0, B_0 e^{\Gamma_0 t} \tanh\left(\frac{x}{a(t)}\right)\right)$
- $\phi_0 = \Gamma_0 x y \Rightarrow \vec{u}(x,y,t) = (-\Gamma_0 x, \Gamma_0 y)$
- This configuration has a shrinking width $a(t) = a_0 e^{-\Gamma_0 t}$, and, by incompressibility, an extending length $L(t) = L_0 e^{\Gamma_0 t}$.

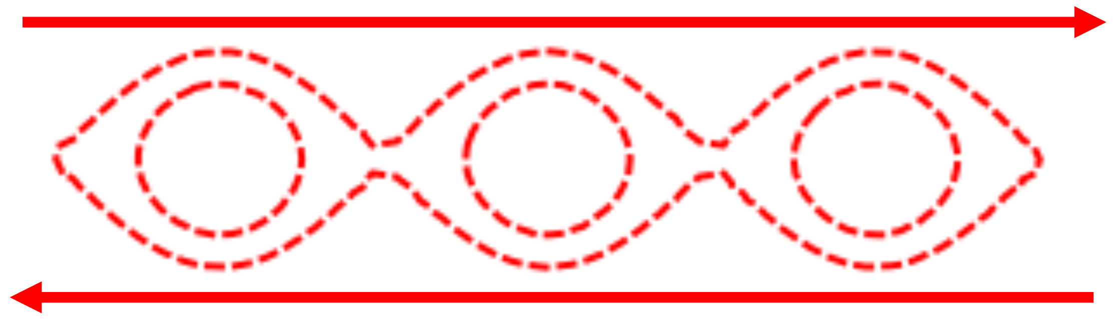
Perturbations of forming current sheet have sheared wave vector

- In the evolving current sheet equilibrium, we study the behavior of small perturbations, so that

$$\psi(x,y,t) = \psi_0(x,t) + \psi_1(x,t)e^{i\,k(t)y}$$
$$\phi(x,y,t) = \phi_0(x,y,t) + \phi_1(x,t)e^{i\,k(t)y}$$

- Define a time dependent k according to $k(t) = k_0 e^{-\Gamma_0 t}$.
- Physically, this means the number of islands characterizing a mode should remain constant:

$$N(t) \sim k(t)L(t) = k_0 e^{-\Gamma_0 t} L_0 e^{\Gamma_0 t} = \text{constant}$$



- With this postulate, perturbed equations lose explicit y-dependence and read:

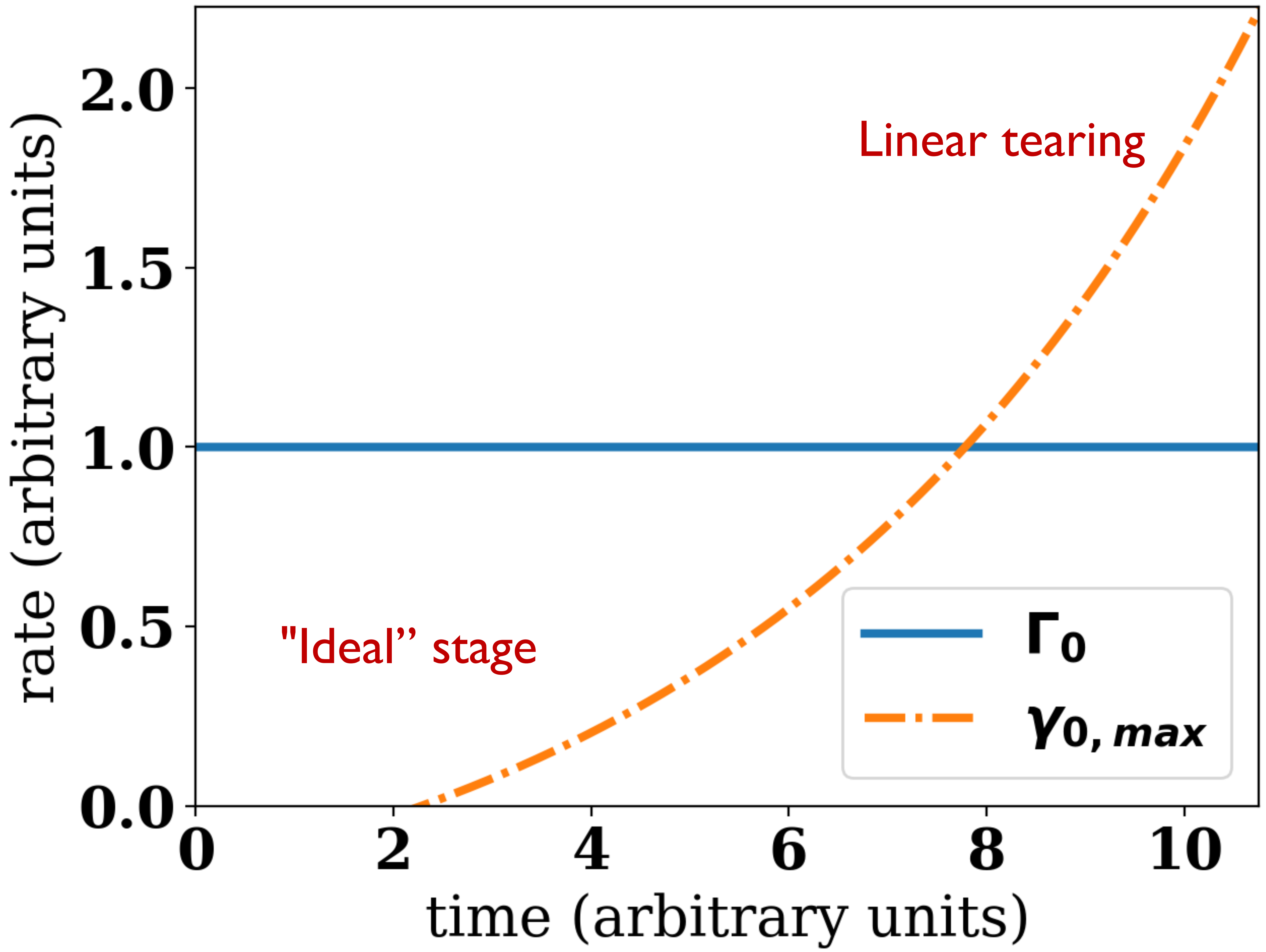
Linearized momentum equation:

$$\left(\partial_x^2 - k^2(t)\right)\partial_t \phi_1 - \Gamma_0 x\, \partial_x\left(\partial_x^2 - k^2(t)\right)\phi_1 + 2\,\Gamma_0 k^2(t)\phi_1 = \partial_x \psi_0(t) i\, k(t)\left(\partial_x^2 - k^2(t)\right)\psi_1 - \partial_x^3 \psi_0(t) i\, k(t)\psi_1$$

Linearized induction equation:

$$\partial_t \psi_1 - \Gamma_0 x\, \partial_x \psi_1 - \partial_x \psi_0(t) k(t)\phi_1 = \eta\left(\partial_x^2 - k^2(t)\right)\psi_1$$

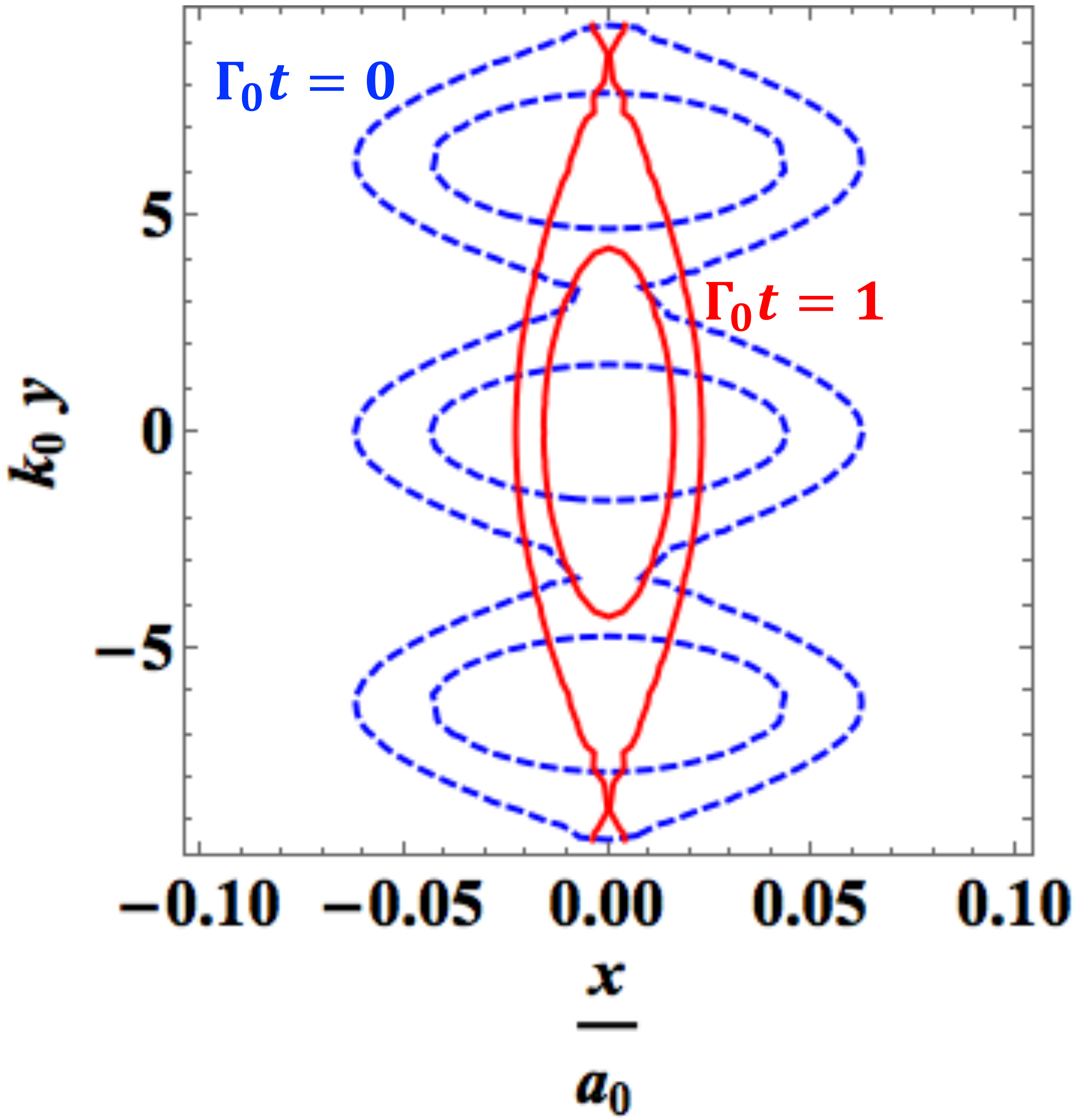
Forming current sheet passes through several stages



- As the sheet forms, it passes through several stages:

1. "Ideal" stage: When the current sheet is still thick, the tearing is weak and the flow is the dominant behavior.
2. Linear tearing: Once the sheet thins sufficiently, tendency to tear will overwhelm the rate of the flow, and linear tearing will occur in an equilibrium that changes under the influence of flow.

During the "ideal stage" islands are reshaped by flow



- When the current sheet is still thick, the tearing is weak and the flow is the dominant behavior.
- Resistive term neglected in linearized induction equation $\partial_t \psi_1 - \Gamma_0 x\, \partial_x \psi_1 - \partial_x \psi_0(t) k(t) \phi_1 = \eta\left(\partial_x^2 - k^2(t)\right)\psi_1$.
- The flow along the sheet will stretch the islands via the time-dependent wave vector.
- The flow into the sheet compresses the islands.

Linear tearing is studied via a multiple scale formulation

- Perturbed equations read:

$$\left(\partial_x^2 - k^2(t)\right)\partial_t\phi_1 - \Gamma_0 x\,\partial_x\left(\partial_x^2 - k^2(t)\right)\phi_1 + 2\,\Gamma_0 k^2(t)\phi_1 = \partial_x\psi_0(t)i\,k(t)\left(\partial_x^2 - k^2(t)\right)\psi_1 - \partial_x^3\psi_0(t)i\,k(t)\psi_1$$

$$\partial_t\psi_1 - \Gamma_0 x\,\partial_x\psi_1 - \partial_x\psi_0(t)k(t)\phi_1 = \eta\left(\partial_x^2 - k^2(t)\right)\psi_1$$

- Since the coefficients of the perturbation are time dependent, this equation cannot be immediately separated as it is in traditional tearing mode analysis.

$$\phi_1 = \Phi_1(x)e^{\gamma t}$$

- Exponential growth must be separated out using a WKB-style multiple scale formulation.

$$\tau \equiv \Gamma_0 t$$

Slow time scale of flow and changes to equilibrium

$$\hat{t} \equiv \gamma_0 t$$

Fast time scale of tearing

$$\epsilon \equiv \frac{\tau}{\hat{t}} = \frac{\Gamma_0}{\gamma_0}$$

- Perturbations can be expanded to separate out exponential growth:

$$\phi_1(x,\tau) = \left[\Phi_{1,0}(x,\tau) + \epsilon\Phi_{1,1}(x,\tau) + \ \ldots\right]\times \exp\left(\frac{1}{\epsilon}\int_0^{\tau}\hat{\gamma}(s)ds\right)$$

In linear tearing, changes to equilibrium enter at same order as flow

- With this expansion, equations for eigenfunctions and growth rate can be separated into terms of order ϵ^0, and order ϵ^1 and higher. For the linearized momentum equation, we have:

$$\gamma(\tau)\left(\partial_x^2 - k^2(\tau)\right)\Phi_{1,0}(x,\tau) - \Gamma_0 x\, \partial_x\left(\partial_x^2 - k^2(\tau)\right)\Phi_{1,0}(x,\tau) + 2\,\Gamma_0 k^2(\tau)\Phi_{1,0}(x,\tau) =$$
$$\partial_x\psi_0(\tau) i\, k(\tau)\left(\partial_x^2 - k^2(\tau)\right)\Psi_{1,0}(x,\tau) - \partial_x^3\psi_0(\tau) i\, k(\tau)\Psi_{1,0}(x,\tau) + \text{terms involving } \partial_\tau\Phi_{1,0},\ \partial_\tau\Psi_{1,0},$$
$$\Phi_{1,1},\ \Psi_{1,1},\ \text{etc.}$$

- Terms explicitly containing Γ_0 are of the same order as changes in the background equilibrium, which prevents them from influencing the separable part of the behavior.
- So exponential part of tearing growth depends *implicitly only* the flow, and can be found by adding time dependence to traditional rates.

- For FKR modes (Furth *et al.* 1963), we have $\gamma_{\text{FKR}}(\tau) = \eta^{3/5} a^{-2}(\tau) B_y^{2/5}(\tau) k^{-2/5}(\tau)/\tau_A(\tau)$; analogously for Coppi modes (Coppi *et al.* 1976).
- This is as assumed in recent work by Uzdensky and Loureiro 2016.

Conclusions

- Understanding reconnection requires model for tearing in forming current sheet.
- A self-consistent model for current sheet formation caused by flows can be found in RMHD.
- During early stages of current sheet formation, any perturbations are ideally reshaped by flow.
- Linear tearing can be studied via a multiple scale formulation, showing that in a changing equilibrium, flow corrections enter only implicitly.
- See JPP article in preparation for more mathematical details and treatment of nonlinear stage.

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