Dependence of Alfvén eigenmode linear stability on device magnetic field strength and consequences for next-generation tokamaks

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PSFC Plasma Science and Fusion Center
Massachusetts Institute of Technology

Outline

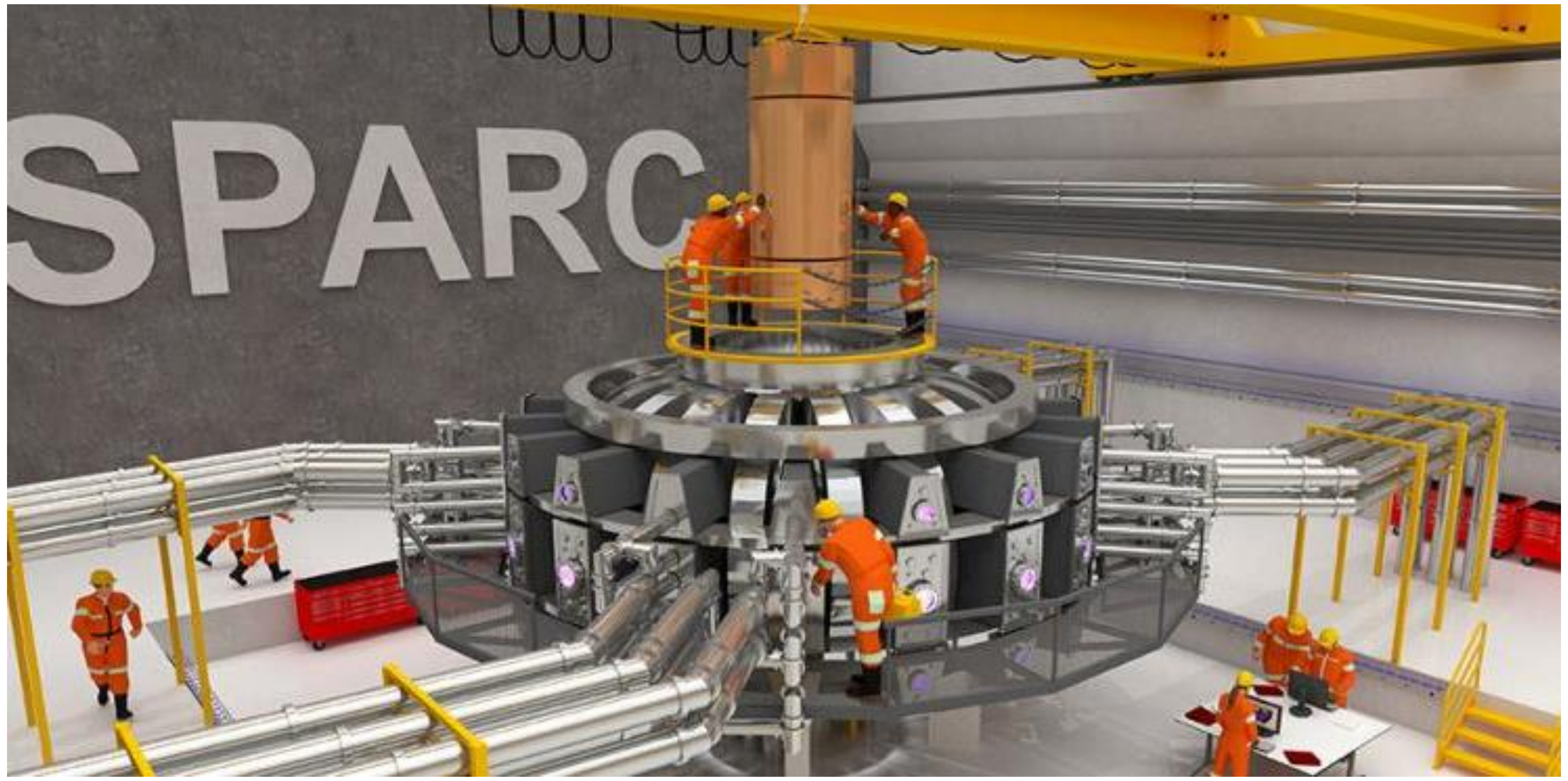
- SPARC as a novel laboratory for alpha particle physics
- Physics of Alfvén Eigenmode (AE) stability
- Advantages of high-performance high-B operation for AE stability
- Physics opportunities enabled by lower Greenwald fraction high-B operation

Much more detailed treatment of high-B AE stability may be found in submitted paper, currently available on arXiv:

"Dependence of Alfvén eigenmode linear stability on device magnetic field strength and consequences for next–generation tokamaks" (https://arxiv.org/abs/1809.07278)

SPARC explores alpha particle physics in different regime than ITER

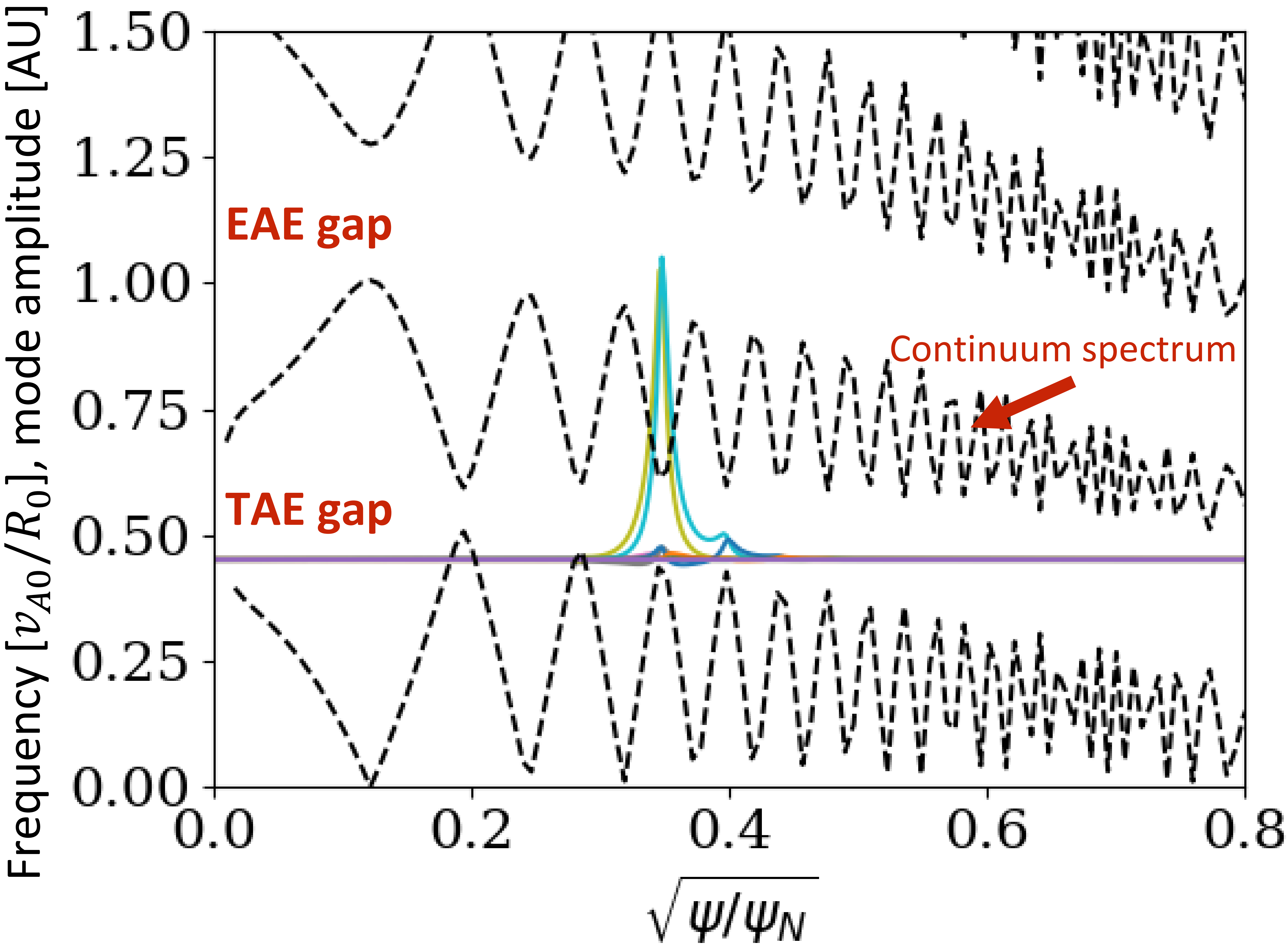
- Alpha particle behavior may be the most interesting physics of Q>1 devices
- SPARC will explore this physics in a different parameter space than ITER
- This parameter space ***may*** present advantages for reactor operation
- This parameter space ***will*** allow increased, complementary understanding of alpha particle physics



Quantity	ITER	SPARC strawman
Major radius	$R_0 = 6.2\ m$	$R_0 = 1.65\ m$
On-axis electron density	$n_e(0) \sim 11 \times 10^{19}\ m^{-3}$	$n_e(0) \sim 40$ - $65 \times 10^{19}\ m^{-3}$
On-axis magnetic field	$B_0 \sim 5.3$ T	$B_0 \sim 12$ T
Alfvén frequency	$v_{A0}/(2\pi R_0) \sim 1.8 \times 10^5\ Hz$	$v_{A0}/(2\pi R_0) \sim 6.3 - 8.0 \times 10^5\ Hz$

AEs are excited by energetic particles, can cause energetic particle loss

- One key part of alpha particle physics is alpha-AE interaction
- AEs are shear Alfvén waves that exist in tokamaks as discrete modes
- Energetic particles, including alphas, destabilize AEs
- AEs can cause transport of alphas to edge, degrading plasma performance and damaging the device



Growth rate determined by thermal species and energetic particles

- AE linear growth rate is determined by sum of drive and damping:

$$\frac{\gamma}{\omega} = \frac{\gamma_\alpha}{\omega} + \sum_j \frac{\gamma_j}{\omega}.$$

- Alpha growth is given approximately by [Betti and Freidberg, 1992]:

$$\frac{\gamma_\alpha}{\omega} \sim q_{AE}^2 \beta_\alpha \, F\left(\frac{v_{A0}}{v_{\alpha 0}}, \frac{1}{p_\alpha}\frac{dp_\alpha}{dr}, q_{AE}, \mathrm{n}, \mathrm{r}_{\mathrm{L}\theta\alpha}\right)$$

- Ion damping is given approximately by [Betti and Freidberg, 1992]:

$$\frac{\gamma_j}{\omega} \sim -q_{AE}^2 \beta_j G\left(\frac{v_{A0}}{v_{thj}}\right)$$

Quantity	Definition
ω	mode frequency
$\gamma_\alpha, \gamma_j, \gamma$	alpha contribution, ion contribution, and overall growth rate
q_{AE}	safety factor at mode location
β_α, β_j	alpha, ion beta
$v_{A0}, v_{\alpha 0}, v_{thj}$	on-axis Alfvén speed, alpha birth speed, ion thermal speed
$\mathrm{n}, \mathrm{r}_{\mathrm{L}\theta\alpha}$	toroidal mode number, poloidal alpha Larmor radius
F	growth from spatial gradient of alphas $\left(\frac{1}{\mathrm{p}_\alpha}\frac{\mathrm{dp}_\alpha}{\mathrm{dr}}\right)$ at resonant velocities
G	damping from energy gradient of ions at resonant velocities

High density in high field machines minimizes slowing down time, AE drive

- Economically, a reactor should achieve a fixed value of fusion power density, $P_f\left[\frac{MW}{m^3}\right] = E_f n_D n_T\langle\sigma v\rangle \sim \mathrm{n_e^2 T_e^2} \equiv C$
- Alpha particle density scales with $n_\alpha \sim \frac{Rate\ of\ \alpha\ production}{Speed\ at\ which\ \alpha's\ slow\ down\ to\ ash} \sim \frac{n_D n_T\langle\sigma v\rangle}{\left(n_e/T_e^{1.5}\right)} \sim n_e T_e^{3.5}$
- Neglecting resonance positions, ratio of AE drive to damping scales like:

$$\frac{Drive}{Damping} \sim \frac{\beta_\alpha}{\beta_j} \sim \frac{\frac{n_\alpha T_\alpha}{B^2}}{\frac{n_e T_e}{B^2}} \sim T_e^{2.5} \sim \frac{C^{1.25}}{n_e^{2.5}}$$

- Higher magnetic field devices have higher currents at a given *q*, which allows higher densities while still obeying Greenwald limit
- Higher field devices thus incur less AE drive for a given power density

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Increasing field (or decreasing density) cuts off resonance

- Recall expression for alpha particle drive:

$$\frac{\gamma_\alpha}{\omega} \sim q_{AE}^2 \beta_\alpha F\left(\frac{v_{A0}}{v_{\alpha 0}}, \frac{1}{p_\alpha}\frac{dp_\alpha}{dr}, q_{AE}, \mathrm{n}, \mathrm{r}_{\mathrm{L}\theta\alpha}\right)$$

F: growth from spatial gradient of alphas at resonant velocities

- The most important TAE resonances are $v_{A0}, v_{A0}/3 \sim B_0/\sqrt{n_0}$

- The D-T fusion alpha particle birth velocity is $v_{\alpha 0} = 1.3 \times 10^7\ m/s$

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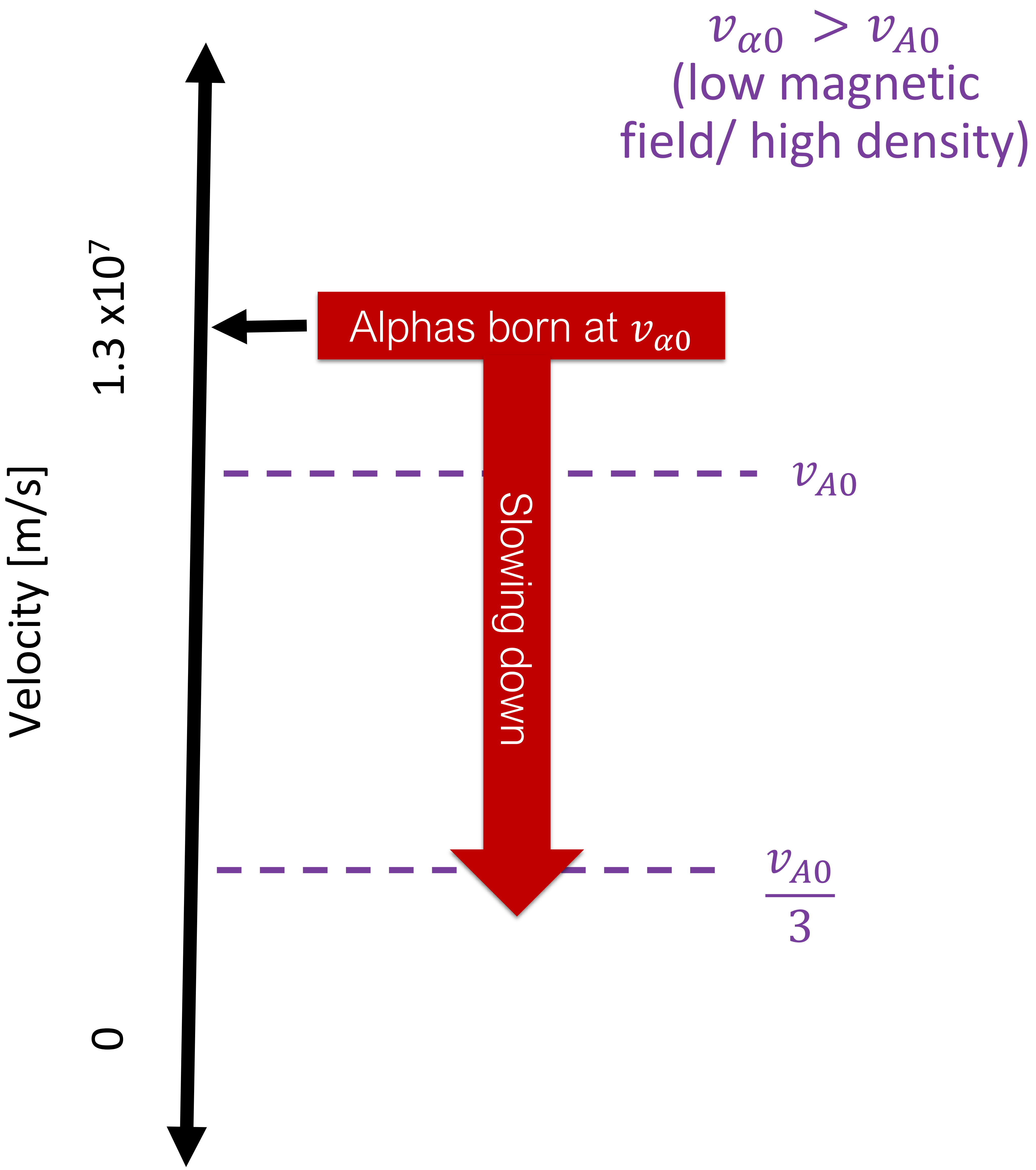

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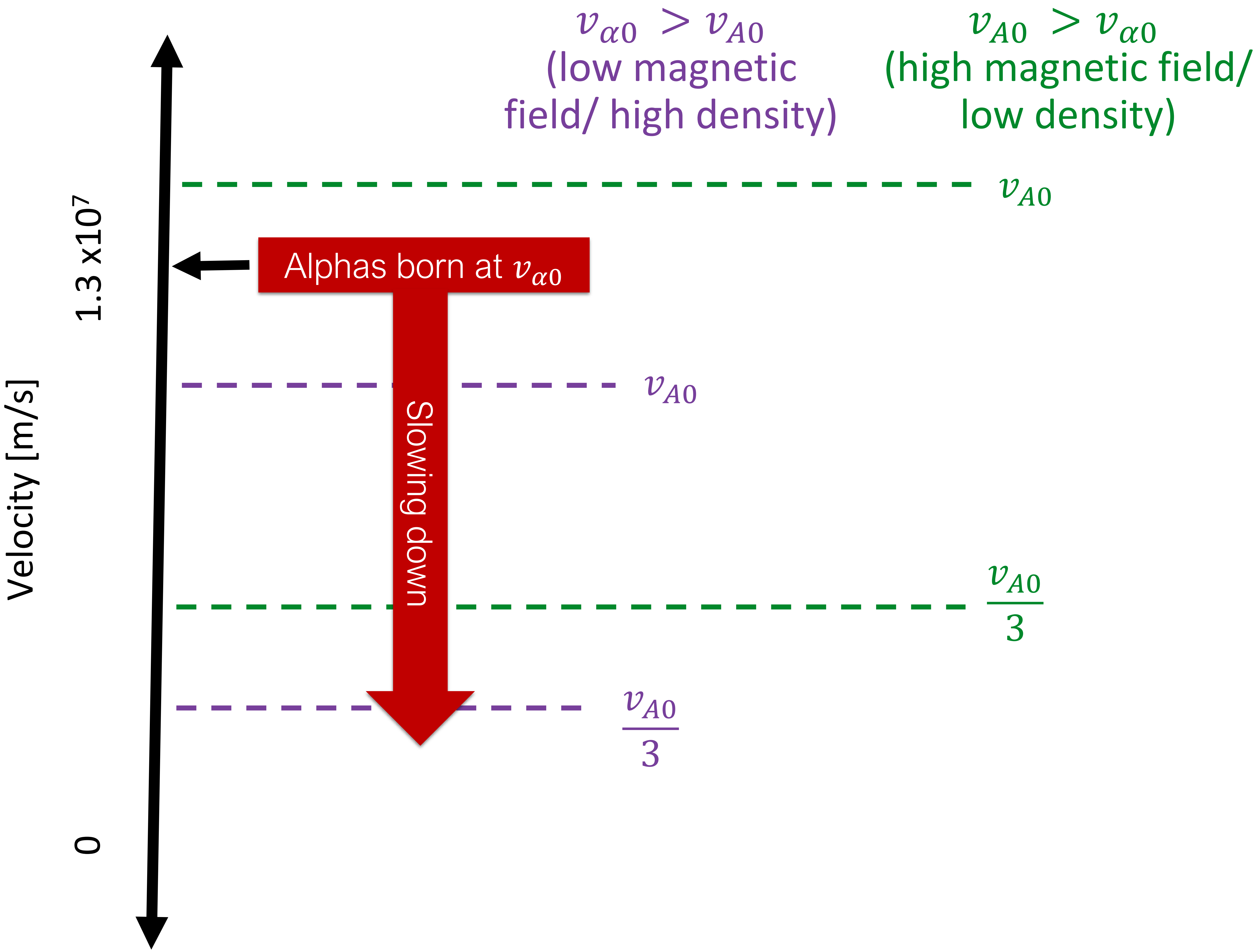


Computational study looks for resonance cutoff in plausible low f_{GW} SPARC shot

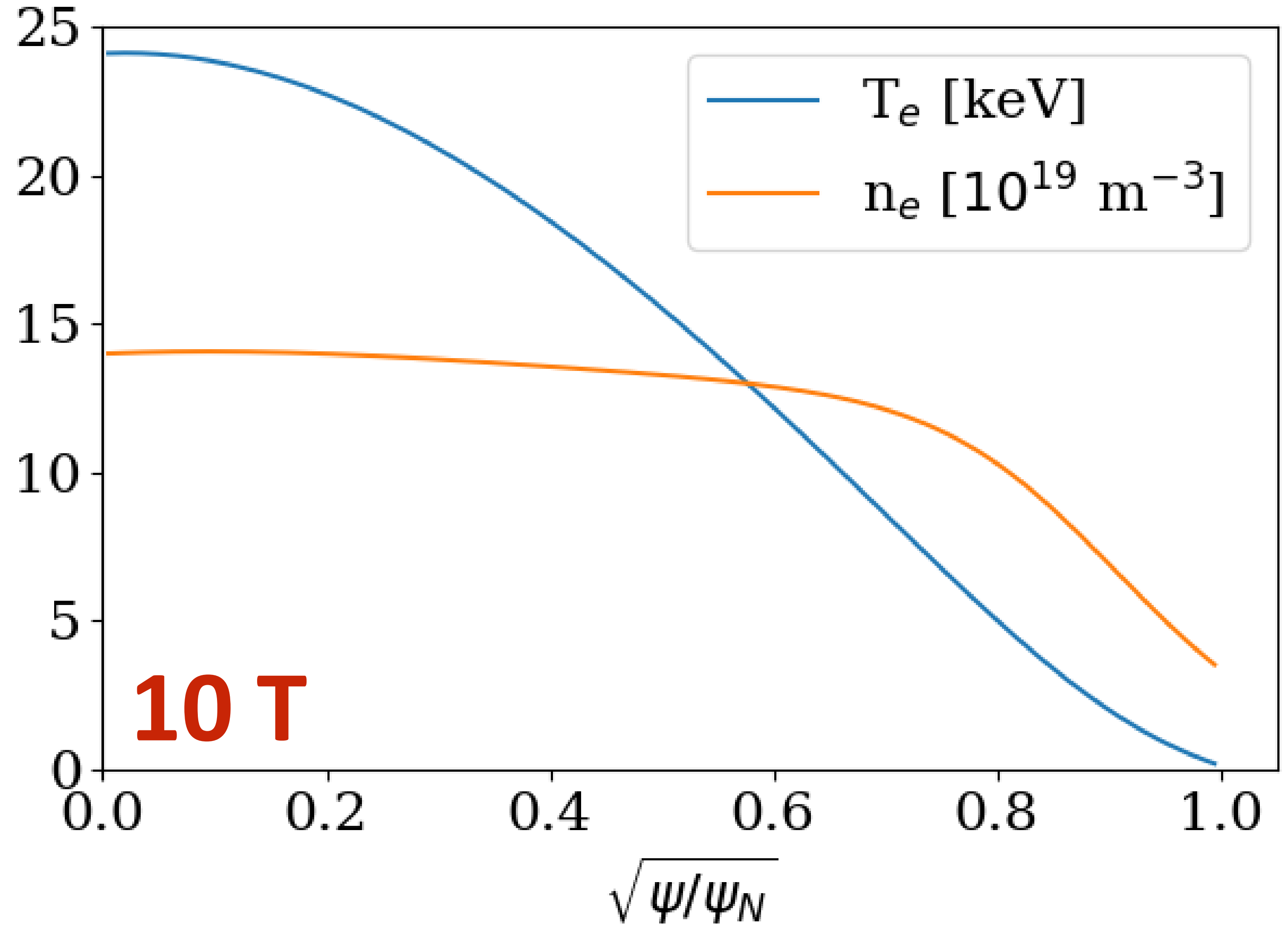
- Take $\beta, f_{GW}, \beta_N, q$ and profile shapes from a high-temperature, low-density C-Mod shot and create a model SPARC-sized D-T equilibrium

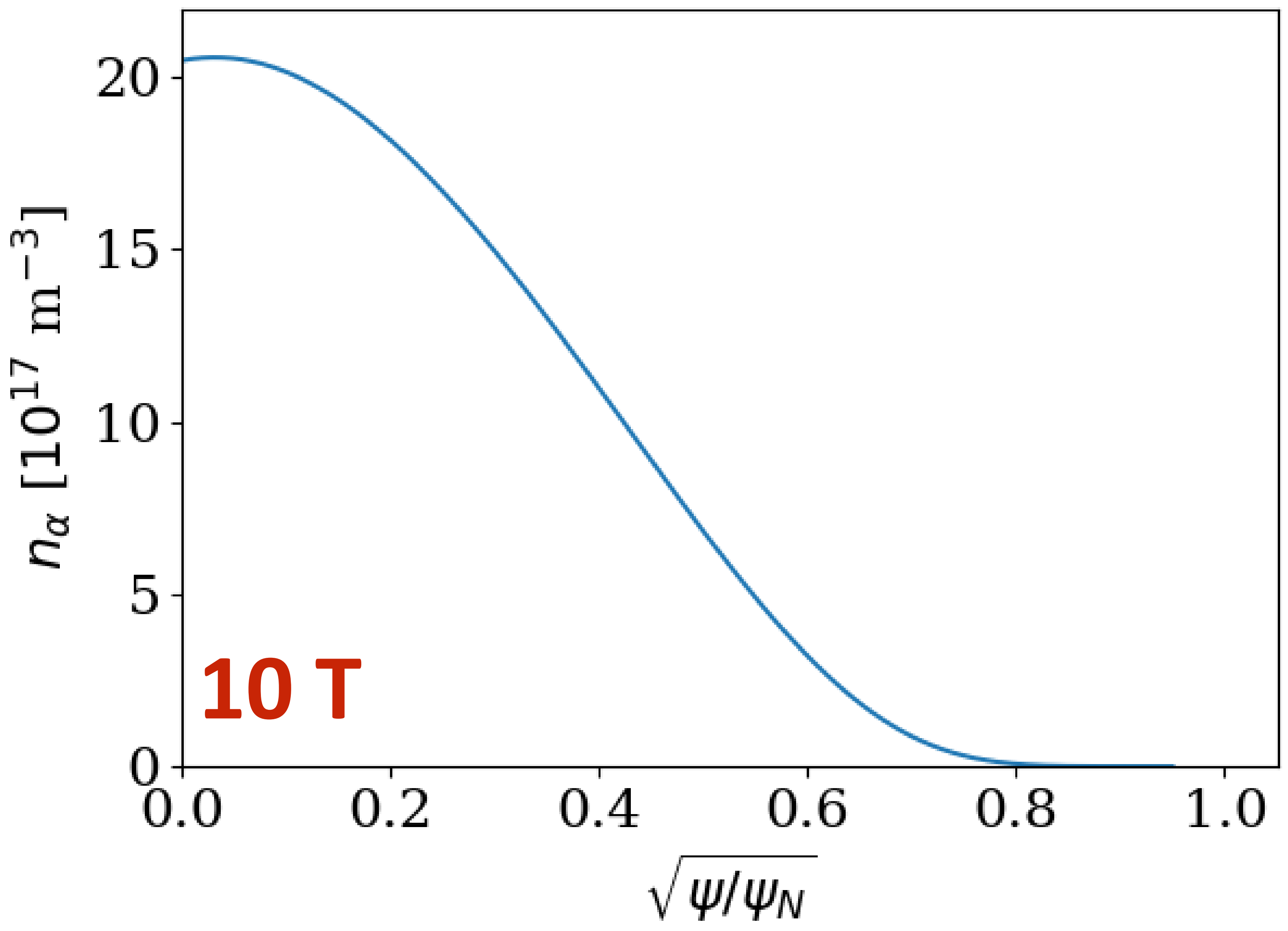
- Scan magnetic field in configuration while keeping $\beta, f_{GW}, \beta_N, q$ and profile shapes constant

- Temperature, density, and fusion power density P_f increase with field

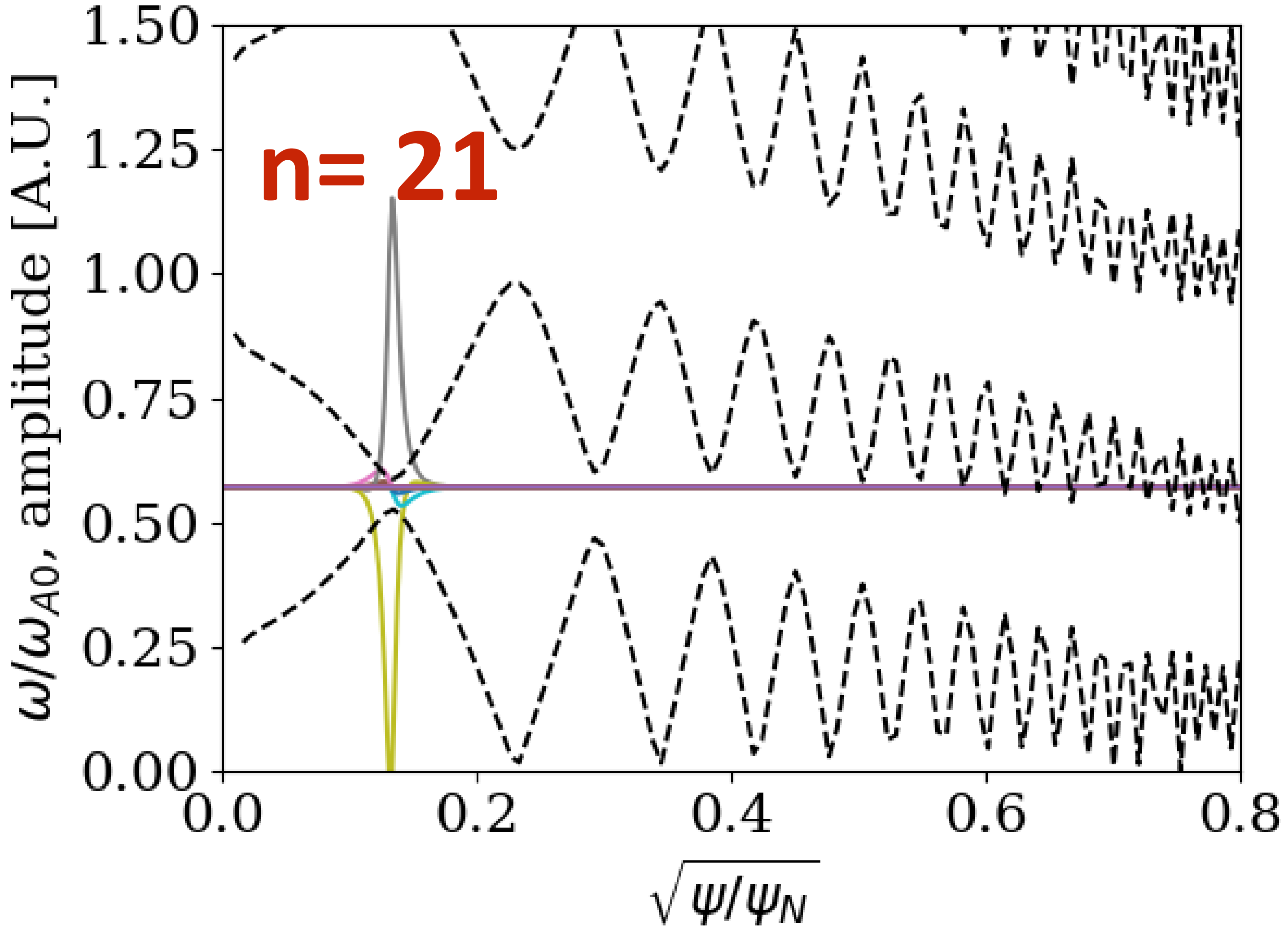
- Use suite of codes (HELENA, MISHKA, CASTOR-K) used to study AE behavior in ITER, JET to find AEs and growth rates as magnetic field is scanned

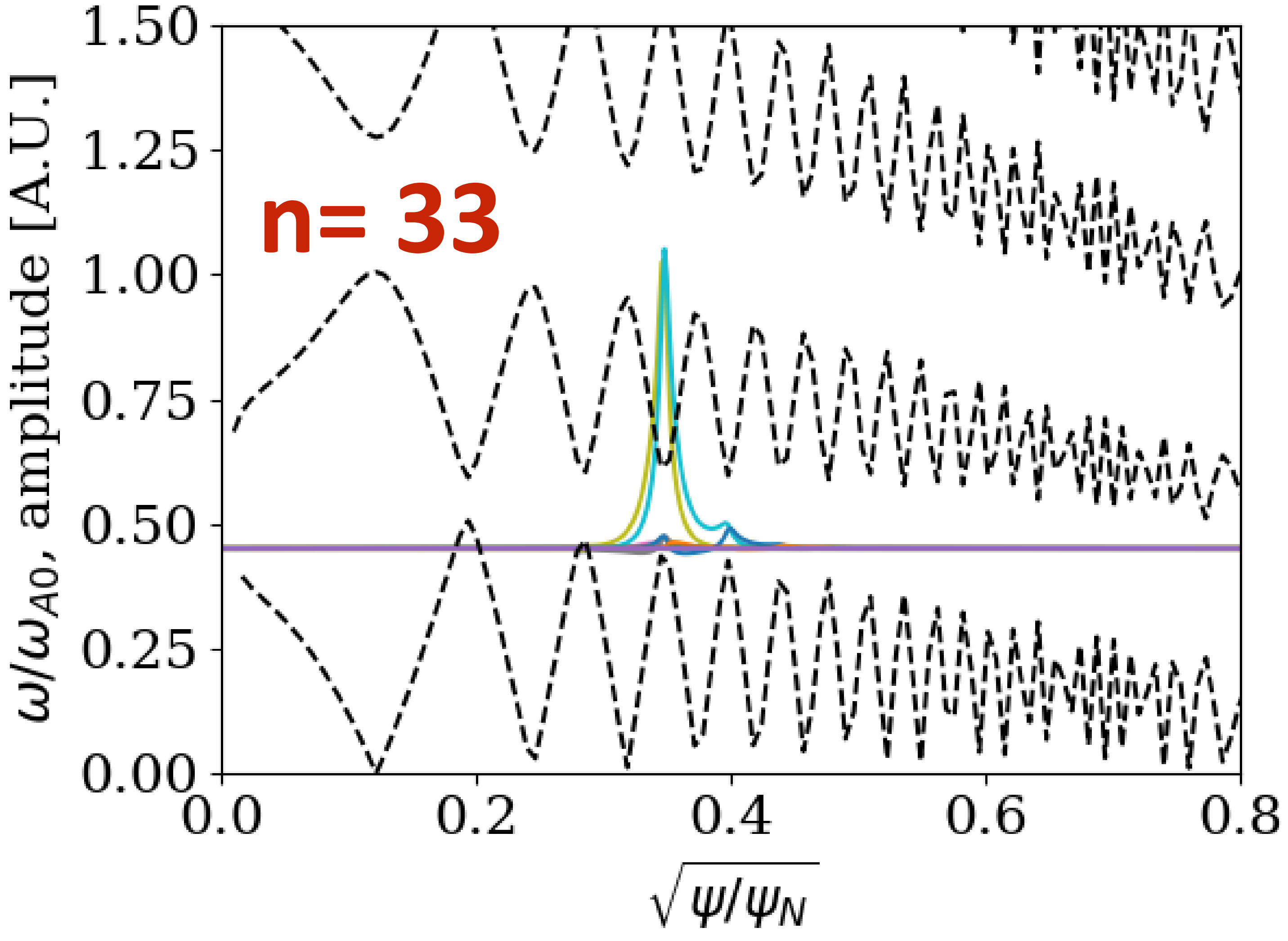
Figure of merit	Value in C-Mod shot and our scan
β	0.75%
f_{GW}	0.18
β_N	0.80
q_{95}	3.5

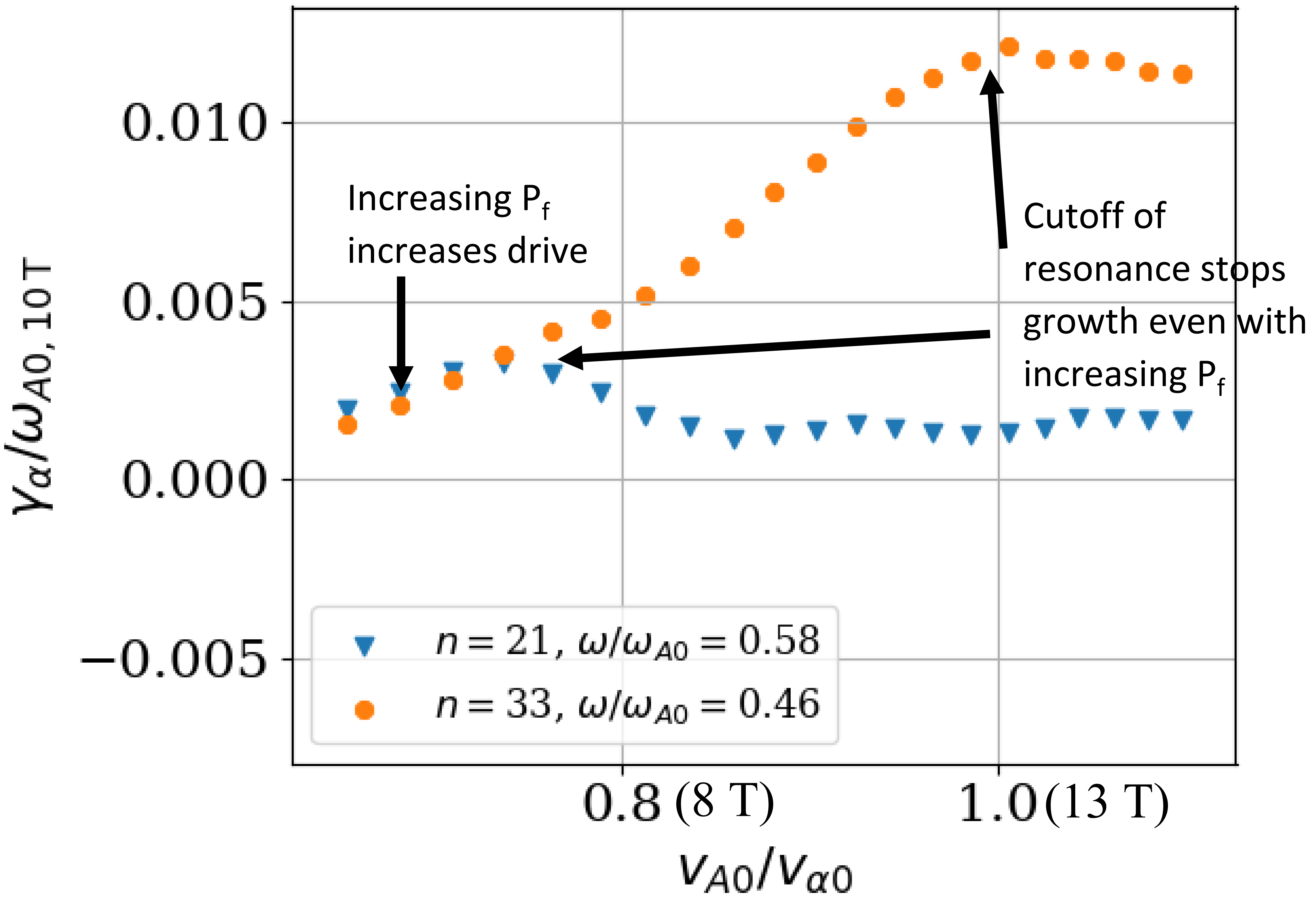




Increased field causes increased power density, drive, then resonance cutoff



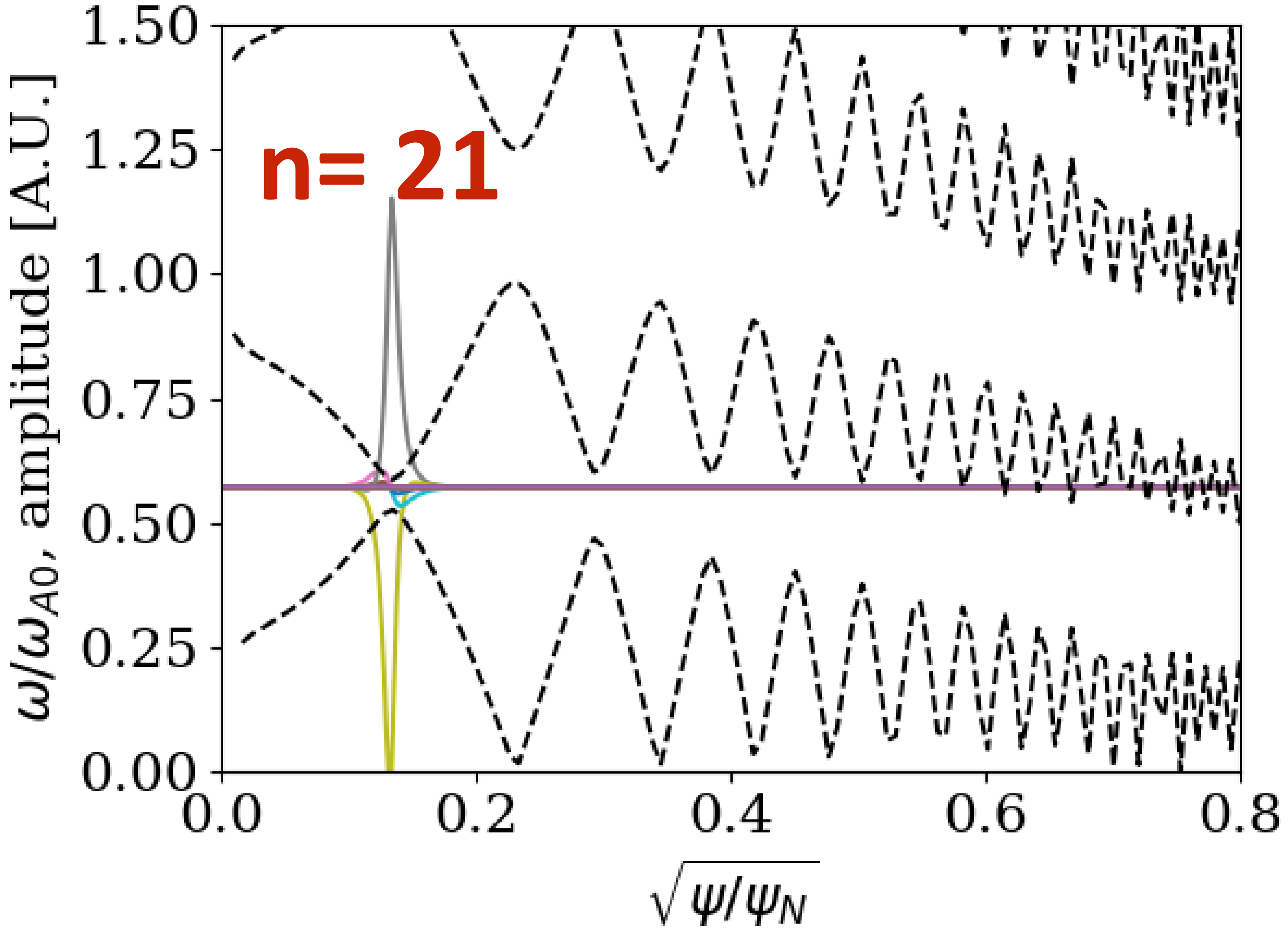


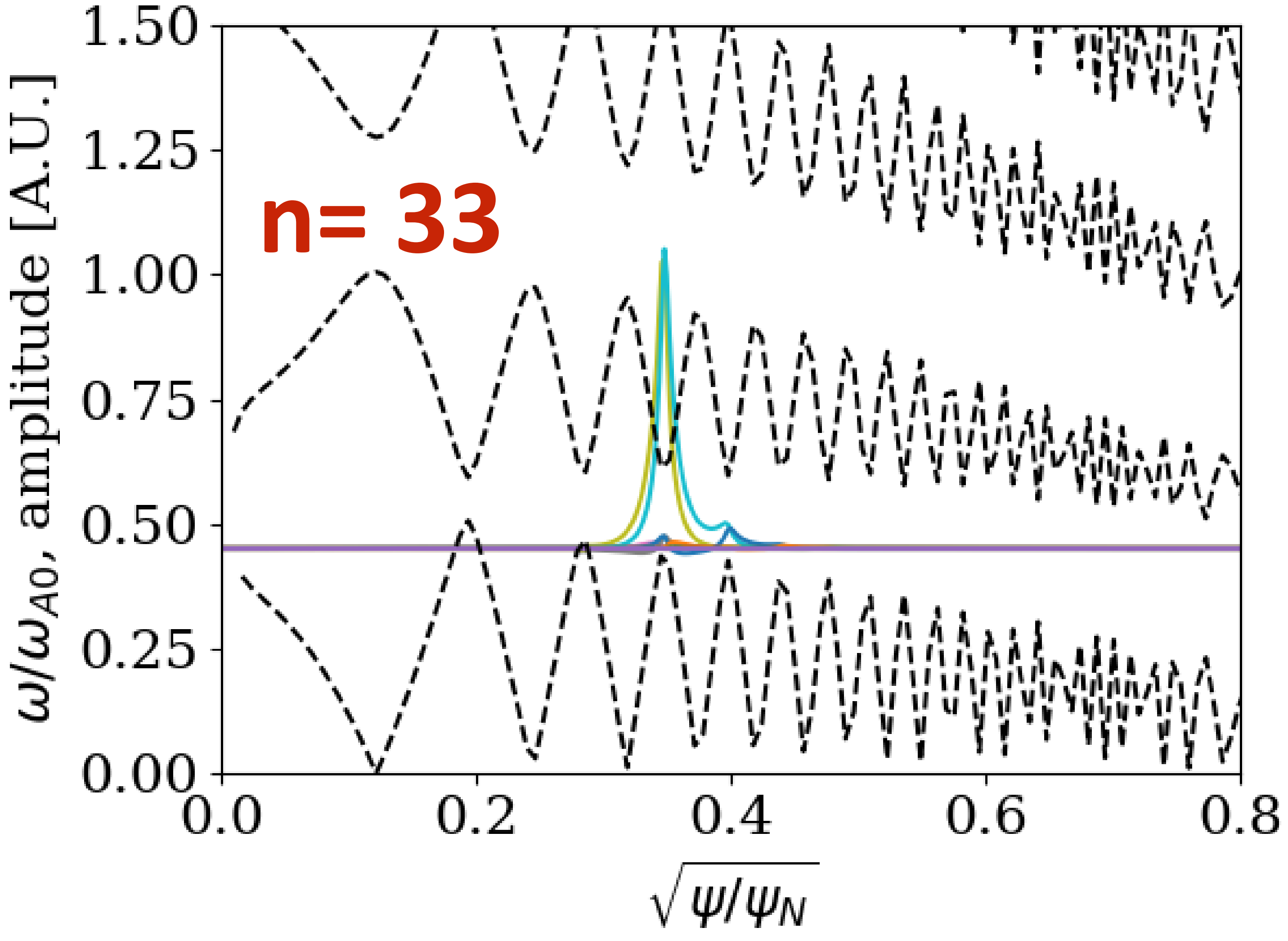


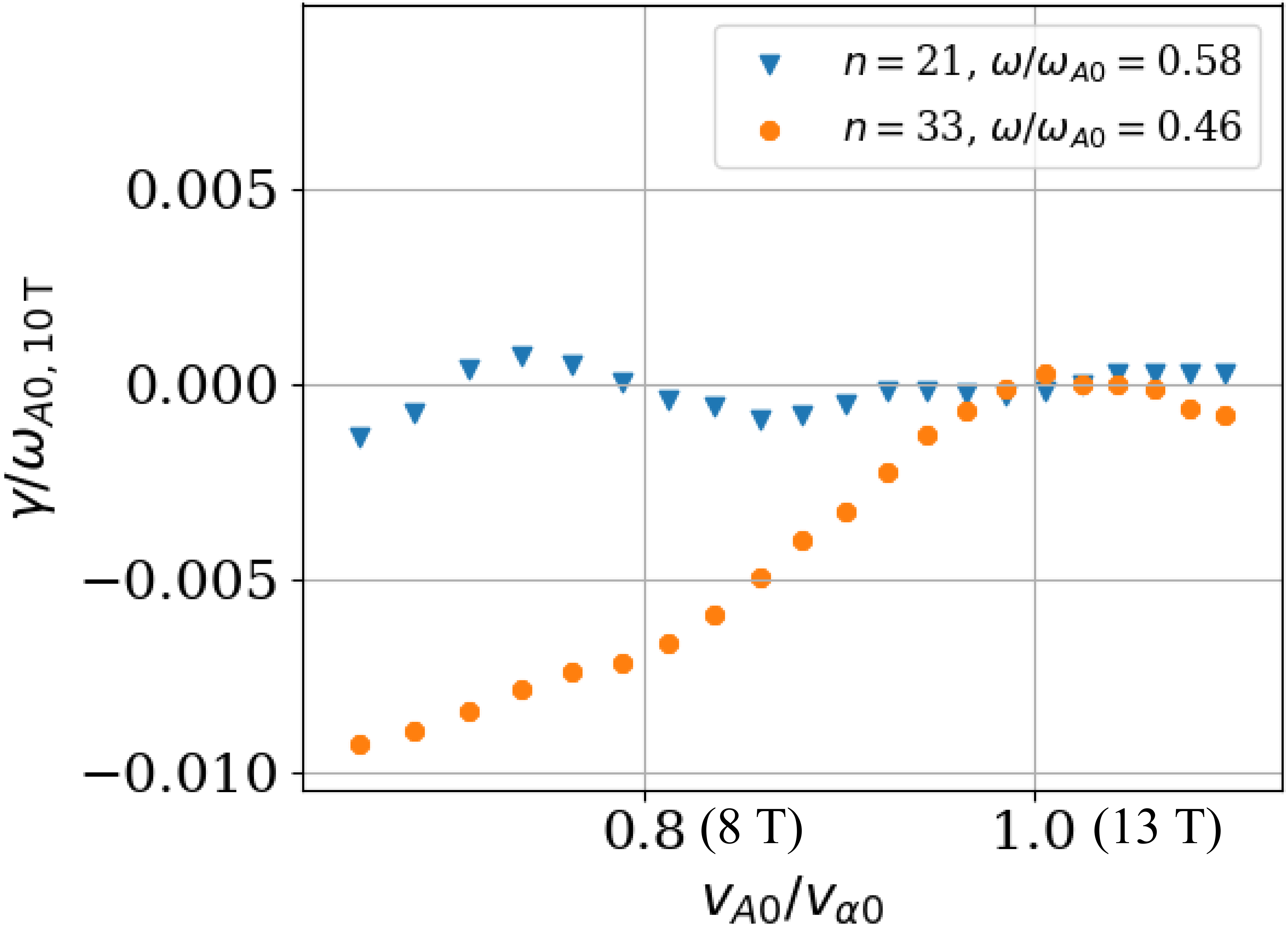
Increasing P_f increases drive

Cutoff of resonance stops growth even with increasing P_f

Cutoff of TAE resonances visible as AE switch from instability to stability







Conclusions

- SPARC will have different alpha particle physics than ITER
- High-performance high-B operation has advantages for stability physics
- Lower Greenwald fraction operation allows novel probe of AE stability physics, performance benefits
- More work to be done in many areas: heating-driven AEs, types of AE beyond TAE/EAE, effect of spectrum shift to higher *n* on nonlinear behavior and transport
- Collaboration by broader community is encouraged!

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